RADIATION OF ACCELERATED CHARGE IN ABSORBING MEDIUM

V.M. Grichine ^{*)}

CERN/IT-API-SI, Geneva 23 CH-1211, Switzerland

Abstract

The radiation produced by an arbitrary moving relativistic charge in an absorbing medium is considered in terms of the radiation energy loss. The energy-angle distributions of emitted photons for Cerenkov radiation, prompt and magnetic bremsstrahlung, uniform motion over finite distance (the Tamm problem), and for simple cases of periodic motion are derived. The Doppler effect in absorbing medium for undulator-like motion is considered. The high-energy, low-angle approximation of the spectra is discussed in terms of the complex formation zone of the radiation produced by an ultra-relativistic charge in an absorbing medium.

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^{*)} e-mail: Vladimir.Grichine@cern.ch, on leave from P.N. Lebedev Physical Institute, Lenin Pr. 53, 119991 Moscow, Russia

1 Introduction. The Poynting's theorem in absorbing medium

Relativistic particle energy loss in a medium can be subdivided into the radiation (RL, $\perp$) and polarisation (PL, $\parallel$) losses, i.e., the work done by the particle against the transverse and longitudinal (relative to the wave vector) components of its electric field in the current particle position, respectively [1, 2]. In terms of a quantum approach, RL and PL are responsible for the generation of transverse (photons in medium) and longitudinal (plasmons, delta-electrons) medium excitations along the particle trajectory, respectively.

RL, reflecting the contribution from the particle electric current, is responsible for the relativistic dependence of the particle energy loss. It results in observable effects that are used for charged particle identification, i.e., soft (non-destructive for the particle momentum) estimation of the particle Lorentz factor. In the visible and ultraviolet ranges of transferred energies RL generalises Cerenkov radiation for the case when medium absorption is taken into account. In the range of atomic frequencies, where the generated photons are absorbed in the vicinity of the particle trajectory resulting in measured ionisation, RL describes the relativistic rise of dE/dx . In the X-ray range, where RL in a uniform medium is suppressed by destructive interference, introduction of proper distributed medium interfaces recovers the generation of X-ray photons. The detection of these photons (X-ray transition radiation) allows one to extend the relativistic dependence of the particle energy loss.

Consideration of the relativistic dependence of the particle energy loss is convenient from the simulation point of view, allowing one to track (involving all background processes) secondary photons from the points of their generation. The GEANT4 Object-Oriented Toolkit for simulation in High Energy Physics [3] provides extended tools for simulation of particle identification processes in complex detector geometries. The aim of the present paper is to discuss the concept of CL and its possible implementations in the framework of GEANT4 for different modes of relativistic charge motion in an absorbing medium.

Let us consider a relativistic charged particle with charge e moving in an absorbing medium with complex dielectric permittivity $\epsilon = \epsilon_1 + i\epsilon_2$. Poynting's theorem can be derived directly from Maxwell's equations [4, 5]:

$$-\mathbf{j}_{ext}\mathbf{E} = \frac{1}{4\pi} \left\{ \mathbf{E} \frac{\partial \mathbf{D}}{\partial t} + \mathbf{H} \frac{\partial \mathbf{B}}{\partial t} \right\} + \text{div} \mathbf{S}, \quad (1)$$

where $\mathbf{E}$ and $\mathbf{D}$ are the electric field and displacement, $\mathbf{H}$ and $\mathbf{B}$ are the magnetic field and induction, respectively, $\mathbf{j}_{ext} = e\mathbf{v}\delta(\mathbf{r} - \mathbf{vt})$ is the current density of the external (for simplicity, point-like) charge ($\mathbf{r}$ is the particle coordinate at the time moment t , and δ is the Dirac delta function), and (in the Gauss system of units):

$$\mathbf{S} = \frac{c}{4\pi} \mathbf{E} \times \mathbf{H},$$

is the Poynting vector expressing the electro-magnetic energy flux density (c is the speed of light in vacuum). Integration of (1) over a large volume V surrounding the particle trajectory results in:

$$-e\mathbf{v}\mathbf{E}(\mathbf{vt}, t) = \frac{1}{4\pi} \int_V \left\{ \mathbf{E} \frac{\partial \mathbf{D}}{\partial t} + \mathbf{H} \frac{\partial \mathbf{B}}{\partial t} \right\} d\mathbf{r} + \oint_S \mathbf{S} ds, \quad (2)$$

where ds is the element of the surface S surrounding V . Equation (2) expresses the energy loss balance; the work done by the particle against its electric field will be distributed between the absorption of the electro-magnetic energy in the surrounding volume V and the energy flux

over the surface S . In an absorbing medium for sufficiently big volume V the second term in the right side of (2) can be neglected, and

$$\frac{d\bar{\Delta}}{dt} = -e\mathbf{v}\mathbf{E}(\mathbf{v}t, t) = \frac{1}{4\pi} \int_V \left\{ \mathbf{E} \frac{\partial \mathbf{D}}{\partial t} + \mathbf{H} \frac{\partial \mathbf{B}}{\partial t} \right\} d\mathbf{r} = \frac{d\bar{U}}{dt}, \quad (3)$$

where $\bar{\Delta}$ and $\bar{U}$ are the mean energy loss of the particle and absorbed energy in surrounding volume.

In the next sections we will derive the energy-angle distributions of the mean number of secondary quasi-particles (like photons in medium, plasmons or δ -electrons) produced by a relativistic charge moving with arbitrary acceleration in a non-transparent medium, $\epsilon_2 > 0$. Instead of using of the Frank-Tamm-Fermi method [6, 7] based on the calculation of the energy loss as the flux of the Poynting vector over a cylindrical surface surrounding the particle trajectory, we will use the Landau method [4]. The latter is based on the calculation of the energy loss of a relativistic charged particle as the work done by the electric field produced by the particle at its current position. This approach allows us to get the energy-angle distribution of primary quasi-particles generated by the incident particle moving in an absorbing medium. The distribution can be used for simulation of the radiation produced by relativistic charged particles in practical high energy physics (HEP) detectors.

2 Radiation produced by non-periodically moving charge

2.1 General consideration

Following the results of [2, 4, 8] we calculate the mean radiation energy loss based on the solution of Maxwell's equations for an electro-magnetic field created by a relativistic charged particle moving in a non-magnetic dielectric medium. It is convenient to introduce the vector $\mathbf{A}$ and scalar ϕ potentials:

$$\mathbf{H} = \nabla \times \mathbf{A}, \quad \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{A}}{\partial t} - \nabla \cdot \phi, \quad (4)$$

in the Coulomb gauge:

$$\nabla \cdot \mathbf{A} = 0. \quad (5)$$

The Maxwell's equations for potentials become:

$$\nabla \cdot (\epsilon \nabla \phi) = -4\pi \varrho(\mathbf{r}, t), \quad (6)$$

$$-\nabla^2 \mathbf{A} = -\frac{1}{c^2} \frac{\partial}{\partial t} \left(\epsilon \frac{\partial \mathbf{A}}{\partial t} \right) - \frac{1}{c} \frac{\partial}{\partial t} (\epsilon \nabla \phi) + \frac{4\pi}{c} \mathbf{j}(\mathbf{r}, t), \quad (7)$$

where $\varrho(\mathbf{r}, t)$ and $\mathbf{j}(\mathbf{r}, t)$ are charge and electric current densities, respectively. These equations can be solved introducing the Fourier transforms for potentials:

$$F(\mathbf{r}, t) = \iint \frac{d\mathbf{k} d\omega}{(2\pi)^4} F(\mathbf{k}, \omega) \exp[i(\mathbf{k}\mathbf{r} - \omega t)], \quad (8)$$

and

$$F(\mathbf{k}, \omega) = \iint d\mathbf{r} dt F(\mathbf{r}, t) \exp[-i(\mathbf{k}\mathbf{r} - \omega t)]. \quad (9)$$

where $\hbar\mathbf{k}$ is the momentum transferred from the incident particle to the medium and $\hbar\omega$ is the energy transfer ($\hbar$ is Planck's constant, $\mathbf{k}$ is the wave vector and ω is the frequency of emitted photons). The Fourier components of the electric fields according to (4) will be:

$$\mathbf{E}_\perp(\mathbf{k}, \omega) = i \frac{4\pi}{c^2} \omega \frac{\mathbf{j}(\mathbf{k}, \omega) - \frac{\mathbf{k}\omega}{k^2} \varrho(\mathbf{k}, \omega)}{k^2 - \epsilon(\mathbf{k}, \omega) \frac{\omega^2}{c^2}}, \quad \mathbf{E}_\perp(\mathbf{k}, \omega) \cdot \mathbf{k} = 0, \quad (10)$$

and

$$\mathbf{E}_\parallel(\mathbf{k}, \omega) = i \frac{4\pi}{k^2} \mathbf{k} \frac{\varrho(\mathbf{k}, \omega)}{-\epsilon(\mathbf{k}, \omega)}, \quad \mathbf{E}(\mathbf{k}, \omega) = \mathbf{E}_\perp(\mathbf{k}, \omega) + \mathbf{E}_\parallel(\mathbf{k}, \omega). \quad (11)$$

We will start with the derivation of the full mean radiation energy loss as a function of the transferred energy and emission angle. Consider the mean radiation energy loss $\Delta_\perp$ produced by a relativistic charged particle arbitrary moving in a non-transparent medium. According to the Landau method [4] it can be expressed as:

$$\begin{aligned} \Delta_\perp &= - \int_{-\infty}^{\infty} dt \int_{R_3} d\mathbf{r} \mathbf{j}(\mathbf{r}, t) \cdot \mathbf{E}_\perp(\mathbf{r}, t) = U_\perp = \\ &= \frac{1}{4\pi} \int_{-\infty}^{\infty} dt \int_{R_3} d\mathbf{r} \mathbf{E}_\perp(\mathbf{r}, t) \cdot \frac{\partial \mathbf{D}_\perp}{\partial t} = \\ &= -\frac{2}{(2\pi)^4} \int_0^\infty d\omega \int_{K_3} d\mathbf{k} \operatorname{Re}\{\mathbf{j}^*(\mathbf{k}, \omega) \cdot \mathbf{E}_\perp(\mathbf{k}, \omega)\}, \end{aligned} \quad (12)$$

where $U_\perp$ and $\mathbf{D}_\perp$ are the full absorbed radiation energy and electrical induction, respectively. Substituting to the latter equation the relation (10), we have:

$$\Delta_\perp = \frac{1}{2\pi^3} \int_0^\infty \omega d\omega \int_{K_3} \frac{d\mathbf{k}}{k^2} \operatorname{Im} \left\{ \frac{|\mathbf{k} \times \mathbf{j}(\mathbf{k}, \omega)|^2}{c^2 \left[k^2 - \epsilon(\mathbf{k}, \omega) \frac{\omega^2}{c^2} \right]} \right\}. \quad (13)$$

The corresponding relation for the longitudinal (Bohr) energy loss will be:

$$\Delta_\parallel = \frac{1}{2\pi^3} \int_0^\infty \omega d\omega \int_{K_3} \frac{d\mathbf{k}}{k^2} \operatorname{Im} \left\{ \frac{|\varrho(\mathbf{k}, \omega)|^2}{-\epsilon(\mathbf{k}, \omega)} \right\}. \quad (14)$$

The latter expression describes the energy loss producing longitudinal excitations (plasmons, δ -electrons) of the medium.

For the point-like charge e moving along the trajectory $\mathbf{r}(t)$ the Fourier component of the electric current density $\mathbf{j}(\mathbf{k}, \omega)$ reads:

$$\mathbf{j}(\mathbf{k}, \omega) = e \int_{\mathbf{v} \neq 0} dt \mathbf{v}(t) \exp\{i[\omega t - \mathbf{k}\mathbf{r}(t)]\}, \quad (15)$$

where $\mathbf{v}(t)$ is the time variable velocity of the charge.

In the next subsections we consider some practical examples illustrating the using of (13). The energy spectrum of emitted photons will be defined as:

$$\frac{d\bar{\Delta}_\perp}{\hbar d\omega} = \hbar\omega \frac{d\bar{N}}{\hbar d\omega},$$

where $\bar{N}$ is the mean number of emitted photons.

2.2 Cerenkov radiation

In the case of Cerenkov radiation the charge e experiences the uniform motion with the constant velocity $\mathbf{v}$ along infinite trajectory. It allows to calculate the Fourier component of the electric current density:

$$\mathbf{j}(\mathbf{k}, \omega) = e\mathbf{v} \int_{-\infty}^{\infty} dt \exp\{i[\omega t - \mathbf{k}\mathbf{v}]t\} = 2\pi e\mathbf{v}\delta(\mathbf{k} \cdot \mathbf{v} - \omega), \quad (16)$$

where δ is the Dirac delta function. Then:

$$|\mathbf{k} \times \mathbf{j}(\mathbf{k}, \omega)|^2 = 2\pi e^2 k^2 v^2 \sin^2 \theta T \delta(\mathbf{k} \cdot \mathbf{v} - \omega),$$

where θ is the angle between $\mathbf{v}$ and $\mathbf{k}$ and T is proportional to the full time of the motion. It is convenient to consider the mean energy loss per unit time:

$$\begin{aligned} \frac{\Delta_{\perp}}{T} \sim \frac{d\Delta_{\perp}}{dt} &= \frac{2e^2}{\pi} \beta^2 \int_0^{\infty} \omega d\omega \int_{-1}^1 \sin^2 \theta d \cos \theta \times \\ &\times \int_0^{\infty} dk \operatorname{Im} \left\{ \frac{1}{1 - \epsilon(\mathbf{k}, \omega) \frac{\omega^2}{c^2 k^2}} \right\} \delta(\mathbf{k} \cdot \mathbf{v} - \omega), \end{aligned} \quad (17)$$

where $\beta = v/c$. The delta function allows to perform the integration in respect of k . Introducing the element of the particle trajectory $dx = v dt$, we have:

$$\frac{d\Delta_{\perp}}{dx} = \frac{2e^2}{\pi c^2} \int_0^{\infty} \omega d\omega \int_0^1 \frac{\sin^2 \theta}{\cos \theta} d \cos \theta \operatorname{Im} \left\{ \frac{1}{1 - \epsilon \beta^2 \cos^2 \theta} \right\}. \quad (18)$$

Expanding the imaginary part we get for the energy-angle spectrum of emitted photons [9]:

$$\frac{d^3 \bar{N}_{\perp}}{\hbar d\omega dx d \cos^2 \theta} = \frac{\alpha}{\pi \hbar c} \frac{(1 - \cos^2 \theta) \Gamma}{(\cos^2 \theta - \cos^2 \theta_o)^2 + \Gamma^2}, \quad (19)$$

where α is the fine structure constant,

$$\Gamma = \frac{\epsilon_2}{\beta^2 |\epsilon|^2}, \quad (20)$$

and

$$\cos^2 \theta_o = \frac{\epsilon_1}{\beta^2 |\epsilon|^2}. \quad (21)$$

The distribution (19) has a sharp peak at $\cos^2 \theta = \cos^2 \theta_o$. Therefore the most probable emission angle θ_o of Cerenkov photons in a non-transparent medium is defined by:

$$\cos \theta_o = \frac{\sqrt{\epsilon_1}}{\beta |\epsilon|} \simeq \frac{1 - \frac{\nu^2}{2}}{\beta \sqrt{\epsilon_1}}, \quad \nu = \frac{\Gamma}{\cos^2 \theta_o} = \frac{\epsilon_2}{\epsilon_1}. \quad (22)$$

The full width at half maximum, $FWHM$, of the peak is equal to:

$$FWHM = 2\Gamma = \frac{2\epsilon_2}{\beta^2 |\epsilon|^2}. \quad (23)$$

The integration of (19) with respect to $\cos^2 \theta$ results in the energy spectrum of the radiation (we consider the dielectric permittivity as a function of the photon frequency only and in the visible-ultraviolet range of interest to be independent of $\cos \theta$ (no spatial dispersion), $\epsilon = \epsilon(\omega)$):

$$\frac{d^2 \bar{N}_\perp}{\hbar d\omega dx} = \frac{\alpha}{\pi \hbar c} \left\{ \left[1 - \frac{\epsilon_1}{\beta^2 |\epsilon|^2} \right] \arg(1 - \beta^2 \epsilon^*) + \frac{\epsilon_2}{\beta^2 |\epsilon|^2} \ln \left(\frac{1}{|1 - \beta^2 \epsilon|} \right) \right\}. \quad (24)$$

In the limit of a transparent medium, when $\Gamma \rightarrow 0$, we note that

$$\frac{\Gamma}{\pi[(\cos^2 \theta - \cos^2 \theta_o)^2 + \Gamma^2]} \rightarrow \delta \left(\cos^2 \theta - \frac{1}{\beta^2 n^2} \right),$$

and get the energy-angle distribution of the mean number of Cerenkov photons [6]:

$$\frac{d^3 \bar{N}_c}{\hbar d\omega dx d \cos^2 \theta} = \alpha \frac{1}{\hbar c} \left(1 - \frac{1}{\beta^2 n^2} \right) \delta \left(\cos^2 \theta - \frac{1}{\beta^2 n^2} \right). \quad (25)$$

This expression fixes explicitly the emission angle to be $\theta_c = \arccos(1/\beta n)$. If $\beta n > 1$, integration of (25) with respect to x results in the well known formula of the Frank-Tamm theory:

$$\frac{d^2 \bar{N}_c}{\hbar d\omega dx} = \frac{\alpha}{\hbar c} \left(1 - \frac{1}{\beta^2 n^2} \right), \quad \beta n > 1. \quad (26)$$

In the optical-ultraviolet range the value of ν can be estimated from:

$$\nu \sim \frac{c}{n\omega l} \sim \frac{\lambda}{l}, \quad (27)$$

where λ is the photon wave length, and l is the photon absorption length. Since for semi-transparent media $\hbar\omega l \gtrsim 2 \text{ eV cm}$ and $\hbar c \sim 2 \cdot 10^{-5} \text{ eV cm}$,

$$\nu \lesssim 10^{-5},$$

and the broadening is in practice very difficult to observe in semi-transparent solids and liquids. Fig. 1 illustrates this showing the angle distributions of Cerenkov photons produced in C_6F_{14} [9] by relativistic particle $v \sim c$ for two values of the photon energy: $\hbar\omega = 6.8 \text{ eV}$ (open circles) and $\hbar\omega = 7.7 \text{ eV}$ (closed circles and upper x-axis). The curves are normalised on the equal most probable values.

For gases the situation can be more promising. Here the Cerenkov angle is always very small, $\theta \ll 1$ or $\cos^2 \theta_o \sim 1$. Then the relative broadening due to the gas absorption $\delta\theta/\theta$ can be estimated from:

$$\frac{\delta\theta}{\theta} \sim \frac{\nu}{\theta^2} \sim \frac{c}{2\eta\omega l} \sim \frac{\lambda}{\eta l}, \quad (28)$$

where $\eta = n - 1$ is the refractivity. In most gases of practical use $\eta \sim 10^{-4}$ and therefore the aberration of Cerenkov radiation due to absorption can be observed in gas Ring Imaging Cerenkov (RICH) detectors [10]. Radio frequency Cerenkov photons (meter range) can be also considered as promising for the observation of the broadening due to absorption since ν is proportional to the ratio of the wave length to the absorption length of the photons.

2.3 Prompt bremsstrahlung

It is interesting to apply the general relation (13) for the description of the semi-infinite motion of the charged particle e with the constant velocity $\mathbf{v}$. This case corresponds, for example, to the beta-decay of a nucleus, when electron starts to move in surrounding medium. The Fourier component of the electric current density reads:

$$\begin{aligned}\mathbf{j}(\mathbf{k}, \omega) &= e\mathbf{v} \int_0^\infty dt \exp\{i[\omega t - \mathbf{k}\mathbf{v}]t\} = \\ &= e\mathbf{v} \left[\pi\delta(\mathbf{k} \cdot \mathbf{v} - \omega) - \text{V.p.} \frac{i}{\mathbf{k} \cdot \mathbf{v} - \omega} \right],\end{aligned}\quad (29)$$

where V.p. means the principal value. Then:

$$|\mathbf{k} \times \mathbf{j}(\mathbf{k}, \omega)|^2 = e^2 k^2 v^2 \sin^2 \theta \left[\pi T \delta(\mathbf{k} \cdot \mathbf{v} - \omega) + \frac{1}{(\mathbf{k} \cdot \mathbf{v} - \omega)^2} \right],$$

where T is proportional to the full time of the motion. The first term in the latter relation corresponds to Cerenkov radiation produced from the semi-infinite trajectory, while the second one is responsible for the radiation from initial prompt acceleration (bremsstrahlung). Let us consider the full mean radiation energy loss due to prompt bremsstrahlung. The corresponding part of relation (13) reads:

$$\begin{aligned}\Delta_\perp &= \frac{e^2}{\pi^2 c^2} \int_0^\infty \omega d\omega \int_0^1 \frac{\sin^2 \theta}{\cos^2 \theta} d\cos \theta \times \\ &\times \int_0^\infty k^2 dk \operatorname{Im} \left\{ \frac{1}{(k^2 - a^2)(k - b)^2} + \frac{1}{(k^2 - a^2)(k + b)^2} \right\},\end{aligned}\quad (30)$$

where the following notation was introduced:

$$a^2 = \epsilon \frac{\omega^2}{c^2}, \quad b = \frac{\omega}{v \cos \theta}.$$

The relation inside the imaginary part is the even function of k . Therefore it is convenient to perform the integration with respect to k in the complex plane z over the contour shown in Fig. 2:

$$\Delta_\perp = \frac{e^2}{\pi^2 c^2} \int_0^\infty \omega d\omega \int_0^1 \frac{\sin^2 \theta}{\cos^2 \theta} d\cos \theta \operatorname{Im} \left\{ \oint_C f(z) dz \right\}, \quad (31)$$

where $f(z)$ is defined by:

$$f(z) = \frac{z^2}{2(z^2 - a^2)} \left[\frac{1}{(z - b)^2} + \frac{1}{(z + b)^2} \right],$$

and:

$$\lim_{R \rightarrow \infty} \int_R f(z) dz = 0.$$

The integral over the contour C is defined by the sum of residues in the singularities of $f(z)$, i.e. the points a and $\pm b$. Since:

$$\operatorname{res}(-b) + \operatorname{res}(b) = 0,$$

the contour integral reads:

$$\begin{aligned} \text{Im} \left\{ \oint_C f(z) dz \right\} &= 2\pi \text{Re} \{ \text{res}(a) \} = \\ &= \frac{\pi v^2 \cos^2 \theta}{2 \omega c} \text{Re} \left\{ \frac{\sqrt{\epsilon}}{(1 - \beta \sqrt{\epsilon} \cos \theta)^2} + \frac{\sqrt{\epsilon}}{(1 + \beta \sqrt{\epsilon} \cos \theta)^2} \right\}. \end{aligned} \quad (32)$$

Substituting the latter relation to equation (31) results in the energy-angle distribution of the prompt bremsstrahlung energy loss:

$$\frac{d^2 \Delta_{\perp}}{\hbar d\omega d \cos \theta} = \frac{\alpha}{2\pi} \beta^2 \sin^2 \theta \text{Re} \left\{ \frac{\sqrt{\epsilon}}{(1 - \beta \sqrt{\epsilon} \cos \theta)^2} \right\}. \quad (33)$$

The latter relation reduces in the case of vacuum to the well known expression (see, for example [11]):

$$\frac{d\Delta_{\perp}}{\hbar d\omega} = \frac{\alpha}{2\pi} \beta^2 \int_0^{\pi} \frac{\sin^3 \theta d\theta}{(1 - \beta \cos \theta)^2}. \quad (34)$$

For the description of the prompt bremsstrahlung, when the charge e instantly change the mode of uniform motion from the velocity $\mathbf{v}_1$ to $\mathbf{v}_2$, one have to investigate the more general version of (30):

$$\frac{d\Delta_{\perp}}{\hbar d\omega} = \frac{\alpha}{2\pi^3} \frac{\omega}{c} \int_{K_3} d\mathbf{k} \text{Im} \left\{ \frac{\left[\frac{\mathbf{n} \times \mathbf{v}_1}{\mathbf{k} \cdot \mathbf{v}_1 - \omega} - \frac{\mathbf{n} \times \mathbf{v}_2}{\mathbf{k} \cdot \mathbf{v}_2 - \omega} \right]^2}{k^2 - \epsilon(\mathbf{k}, \omega) \frac{\omega^2}{c^2}} \right\}, \quad (35)$$

where $\mathbf{n}$ is the unit vector in the direction of $\mathbf{k}$.

2.4 Uniform motion over finite distance. The Tamm problem

Let us consider now the uniform motion with the velocity $\mathbf{v}$ along a finite distance Fig.3 [12].

$$\mathbf{j}(\mathbf{k}, \omega) = 2e\mathbf{v} \frac{\sin \left[(\mathbf{k} \cdot \mathbf{v} - \omega) \frac{\tau}{2} \right]}{\mathbf{k} \cdot \mathbf{v} - \omega}, \quad (36)$$

where τ is the full time duration of the motion. The full mean radiation energy loss for the uniform motion along finite distance then reads:

$$\begin{aligned} \Delta_{\perp} &= \frac{4e^2}{\pi^2 c^2} \int_0^{\infty} \omega d\omega \int_0^1 \frac{\sin^2 \theta}{\cos^2 \theta} d \cos \theta \times \\ &\times \int_0^{\infty} k^2 dk \text{Im} \left\{ \frac{\sin^2[d(k-b)]}{(k^2 - a^2)(k-b)^2} + \frac{\sin^2[d(k+b)]}{(k^2 - a^2)(k+b)^2} \right\}, \end{aligned} \quad (37)$$

where we introduced the following notation:

$$a^2 = \epsilon \frac{\omega^2}{c^2}, \quad b = \frac{\omega}{v \cos \theta}, \quad d = \frac{v\tau \cos \theta}{2}.$$

The relation inside the imaginary part is the even function of k . Therefore it is convenient to perform the integration with respect to k in the complex plane z over the contour shown in Fig. 4:

$$\Delta_{\perp} = \frac{4e^2}{\pi^2 c^2} \int_0^{\infty} \omega d\omega \int_0^1 \frac{\sin^2 \theta}{\cos^2 \theta} d \cos \theta \operatorname{Im} \left\{ \oint_C g(z) dz \right\}, \quad (38)$$

where $g(z)$ is defined by:

$$g(z) = \frac{z^2}{2(z^2 - a^2)} \left\{ \frac{\sin^2[d(z - b)]}{(z - b)^2} + \frac{\sin^2[d(z + b)]}{(z + b)^2} \right\},$$

and:

$$\lim_{R \rightarrow \infty} \int_R g(z) dz = 0.$$

The integral over the contour C is defined by the residue in the singularity of $g(z)$, i.e. in the point a . The contour integral reads:

$$\begin{aligned} \operatorname{Im} \left\{ \oint_C g(z) dz \right\} &= 2\pi \operatorname{Re} \{ \operatorname{res}(a) \} = \frac{\pi v^2 \cos^2 \theta}{2 \omega c} \times \\ &\times \operatorname{Re} \left\{ \frac{\sqrt{\epsilon} \sin^2 \left[\frac{\omega \tau}{2} (1 - \beta \sqrt{\epsilon} \cos \theta) \right]}{(1 - \beta \sqrt{\epsilon} \cos \theta)^2} + \frac{\sqrt{\epsilon} \sin^2 \left[\frac{\omega \tau}{2} (1 + \beta \sqrt{\epsilon} \cos \theta) \right]}{(1 + \beta \sqrt{\epsilon} \cos \theta)^2} \right\}. \end{aligned} \quad (39)$$

Substituting the latter relation to equation (38) results in the energy-angle distribution of the radiation energy loss for the Tamm problem in absorbing medium:

$$\frac{d^2 \Delta_{\perp}}{\hbar d\omega d \cos \theta} = \frac{2\alpha}{\pi} \beta^2 \sin^2 \theta \operatorname{Re} \left\{ \frac{\sqrt{\epsilon} \sin^2 \left[\frac{\omega \tau}{2} (1 - \beta \sqrt{\epsilon} \cos \theta) \right]}{(1 - \beta \sqrt{\epsilon} \cos \theta)^2} \right\}. \quad (40)$$

In transparent medium the latter expression is reduced to the results of [12]. In the limit $\omega \tau \gg 1$ the latter relation reduces in the case of transparent medium to the well known expression describing Cerenkov radiation (see, for example):

$$\frac{d^3 \Delta_{\perp}}{\hbar d\omega dx d \cos \theta} = \alpha \frac{\omega}{c} \left(1 - \frac{1}{\beta^2 n^2} \right) \delta \left(\cos \theta - \frac{1}{\beta n} \right), \quad (41)$$

where $dx = v d\tau$ is the trajectory element and $n = \sqrt{\epsilon_1}$ is the refractive index of the medium.

2.5 Undulator-like radiation. Doppler effect in absorbing medium

Let the charge v move according the following law:

$$\mathbf{r}_o(t) = \mathbf{v}t + \mathbf{a} \sin(\omega_o t), \quad \mathbf{v}_o(t) = \mathbf{v} + \omega_o \mathbf{a} \cos(\omega_o t),$$

where $\mathbf{v}$ is the velocity of uniform motion, and $\mathbf{a}$ and ω_o are the amplitude and the frequency of the periodic part of the motion, respectively. We consider $\mathbf{a}$ to be small (dipole approximation,

$a = |\mathbf{a}| \ll v/\omega_0$, so that in the expression for the Fourier component of the electric current density we can expand the exponential to series:

$$\begin{aligned} \mathbf{j}(\mathbf{k}, \omega) &= e \int_{-\infty}^{\infty} dt \mathbf{v}_o(t) \exp\{i[\omega t - \mathbf{k} \cdot \mathbf{r}_o(t)]\} \simeq \\ &\simeq e \int_{-\infty}^{\infty} dt \mathbf{v}_o(t) [1 - i\mathbf{k} \cdot \mathbf{a} \sin(\omega_o t)] \exp\{i[\omega t - \mathbf{k} \cdot \mathbf{v}] t\} = \\ &= 2\pi e \mathbf{v} \left\{ \delta(\mathbf{k} \cdot \mathbf{v} - \omega) - \right. \\ &\quad \left. - \frac{\mathbf{k} \cdot \mathbf{a}}{2} [\delta(\mathbf{k} \cdot \mathbf{v} - \omega - \omega_o) - \delta(\mathbf{k} \cdot \mathbf{v} - \omega + \omega_o)] \right\}. \end{aligned} \quad (42)$$

Then we have:

$$\begin{aligned} |\mathbf{k} \times \mathbf{j}(\mathbf{k}, \omega)|^2 &= 2\pi e^2 v^2 \sin^2 \theta T \left\{ \delta(\mathbf{k} \cdot \mathbf{v} - \omega) - \right. \\ &\quad \left. - \frac{\mathbf{k} \cdot \mathbf{a}}{2} [\delta(\mathbf{k} \cdot \mathbf{v} - \omega - \omega_o) - \delta(\mathbf{k} \cdot \mathbf{v} - \omega + \omega_o)] \right\}. \end{aligned} \quad (43)$$

Let $\mathbf{a} \parallel \mathbf{v}$, this case corresponds, for example, to the movement through the stack of charged capacities. Repeating the consideration of section (2.2), we get the energy-angle spectrum for the mean number of photons emitted per unit trajectory length $dx = v dt$:

$$\begin{aligned} \frac{d^3 \bar{N}_\perp}{\hbar d\omega dx d \cos^2 \theta} &= \frac{\alpha}{\pi \hbar c} (1 - \cos^2 \theta) \left\{ \frac{\Gamma}{(\cos^2 \theta - \cos^2 \theta_o)^2 + \Gamma^2} + \right. \\ &\quad + \frac{\left(\frac{a\omega}{2vd_+} \right)^2 \Gamma_+}{(\cos^2 \theta - \cos^2 \theta_+)^2 + \Gamma_+^2} + \\ &\quad \left. + \frac{\left(\frac{a\omega}{2vd_-} \right)^2 \Gamma_-}{(\cos^2 \theta - \cos^2 \theta_-)^2 + \Gamma_-^2} \right\}, \end{aligned} \quad (44)$$

where the following notation was introduced:

$$d_\pm^2 = \left(\frac{\omega}{\omega \pm \omega_o} \right)^2, \quad \Gamma = \frac{\epsilon_2}{\beta^2 |\epsilon|^2}, \quad \Gamma_\pm = \frac{\epsilon_2}{\beta^2 |\epsilon|^2 d_\pm^2}, \quad (45)$$

and

$$\cos^2 \theta_o = \frac{\epsilon_1}{\beta^2 |\epsilon|^2}, \quad \cos^2 \theta_\pm = \frac{\epsilon_1}{\beta^2 |\epsilon|^2 d_\pm^2}. \quad (46)$$

This expression defines the Doppler effect (second and third terms) for the undulator radiation in the dipole approximation in absorbing medium. It is seen that because of the absence of the singularity at the Cerenkov cone, the Doppler effect does not subdivided to the regions of the normal and anomalous effects. In the limit of transparent medium $\epsilon_2 \rightarrow 0$ equation (44) will be reduced to:

$$\begin{aligned} \frac{d^3 \bar{N}_\perp}{\hbar d\omega dx d \cos^2 \theta} &= \frac{\alpha}{\hbar c} (1 - \cos^2 \theta) \left\{ \delta(\cos^2 \theta - \cos^2 \theta_c) + \right. \\ &\quad + \left(\frac{a\omega}{2vd_+} \right)^2 \delta(\cos^2 \theta - \cos^2 \theta_+) + \\ &\quad \left. + \left(\frac{a\omega}{2vd_-} \right)^2 \delta(\cos^2 \theta - \cos^2 \theta_-) \right\}, \end{aligned} \quad (47)$$

where the first term is responsible for Cerenkov radiation, while the second and third terms describe the Doppler effect in transparent medium:

$$\cos \theta_c = \frac{1}{\beta n}, \quad \cos \theta_{\pm} = \frac{\omega \pm \omega_o}{\beta n \omega} = \left(1 \pm \frac{\omega_o}{\omega}\right) \cos \theta_c.$$

The singularity of the angle distribution at $\cos \theta_c = 1/\beta n$ subdivides the regions of anomalous:

$$\omega = \frac{\omega_o}{\beta n \cos \theta - 1}, \quad \text{for, } \cos \theta > \cos \theta_c,$$

and normal:

$$\omega = \frac{\omega_o}{1 - \beta n \cos \theta}, \quad \text{for, } \cos \theta < \cos \theta_c,$$

Doppler effects (see, Fig.5, reviews [5, 13, 14] and references therein). It should be emphasised that this simple picture of the Doppler effect is valid under the dipole radiation condition (42) only. In the case of bigger a it will appear additional angular maxima corresponding to higher harmonics at:

$$\cos \theta_{\pm\nu} = \frac{\omega \pm \nu \omega_o}{\beta n \omega},$$

where $\nu > 1$ is the arbitrary integer number.

3 Radiation produced by periodically moving charge

3.1 General consideration

For periodic motions with the frequency ω_o it is convenient to use the series, rather than integral, Fourier:

$$f(t) = \sum_{\nu=-\infty}^{\infty} f_{\nu} \exp(-i\nu\omega_o t), \quad f_{\nu} = \frac{1}{T} \int_{-T/2}^{T/2} dt f(t) \exp(i\nu\omega_o t), \quad (48)$$

where ν is the arbitrary integer number and $T = 2\pi/\omega_o$ is the period of the motion. Then the full (in all space) averaged over the period of motion radiation intensity loss reads ($\epsilon_{\nu}(\mathbf{k}) = \epsilon(\mathbf{k}, \nu\omega_o)$):

$$\left\langle \frac{d\Delta_{\perp}}{dt} \right\rangle = \frac{1}{\pi^2} \sum_{\nu=1}^{\infty} \nu \omega_o \int_{K_3} \frac{d\mathbf{k}}{k^2} \text{Im} \left\{ \frac{|\mathbf{k} \times \mathbf{j}_{\nu}(\mathbf{k})|^2}{c^2 \left[k^2 - \epsilon_{\nu}(\mathbf{k}) \left(\frac{\nu\omega_o}{c} \right)^2 \right]} \right\}. \quad (49)$$

The polarisation (Bohr) energy loss intensity averaged over the period of motion will be:

$$\left\langle \frac{d\Delta_{\parallel}}{dt} \right\rangle = \frac{1}{\pi^2} \sum_{\nu=1}^{\infty} \nu \omega_o \int_{K_3} \frac{d\mathbf{k}}{k^2} \text{Im} \left\{ \frac{|\rho_{\nu}(\mathbf{k})|^2}{-\epsilon_{\nu}(\mathbf{k})} \right\}. \quad (50)$$

For the point-like charge e moving along the trajectory $\mathbf{r}(t)$ the Fourier component of the electric current density $\mathbf{j}(\mathbf{k}, \omega)$ reads:

$$\mathbf{j}_{\nu}(\mathbf{k}) = \frac{e}{T} \int_{-T/2}^{T/2} dt \mathbf{v}(t) \exp\{i[\nu\omega_o t - \mathbf{k}\mathbf{r}(t)]\}. \quad (51)$$

In the next subsections we consider some practical examples illustrating the applications of (49).

3.2 Oscillating charge in absorbing medium

Let us consider the charge e oscillating in absorbing medium according the following law:

$$\mathbf{r}_o(t) = \mathbf{a} \sin(\omega_o t), \quad \mathbf{v}_o(t) = \omega_o \mathbf{a} \cos(\omega_o t),$$

where the relativistic condition $v_o < c$ limits the range of amplitudes $a = |\mathbf{a}| < c/\omega_o$. The Fourier components of the electric current density are:

$$\mathbf{j}_\nu(\mathbf{k}) = \nu \frac{e\omega_o \mathbf{a}}{\mathbf{k} \cdot \mathbf{a}} J_{-\nu}(-\mathbf{k} \cdot \mathbf{a}), \quad \mathbf{j}_\nu^*(\mathbf{k}) = \nu \frac{e\omega_o \mathbf{a}}{\mathbf{k} \cdot \mathbf{a}} J_\nu(\mathbf{k} \cdot \mathbf{a}), \quad (52)$$

where J_ν is the Bessel function of the integer order. The mean radiation intensity averaged over the period of motion reads:

$$\begin{aligned} \left\langle \frac{d\Delta_\perp}{dt} \right\rangle &= \frac{2e^2}{\pi c^2} \sum_{\nu=1}^{\infty} (\nu\omega_o)^3 \int_{-1}^1 \frac{\sin^2 \theta}{\cos^2 \theta} d \cos \theta \int_0^\infty dk \times \\ &\times \text{Im} \left\{ \frac{J_\nu^2(ka \cos \theta)}{k^2 - \epsilon_\nu(\mathbf{k}) \left(\frac{\nu\omega_o}{c} \right)^2} \right\}. \end{aligned} \quad (53)$$

Since the expression inside the imaginary part is the even function of k , the latter equation can be transformed to contour integral (see, Fig.6):

$$\left\langle \frac{d\Delta_\perp}{dt} \right\rangle = \frac{2e^2}{\pi c^2} \sum_{\nu=1}^{\infty} (\nu\omega_o)^3 \int_{-1}^1 \frac{\sin^2 \theta}{\cos^2 \theta} d \cos \theta \text{Im} \left\{ \oint_C F(z) dz \right\}, \quad (54)$$

where $F(z)$ is defined by the following equation:

$$F(z) = \frac{1}{2} \frac{J_\nu^2(za \cos \theta)}{z^2 - \epsilon_\nu \left(\frac{\nu\omega_o}{c} \right)^2}.$$

The integration over z is reduced to the calculation of the residue at $z = \nu\omega_o \sqrt{\epsilon_\nu}/c$. The relation (53) finally reads:

$$\left\langle \frac{d^2 \Delta_\perp}{dt d \cos \theta} \right\rangle = \frac{e^2}{c^2} \tan^2 \theta \sum_{\nu=1}^{\infty} (\nu\omega_o)^2 \text{Re} \left\{ \frac{J_\nu^2 \left[\frac{\nu\omega_o a}{c} \sqrt{\epsilon_\nu} \cos \theta \right]}{\sqrt{\epsilon_\nu}} \right\}. \quad (55)$$

In the dipole approximation, when the argument of the Bessel function is small, we leave only first term of the Bessel function expansion at small arguments $J_1(z) \sim z/2$. Then the intensity in transparent medium will be described by the following well-known expression [5]:

$$\left\langle \frac{d^2 \Delta_\perp}{dt} \right\rangle_1 = \frac{e^2 a^2 \omega_o^2}{8\pi c^3} n(\omega_o) \sin^2 \theta d\Omega, \quad (56)$$

where $n(\omega_o)$ is the refractive index at frequency ω_o , and $d\Omega$ is the element of the solid angle at the direction of the radiation. The latter expression is reduced to the vacuum case, when one put $n(\omega_o) = 1$.

3.3 Saw-tooth motion

We consider now the saw-tooth motion according the following law:

$$\mathbf{v}_o(t) = \begin{cases} v & 0 < t < \frac{T}{2}, \\ -v & \frac{T}{2} < t < T, \end{cases}$$

where T is the period of motion (see, Fig.7). The vector product of the wave vector and the current density reads:

$$\begin{aligned} |\mathbf{k} \times \mathbf{j}_\nu(\mathbf{k})|^2 = & \frac{4e^2 k^2 v^2 \sin^2 \theta}{T^2} \left\{ \frac{\sin^2 \left[\frac{T}{4}(\mathbf{k} \cdot \mathbf{v} - \nu \omega_\nu) \right]}{(\mathbf{k} \cdot \mathbf{v} - \nu \omega_\nu)^2} + \right. \\ & + \frac{\sin^2 \left[\frac{T}{4}(\mathbf{k} \cdot \mathbf{v} + \nu \omega_\nu) \right]}{(\mathbf{k} \cdot \mathbf{v} + \nu \omega_\nu)^2} - \\ & \left. - 2 \cos \left[\frac{\nu \omega_o T}{2} \right] \frac{\sin \left[\frac{T}{4}(\mathbf{k} \cdot \mathbf{v} - \nu \omega_\nu) \right]}{(\mathbf{k} \cdot \mathbf{v} - \nu \omega_\nu)} \frac{\sin \left[\frac{T}{4}(\mathbf{k} \cdot \mathbf{v} + \nu \omega_\nu) \right]}{(\mathbf{k} \cdot \mathbf{v} + \nu \omega_\nu)} \right\}. \end{aligned} \quad (57)$$

The mean radiation intensity loss will then be:

$$\begin{aligned} \left\langle \frac{d\Delta_\perp}{dt} \right\rangle = & \frac{8e^2}{\pi^2 T^2 c^2} \sum_{\nu=1}^{\infty} \omega \int_{-1}^1 \frac{\sin^2 \theta}{\cos^2 \theta} d \cos \theta \times \\ & \times \int_0^\infty k^2 dk \operatorname{Im} \left\{ \frac{\sin^2[d(k - b_\nu)]}{(k^2 - a_\nu^2)(k - b_\nu)^2} + \frac{\sin^2[d(k + b_\nu)]}{(k^2 - a_\nu^2)(k + b_\nu)^2} - \right. \\ & \left. - 2 \cos \left[\frac{\nu \omega_o T}{2} \right] \frac{\sin[d(k - b_\nu)]}{(k^2 - a_\nu^2)(k - b_\nu)} \frac{\sin[d(k + b_\nu)]}{(k + b_\nu)} \right\}, \end{aligned} \quad (58)$$

where we introduced the following notation:

$$a_\nu^2 = \epsilon \left(\frac{\nu \omega_o}{c} \right)^2, \quad b_\nu = \frac{\nu \omega_o}{v \cos \theta}, \quad d = \frac{vT \cos \theta}{4}.$$

We consider the radiation intensity loss as the sum of harmonics at $\omega = \nu \omega_o$:

$$\left\langle \frac{d\Delta_\perp}{dt} \right\rangle = \sum_{\nu=1}^{\infty} \left\langle \frac{d\Delta_{\perp\nu}}{dt} \right\rangle.$$

Repeating the calculation considered in section (2.4) for the Tamm problem, we have for the ν -harmonics:

$$\begin{aligned} \left\langle \frac{d^2 \Delta_{\perp \nu}}{dt d \cos \theta} \right\rangle = & \frac{4e^2 \beta^2}{T^2 c} \sin^2 \theta \operatorname{Re} \left\{ \frac{\sqrt{\epsilon} \sin^2 \left[\frac{\nu \omega_o T}{4} (1 - \beta \sqrt{\epsilon} \cos \theta) \right]}{(1 - \beta \sqrt{\epsilon} \cos \theta)^2} + \right. \\ & + \frac{\sqrt{\epsilon} \sin^2 \left[\frac{\nu \omega_o T}{4} (1 + \beta \sqrt{\epsilon} \cos \theta) \right]}{(1 + \beta \sqrt{\epsilon} \cos \theta)^2} - \\ & - 2 \cos \left[\frac{\nu \omega_o T}{2} \right] \frac{\sqrt{\epsilon} \sin \left[\frac{\nu \omega_o T}{4} (1 - \beta \sqrt{\epsilon} \cos \theta) \right]}{(1 - \beta \sqrt{\epsilon} \cos \theta)} \times \\ & \left. \times \frac{\sqrt{\epsilon} \sin \left[\frac{\nu \omega_o T}{4} (1 + \beta \sqrt{\epsilon} \cos \theta) \right]}{(1 + \beta \sqrt{\epsilon} \cos \theta)} \right\}. \end{aligned} \quad (59)$$

The first two terms in the latter expressions describe the contributions from the motions with $\mathbf{v}$ and $-\mathbf{v}$, respectively. The third term is responsible for the prompt bremsstrahlung $\pm \mathbf{v} \rightarrow \mp \mathbf{v}$.

3.4 Magnetic bremsstrahlung

Let us consider now the motion in the external magnetic field H_o . We put for simplicity that relativistic charge rotates in the plane perpendicular to the direction of the magnetic field. The law of motion is the following:

$$x_o(t) = R \cos(\omega_o t), \quad y_o(t) = R \sin(\omega_o t), \quad \omega_o = \frac{v}{R} = \frac{e H_o}{m c \gamma},$$

where m is the mass of relativistic charge. The ν -component of the radiation intensity I_ν can be expressed by:

$$\frac{dI_\nu}{d\Omega} = \frac{\omega_o^2 \nu^2}{2\pi c^3} \operatorname{Re} \left\{ \sqrt{\epsilon_\nu} |\mathbf{n} \times \mathbf{j}_\nu|^2 \right\}, \quad (60)$$

where $\mathbf{n}$ is the unit vector in the direction of radiation, and $\mathbf{j}_\nu$ is considered in the radiation pole:

$$k^2 = \epsilon_\nu \frac{\nu^2 \omega_o^2}{c^2}, \quad \epsilon_\nu = \epsilon(\nu \omega_o).$$

After transformations utilising the properties of Bessel functions J_ν [17] we have:

$$\mathbf{j}_\nu = e v \left[i \mathbf{e}_\varphi J'_\nu(\nu \beta \sqrt{\epsilon_\nu} \sin \theta) + \mathbf{e}_\rho \frac{J_\nu(\nu \beta \sqrt{\epsilon_\nu} \sin \theta)}{\beta \sqrt{\epsilon_\nu} \sin \theta} \right], \quad (61)$$

where $\mathbf{e}$ is the unit vector in the direction of radiating charge. The latter expression allows us to calculate the vector product of $\mathbf{n}$ and $\mathbf{j}_\nu$. Then we have for the angular distribution on the ν -harmonic:

$$\begin{aligned} I_\nu = & \frac{e^2 \omega_o^2 \nu^2 \beta^2}{c} \operatorname{Re} \left\{ \sqrt{\epsilon_\nu} \int_0^\pi \sin \theta d\theta \left[J'^2_\nu(\nu \beta \sqrt{\epsilon_\nu} \sin \theta) + \right. \right. \\ & \left. \left. + (1 - \sin^2 \theta) \frac{J_\nu^2(\nu \beta \sqrt{\epsilon_\nu} \sin \theta)}{\beta^2 \epsilon_\nu \sin^2 \theta} \right] \right\}. \end{aligned} \quad (62)$$

The integration in respect of θ results in:

$$I_\nu = \frac{2e^2\omega_o^2}{v} \text{Re} \left\{ \nu\beta^2 J_{2\nu}(2\nu\beta\sqrt{\epsilon_\nu}) - \left(\frac{1}{\epsilon_\nu} - \beta^2 \right) \int_0^{2\nu\beta\sqrt{\epsilon_\nu}} J_{2\nu}(x) dx \right\}. \quad (63)$$

The latter expression reduces in transparent medium to the well-known relation [18, 19]:

$$P(\omega) = \frac{I_\nu}{\omega_o} = \frac{2e^2\omega_o\nu}{v n_\nu^2} \left\{ 2n_\nu^2\beta^2 J_{2\nu}(2\nu\beta n_\nu) - (1 - n^2\beta^2) \int_0^{2\nu\beta n_\nu} J_{2\nu}(x) dx \right\}, \quad (64)$$

where $n_\nu = \sqrt{\epsilon_\nu}$ is the refractive index corresponding to the frequency $\omega = \nu\omega_o$. The details of the analysis in the range of visible light and X-rays where $\nu \gg 1$ will be published elsewhere.

4 Discussion. High-energy, low-angle approximation

It is interesting to consider the X-ray range of the radiation produced by relativistic charged particles in absorbing medium. For the frequencies more than K -shell excitation potential we can use the standard high frequency approximation for the dielectric permittivity:

$$\epsilon = 1 - \frac{\omega_p^2}{\omega^2} + i \frac{c}{\omega l}, \quad (65)$$

where ω_p and l are the plasma frequency and the photon absorption length in the medium, respectively. Then the mean number of X-ray Cerenkov photons (see also [15]) emitted from the distance X will be (the small angle approximation of (19)):

$$\frac{d^3 \bar{N}_{xcr}}{\hbar d\omega dx d\theta^2} = \frac{\alpha}{\pi \hbar c} \frac{\omega}{c} \theta^2 \text{Im} \{Z\}, \quad (66)$$

where we introduce the *complex formation zone*, Z , of X-ray Cerenkov radiation in the medium:

$$Z = \frac{L}{1 - i \frac{L}{l}}. \quad (67)$$

In the case of a transparent medium $l = \infty$, the complex formation zone is reduced to the *coherence length* L of XCR:

$$L = \frac{c}{\omega} \left[\gamma^{-2} + \frac{\omega_p^2}{\omega^2} + \theta^2 \right]^{-1}, \quad \gamma^{-2} = 1 - \beta^2. \quad (68)$$

For the case of the prompt bremsstrahlung the number of X-ray photons produced by ultra-relativistic $\beta \sim 1$ charge reads:

$$\frac{d^3 \bar{N}_{xbr}}{\hbar d\omega dx d\theta^2} = \frac{\alpha}{\pi \hbar c} \frac{\omega}{c} \theta^2 \text{Re} \{Z^2\}. \quad (69)$$

Note that this relation is quite similar to the the mean number of X-ray transition radiation photons, emitted when the charge e crosses the interface between two media with different dielectric properties:

$$\frac{d^3 \bar{N}_{xtr}}{\hbar d\omega dx d\theta^2} = \frac{\alpha}{\pi \hbar c} \frac{\omega}{c} \theta^2 \text{Re} \{ (Z_1 - Z_2)^2 \}, \quad (70)$$

where Z_1 and Z_2 are the complex formation zones in the first and the second medium, respectively [16]. The similar expressions can be easily derived for other modes of motion of relativistic charge in absorbing medium. It means, therefore, that the complex formation zone defines the main properties of the X-ray (gamma) radiation produced by relativistic charged particle in absorbing medium.

The examples of the radiation energy loss produced by relativistic charged particle moving with arbitrary acceleration in absorbing medium allow to make the following conclusions:

1. The approach based on the calculation of radiation energy loss looks like more simple and powerful as compared to the estimation of Poynting vector. The calculation of the radiation energy loss allows to apply the powerful apparatus of the analysis on the complex plane.
2. In the high-energy low-angle approximation the radiation energy loss can be expressed in terms of the complex formation zone. The latter covers both coherence and absorption effects in the X-ray and gamma range
3. The radiation energy loss approach allows to simulate the emitted photons just along the trajectory of initial charged particle. It is convenient for the simulation of the radiation produced by relativistic charged particle moving in detectors with complex geometry.

The approach described in this paper is planned to implement in the framework of GEANT4 toolkit for simulation in high energy physics, astrophysics and medical imaging [3].

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Figure Captions

- Fig. 1.** The angle distributions of Cerenkov photons produced in C_6F_{14} by relativistic particle $v \sim c$ for two values of the photon energy: $\hbar\omega = 6.8$ eV (open circles) and $\hbar\omega = 7.7$ eV (closed circles and upper x-axis). The curves are normalised on the equal most probable values.
- Fig. 2.** The contour of integration for the prompt bremsstrahlung.
- Fig. 3.** Uniform motion along finite distance (the Tamm problem).
- Fig. 4.** The contour of integration for the Tamm problem.
- Fig. 5.** The regions of anomalous and normal Doppler effects in transparent medium.
- Fig. 6.** The contour of integration for the oscillating motion.
- Fig. 7.** Saw-tooth motion.

Angle distribution of Cerenkov photons in C₆F₁₄

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7