Cosmological consequences of MSSM flat directions

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September 24, 2002

Abstract

We review the cosmological implications of the flat directions of the Minimally Supersymmetric Standard Model (MSSM). We describe how field condensates are created along the flat directions because of inflationary fluctuations. The post-inflationary dynamical evolution of the field condensate can charge up the condensate with $B$ or $L$ in a process known as Affleck-Dine baryogenesis. Condensate fluctuations can give rise to both adiabatic and isocurvature density perturbations and could be observable in future cosmic microwave experiments. In many cases the condensate is however not the state of lowest energy but fragments, with many interesting cosmological consequences. Fragmentation is triggered by inflation-induced perturbations and the condensate lumps will eventually form non-topological solitons, known as $Q$-balls. Their proper-
ties depend on how supersymmetry breaking is transmitted to the MSSM; if by
gravity, then the $Q$-balls are semi-stable but long-lived and can be the source
of all the baryons and LSP dark matter; if by gauge interactions, the $Q$-balls
can be absolutely stable and form dark matter that can be searched for directly.
We also discuss some cosmological applications of generic flat directions and $Q$-
balls in the context of self-interacting dark matter, inflatonic solitons and extra
dimensions.

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1 Introduction

The interplay between particle physics and cosmology plays an increasing role in understanding the physics beyond the Standard Model (SM) and the early Universe before the era of Big Bang Nucleosynthesis (BBN). On both fronts we currently lack hard data. Above the electroweak scale $E \sim \mathcal{O}(100)$ GeV, the particle content is largely unknown, while beyond the BBN scale $T \sim \mathcal{O}(1)$ MeV, there is no direct information about the thermal history of the Universe. However, there are some observational hints, as well as a number of theoretical considerations, which seem to be pointing towards a wealth of new physics both at small distances and in the very early Universe. Perhaps most importantly, new data is expected soon from accelerator experiments such as LHC and from cosmological measurements carried out by satellites such as MAP and Planck.

In cosmology the recent observations of the cosmic microwave background (CMB) radiation, which has a temperature $\sim 2.728 \pm 0.004$ K, have given rise to an era of precision cosmology. The Cosmic Background Explorer (COBE) satellite first detected in a full-sky map a temperature perturbation of one part in $10^5$ at scales larger than 7 degrees. The irregularities are present at a scale larger than the size of the horizon at the time when the microwave photons were generated and cannot be explained within the traditional hot Big Bang model. The recent balloon experiments BOOMERANG and MAXIMA, together with the ground-based DASI experiment have established the existence of the first few acoustic peaks in the positions predicted by cosmic inflation. Inflation, a period of exponential expansion in the very early Universe, is a direct link to physics at energy scales that will not be accessible to Earth-bound experiments for any foreseeable future. Inflation could occur because a slowly rolling scalar field, the inflaton, dynamically gives rise to an epoch dominated by a false vacuum. During inflation quantum fluctuation are imprinted on space-time as energy perturbations which then are stretched outside the causal horizon. These primordial fluctuations eventually re-enter our horizon, whence their form can be extracted from the CMB (for a review, see [?]).
Inflation can be considered as a model for the origin of matter since all matter arises from the vacuum energy stored in the inflaton field. However, the present models do not give clear predictions as to what sort of matter there is to be found in the Universe. From observations we know that baryons constitute about 3% of the total mass \[^{[?]}\], whereas relic diffuse cosmic ray background virtually excludes any domains of antibaryons in the visible Universe \[^{[?]}\]. Almost 30% of the total energy density is in non-luminous, non-baryonic dark matter \[^{[?]}\]. Its origin and nature is unknown, although various simulations of large scale structure formation suggest that there must be at least some cold dark matter (CDM), comprising of particles with negligible velocity, although there may also be a component of hot dark matter (HDM), comprising of particles with relativistic velocities \[^{[?]}\]. The rest of the energy density is in the form of dark energy \[^{[?]}\].

The striking asymmetry in the baryonic matter has existed at least since the time of BBN and plays an important role in providing the right abundances for the light elements. The present Helium \(^3\text{He}\), Deuterium \(\text{D}\) and Lithium abundances suggest a baryon density and an asymmetry relative to photon density of order \(10^{-10}\) \[^{[?]}\]. Such an asymmetry is larger by a factor of \(10^9\) than what it should have been by merely assuming a initially baryon symmetric hot Big Bang \[^{[?]}\]. Therefore, baryon asymmetry must have been created dynamically in the early Universe.

The origin of baryon asymmetry and dark matter bring cosmology and particle physics together. Within SM all the three Sakharov conditions for baryogenesis \[^{[?]}\] are in principle met; there is baryon number violation, \(C\) and \(CP\) violation, and an out-of-equilibrium environment during a first-order electroweak phase transition. However, it has turned out that within SM the electroweak phase transition is not strong enough \[^{[?]}\], \[^{[?]}\], \[^{[?]}\], \[^{[?]}\], and therefore the existence of baryons requires new physics. Regarding HDM, light neutrinos are a possible candidate \[^{[?]}\], \[^{[?]}\], but there is no candidate for CDM in the SM. HDM alone cannot lead a successful structure formation because of HDM free streaming length \[^{[?]}\], \[^{[?]}\]. Therefore, one must resort to physics beyond the SM also to find a candidate for CDM \[^{[?]}\].

The tangible evidence for small but non-vanishing neutrino masses as indicated by
the neutrino oscillations observed by the Super-Kamiokande [?] and SNO collaborations [?] is definitely another indication for new physics beyond the SM. The main sources of neutrino mass could be either Dirac or Majorana. A Dirac neutrino would require a large fine tuning in the Yukawa sector (one part in $10^{11}$) while a Majorana mass would appear to require a scale much above the electroweak scale together with an extension of the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. In the Majorana case the lightness of the neutrino could be explained via the see-saw mechanism [?, ?].

A theoretical conundrum is that the mass scale of SM is $\sim O(100)$ GeV, much lower than the scale of gravity $M_P = (8\pi G_N)^{-1/2} = 2.436 \times 10^{18}$ GeV, and not protected from quantum corrections. The most popular remedy is of course supersymmetry (for a review, see [?, ?, ?]), despite the fact that so far supersymmetry has evaded all observations [?]. The minimal supersymmetric extension of the SM is called the MSSM. Supersymmetry must be broken at a scale $\sim O(1)$ TeV, presumably in some hidden sector from which breaking is transmitted to the MSSM, e.g., by gravitational [?, ?] or gauge interactions [?].

In the MSSM the number of degrees of freedom are increased by virtue of the supersymmetric counterparts of the SM bosons and fermions. One of them, known as the lightest supersymmetric particle (LSP), could be absolutely stable with a mass of the order of supersymmetry breaking scale. LSP would be a natural candidate for CDM (see e.g. [?]). In addition, because of the larger parameter space, electroweak baryogenesis in MSSM in principle has a much better chance to succeed. However, there are a number of important constraints, and lately Higgs searches at LEP have narrowed down the parameter space to the point where it has all but disappeared [?, ?, ?].

Electroweak baryogenesis within MSSM thus appears to be heading towards deep trouble. Moreover, although MSSM can provide CDM, there is no connection between dark matter and electroweak baryogenesis. On the other hand, by virtue of supersymmetry, MSSM has the intriguing feature that there are directions in the field space which have virtually no potential. They are usually known as flat directions, which are made up of squarks and sleptons and therefore carry baryon number and/or lepton
number. The MSSM flat directions have been all classified [?].

Because it does not cost anything in energy, during inflation squarks and sleptons are free to fluctuate along the flat directions and form scalar condensates. Because inflation smoothes out all gradients, only the homogeneous condensate mode survives. However, like any massless scalar field, the condensate is subject to inflaton-induced zero point fluctuations which impart a small, and in inflation models a calculable, spectrum of perturbations on the condensate. After inflation the dynamical evolution of the condensate can charge the condensate up with a large baryon or lepton number, which can then released into the Universe when the condensate decays, as was first discussed by Affleck and Dine [?].

The potential along the MSSM flat direction is not completely flat because of supersymmetry breaking. In addition to the usual soft supersymmetry breaking, the non-zero energy density of the early Universe also breaks supersymmetry, in particular during inflation when the Hubble expansion dominates over any low energy supersymmetry breaking scale [?, ?]. Flatness can also be spoiled by higher-order non-renormalizable terms, and the details of the condensate dynamics depend on these.

In most cases, the MSSM condensate along a flat direction is however not the state of lowest energy. The condensate typically has a negative pressure, which causes the inflation-induced perturbations to grow. Because of this the condensate fragments, usually when the Hubble scale equals the supersymmetry breaking scale [?, ?]. Flatness can also be spoiled by higher-order non-renormalizable terms, and the details of the condensate dynamics depend on these.

The properties of $Q$-balls depend on supersymmetry breaking. If transmitted to MSSM by gravity, the $Q$-balls turn out to be only semistable but nevertheless long-lived compared with the time scales of the very early Universe [?]. When they decay, they may provide not only the baryonic matter but also dark matter LSPs [?]. If supersymmetry breaking is transmitted from the hidden sector to MSSM by gauge interactions, the resulting $Q$-balls would be stable and could exist at present as a form of dark matter [?]. In this case one can make direct searches for their existence [?]. In
both cases there is a prediction for the relation between the baryon and dark matter densities. Moreover, the condensate perturbations are inherited by the $Q$-balls, and can thus be a source of both isocurvature and adiabatic density perturbations [?], [?], [?].

This review is organized as follows. In Section 2, we recapitulate some basic cosmology, and in particular baryogenesis. We briefly discuss various popular schemes of baryogenesis and describe the original Affleck-Dine baryogenesis. In Section 3, we present some background material for inflation, mainly concentrating on supersymmetric models. Quantum fluctuations and reheating are also discussed. In Section 4, we present the MSSM flat directions and discuss their properties. Various contributions to the flat direction potential in the early Universe are listed. Low energy supersymmetry breaking schemes, such as gravity and gauge mediation, are also discussed. In Section 5, we discuss the dynamical properties of flat directions and the running of the flat direction potential due to gauge and Yukawa interactions. Leptogenesis along $LH_u$ flat direction, and the condensate evaporation in a thermal bath, is also described. We discuss fragmentation of the condensates for both gravity and gauge mediated supersymmetry breaking and present the relevant numerical studies. In Section 6, $Q$-ball properties are presented in detail. We describe various types of $Q$-balls, their interactions and their behavior at finite temperature. We discuss surface evaporation, diffusion, and dissociation of charge from $Q$-balls in a thermal bath. In Section 7, we focus on the cosmological consequences of $Q$-balls. We consider $Q$-ball baryogenesis and non-thermal dark matter generation through charge evaporation for different types of $Q$-balls. We discuss $Q$-balls as self-interacting dark matter and present experimental and astrophysical constraints on stable $Q$-balls. In Section 8, we briefly survey beyond-the-MSSM-condensates by considering inflatonic $Q$-balls and Affleck-Dine mechanism without MSSM flat directions. We also describe solitosynthesis, a process of accumulating large $Q$-balls in a charge asymmetric Universe.
2 Baryogenesis

2.1 Baryon asymmetric Universe

There are only insignificant amounts of anti-particles within the solar system. Cosmic ray showers contain $\sim 10^{-4}$ anti-protons for each proton \[\text{?}\], but the anti-protons are by-products of the interaction of the primary beam with the interstellar dust medium. This strongly suggest that galaxies and intergalactic medium is made up of matter rather than anti-matter, and if there were any anti-matter, the abundance has to be smaller than one part in $10^4$. The absence of annihilation radiation from the Virgo cluster shows that little anti-matter is to be found within a 20 Mpc sphere, and the relic diffuse cosmic ray background virtually excludes domains of anti-matter in the visible Universe \[\text{?}\].

The best present estimation for the baryon density comes from BBN \[\text{?}\] combined with the CMB experiments and it is given by \[\text{?}\]

$$0.010 \leq \Omega_b h^2 \leq 0.022,$$

where $\Omega_b \equiv \rho_b / \rho_c$ defines the fractional baryon density $\rho_b$ with respect to the critical energy density of the Universe: $\rho_c = 1.88 \, h^2 \times 10^{-29} \, \text{g cm}^{-3}$. The observational uncertainties in the present value of the Hubble constant; $H_0 = 100 \, h \, \text{km s}^{-1} \cdot \text{Mpc}^{-1} \approx (h/3000)\text{Mpc}^{-1}$ are encoded in $h$. Various considerations such as Hubble Space Telescope observations and type Ia supernova data suggest that $h = 0.70$ \[\text{?}\]. However, from the age of the globular cluster which comes out to be 11 Gyr, $h$ seems to take lower value of about 0.5 \[\text{?}\]. The present convention is to take $0.5 \leq h \leq 0.8$. In terms of the baryon and photon number densities we may write

$$\eta \equiv \frac{n_b - n_{\bar{b}}}{n_\gamma} = 2.68 \times 10^{-8} \Omega_b h^2,$$

where $n_b$ is the baryon number density and $n_{\bar{b}}$ is for anti-baryons. The photon number density is given by $n_\gamma \equiv (2\zeta(3)/\pi^2)T^3$. Observations of the deuterium abundance in quasar absorption lines suggest \[\text{?}\]

$$4(3) \times 10^{-10} \leq \eta \leq 7(10) \times 10^{-10}.$$
The conservative bounds are in parenthesis.

Often in the literature the baryon asymmetry is given in relation to the entropy density \( s = 1.8 g_* n_\gamma \), where \( g_* \) measures the effective number of relativistic species which itself a function of temperature. At the present time \( g_* \approx 3.36 \), while during BBN \( g_* \approx 10.11 \), rising up to 106.75 at \( T \gg 100 \text{ GeV} \). In the presence of supersymmetry at \( T \gg 100 \text{ GeV} \), the number of effective relativistic species are doubled to 213.30.

The baryon asymmetry \( B \), defined as the difference of baryon and anti-baryon number densities relative to the entropy density, is bounded by

\[
5.7(4.3) \times 10^{-11} \leq B \equiv \frac{n_b - n_{\bar{b}}}{s} \leq 9.9(14) \times 10^{-11},
\]

where the numbers in parenthesis are conservative bounds [?]. If at the beginning \( \eta = 0 \), then the origin of this small number can not be understood in a CPT invariant Universe by a mere thermal decoupling of nucleons and anti-nucleons at \( T \sim 20 \text{ MeV} \). The resulting asymmetry would be too small by at least nine orders of magnitude, see [?].

\section*{2.2 Thermal history of the Universe}

\subsection*{2.2.1 Expanding Universe}

The hot Big Bang cosmology assumes that the Universe is spatially homogeneous and isotropic and can be described by the Friedmann-Robertson-Walker (FRW) metric

\[
ds^2 = g_{\mu\nu} dx^\mu dx^\nu = dt^2 - a^2(t) \left[ \frac{dr^2}{1 - Kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right],
\]

where \( a(t) \) is the scale factor that determines the expansion or the contraction of the Universe; the constant \( K \) defines the spatial geometry. If \( K = 0 \), the Universe is flat and has Euclidean geometry, otherwise there is a spatial curvature corresponding either to a closed elliptic \((K = +1)\) or an open hyperbolic \((K = -1)\) geometries. The value of \( K \) cannot however fix the global topology; for instance, an Euclidean topology can be flat and infinite \( R^3 \), or a surface of a 3-torus \( T^3 \). However, topology has other observable consequences, e.g., for the pattern of CMB temperature fluctuations [?].
There are two characteristic scales corresponding to the homogeneous and isotropic Universe: the curvature scale \( r_{\text{curv}} = a(t)|K|^{-1/2} \), and the Hubble scale

\[
H^{-1} = \left[ \frac{\dot{a}(t)}{a(t)} \right]^{-1},
\]

where dot denotes derivative w.r.t. \( t \). The Hubble time is denoted by

\[
t_{\text{Hub}} = \int_{t_i}^{t_f} \frac{dt}{H^{-1}} = \ln \left( \frac{a_f}{a_i} \right).
\]

The behavior of the scale factor depends on the energy momentum tensor of the Universe. For a perfect fluid

\[
T_{\mu \nu} = p g_{\mu \nu} + (\rho + p) u_\mu u_\nu,
\]

where \( \rho \) is the energy density and \( p \) is the pressure of a fluid and the four velocity \( u_\mu \equiv dx_\mu/ds \). For the FRW metric and for the perfect fluid the equations of motion gives the Friedmann-Lemaitre equation

\[
H^2 = \frac{\rho}{3M_p^2} - \frac{K}{a(t)^2},
\]

also known as the Hubble equation. The acceleration equation is given by

\[
\frac{\ddot{a}(t)}{a(t)} = -\frac{\rho}{6M_p^2} (\rho + 3p),
\]

and the conservation of the energy momentum tensor \( T_{\mu \nu}^{\mu \nu} = 0 \) gives

\[
\frac{d(\rho a^3)}{da} = -3pa^2.
\]

Note that \( \rho a^3 \) is constantly decreasing in an expanding Universe for a positive pressure.

The early Universe is believed to have been radiation dominated with \( p = \rho/3 \) and \( a(t) \propto t^{1/2} \), followed by a matter dominated era with \( p = 0 \) and \( a(t) \propto t^{2/3} \). The early Universe might also have had an era of acceleration, known as the inflationary phase, which could have happened only if

\[
\dot{a} > 0 \iff \rho + 3p < 0.
\]

A geometric way of defining inflation is \([?]\)

\[
\frac{d(H^{-1}/a(t))}{dt} < 0,
\]
which states that the Hubble length as measured in comoving coordinates decreases during inflation. We will use this particular definition of inflation very often while discussing the number of e-foldings and density perturbations.

The Hubble expansion rate is related to the temperature by

\[ H = \sqrt{\frac{\rho}{3M_p^2}} = 1.66 \times g_*^{1/2} \frac{T^2}{M_p}, \tag{14} \]

where \( g_* \) is the total number of relativistic degrees of freedom and it is given by

\[ g_*(T) = \sum_{i=b} g_i \left( \frac{T_i}{T} \right)^4 + \frac{7}{8} \sum_{i=f} g_i \left( \frac{T_i}{T} \right)^4. \tag{15} \]

Here \( T_i \) denotes the effective temperature of species \( i \), which decouples at a temperature \( T = T_D \).

During the radiation era when \( H = (1/2t) \), one finds

\[ \frac{t}{1s} \approx 2.42 g_*^{-1/2} \left( \frac{1 \text{ MeV}}{T} \right)^2. \tag{16} \]

### 2.2.2 Entropy

An ideal gas of particles respects the Fermi-Dirac or Bose-Einstein distributions

\[ f_i(p, \mu, T) = \left[ \exp \left( \frac{(E_i - \mu_i)}{T} \right) \mp 1 \right]^{-1}, \tag{17} \]

where \( E_i^2 = |p|^2 + m^2 \), \( \mu_i \) represents the chemical potential of the species \( i \), \(-/+\) corresponds to Bose/Fermi statistics. The value of \( \mu \) is equal and opposite for particles and anti-particles. Therefore, in the early Universe a finite net chemical potential is proportional to the particle anti-particle asymmetry. The bound on charge asymmetry relative to the photon number density is severe, less than one part in \( 10^{43} \) at temperatures close to BBN [?], while baryon asymmetry is comparatively larger, but still small enough for \( \mu_e, \mu_b \approx 0 \) to be an excellent approximation. Neutrinos may however carry a net \( B-L \) charge which need not be vanishingly small at early times, although a large enough neutrino chemical potential can affect nucleosynthesis, for example, see [?].

The number density \( n \), energy density \( \rho \), and pressure \( p \) can be expressed in terms of temperature, and \( g_i \) is the number of internal degrees of freedom [?]

\[ n_i(T) = \frac{g_i}{2\pi^3} \int f_i(p, \mu, T)dp = \frac{g_i}{2\pi^2} T^3 I_i^{11}(\mp), \]

10
\[ \rho_i(T) = \frac{g_i}{2\pi^3} \int E_i(p)f_i(p, \mu, T)d^3p = \frac{g_i}{2\pi^2} T^4 I_i^{21}(\mp), \]
\[ p_i(T) = \frac{g_i}{2\pi^3} \int \frac{|p|^2}{E_i(p)}f_i(p, \mu, T)d^3p = \frac{g_i}{6\pi^2} T^4 I_i^{03}(\mp), \] (18)

where
\[ I_i^{ab}(\mp) = \int_{x_i}^\infty \frac{y^a(y^2 - x_i^2)^{b/2}}{(e^y + 1)}dy, \quad x_i = \frac{m_i}{T}. \] (19)

For a relativistic case with \( x_i \ll 1, \)
\[ I_i^{11}(\mp) = 2\zeta(3), \quad I_i^{21}(\mp) = I_i^{03}(\mp) = \frac{\pi^4}{15}, \quad \text{for bosons}, \]
\[ I_i^{11}(\pm) = \frac{3\zeta(3)}{2}, \quad I_i^{21}(\pm) = I_i^{03}(\pm) = \frac{7\pi^4}{120}, \quad \text{for fermions}, \] (20)

where \( \zeta \) denotes the Riemann Zeta function and \( \zeta(3) = 1.202. \) Thus the energy density of radiation reads
\[ \rho_r = \frac{\pi^2}{30} g_* T^4. \] (21)

For non-relativistic particles with \( x_i \gg 1, \) one obtains for both bosons and fermions
\[ n_{nr}(T) = \frac{\rho_{nr}}{m} = g_i \left( \frac{m_i}{2\pi} \right)^{3/2} e^{-m_i/T}, \quad p_{nr} = 0. \] (22)

If the chemical potential is non-zero, the exponential Eq. (22) also includes a factor \( e^{+\mu_i/T}. \)

The entropy density is defined as
\[ s \equiv \frac{S}{T} = \frac{\rho_i + p_i}{T}, \] (23)

where \( d(sa^3) = 0 \) is a thermodynamically conserved quantity. The decoupling temperature can be expressed as [?]
\[ \frac{T_D}{T} = \left( \frac{g_* S_A(T_D)}{g_* S_A(T)} \frac{g_* S_{-S_A}(T)}{g_* S_{-S_A}(T_D)} \right)^{1/3}, \] (24)

where \( S \) is the total entropy and \( S_A \) the entropy in the degrees of freedom that have decoupled at \( T_D. \)
2.2.3 Nucleosynthesis

According to BBN (for reviews see [? , ?]) the light elements \(^2\text{H}, \, ^3\text{He}, \, ^4\text{He},\) and \(^7\text{Li}\) have been synthesized during the first few hundred seconds. The abundances depend on the baryon-to-photon ratio

\[ \eta \equiv \frac{n_B}{n_\gamma} \]

(25)

All the relevant physical processes take place essentially in the range from a few MeV \(\sim 0.1 \text{ sec}\) down to 60 – 70 KeV \(\sim 10^3 \text{ sec}\). During this period only photons, \(e^\pm\) pairs, and the three neutrino flavors contribute significantly to the energy density. Any additional energy density may be parameterized in terms of the effective number of light neutrino species \(N_\nu\), so that

\[ g_* = 10.75 + \frac{7}{4}(N_\nu - 3) \]

(26)

Nucleosynthesis starts off with a freezing out of the weak interaction between neutron and proton at \(T_D \approx 0.8 \text{ MeV}\). Free neutrons keep decaying until deuterium begins to form through \(n+p \rightarrow d+\gamma\). Deuterium synthesis is over by \(T_D \approx 0.086 \text{ MeV}\) (assuming \(\eta = 5 \times 10^{-10}\)). At \(T_D\), neutron abundance has been depleted to \(X_n(t_D) \equiv n/(n+p) \approx 0.122\). All the surviving neutrons are now captured through \(n+D \rightarrow (^3\text{H}, ^3\text{He})\), and subsequently by virtue of the process \((^3\text{H}, ^3\text{He}) + n \rightarrow ^4\text{He}\), which has a binding energy of 28.3 MeV. The total mass fraction of primordial helium, which is denoted by \(Y_P(^4\text{He})\), is given by

\[ Y_P(^4\text{He}) \approx 2X_n(T_D) = 0.245 \]

(27)

Adopting the experimentally allowed range of \(0.22 < Y_P < 0.26\), one can constraint that number of light neutrino species by [?]

\[ N_\nu \leq 4 \]

(28)

The four LEP experiments combined give the best fit as [?]

\[ N_\nu = 2.994 \pm 0.12 \]

(29)

Nucleosynthesis also constrains many non-conventional ideas, for instance alternative theories of gravity such as scalar-tensor theories [?].
Besides $^4He$, D and $^3He$ are produced at the level of $10^{-5}$, and $^7Li$ at the level of $10^{-10}$. The theoretical prediction has some slight problems in fitting the observed $^4He$ and $^7Li$ abundances. Both seem to indicate $1.7 \times 10^{-10} < \eta < 4.7 \times 10^{-10}$, corresponding to $0.006 < \Omega_b h^2 < 0.017$ with a central value $\Omega_b h^2 = 0.009$ \footnote{There have been attempts \footnote{?, ?} for baryogenesis via a repulsive interaction between baryons and anti-baryons which would lead to their spatial separation before thermal decoupling of nucleons and anti-nucleons. However, at such early times the causal horizon contained only a very small fraction of the solar mass so that the asymmetry could not be smooth at distances greater than the galactic size.}. The abundance ratio $D/H$ is comparable with $^4He$ and $^7Li$ abundances at the $2\sigma$ level in the range $4.7 \times 10^{-10} < \eta < 6.2 \times 10^{-10}$, which corresponds to $0.017 < \Omega_b h^2 < 0.023$. The likelihood analysis which includes all the three elements (D, $^4He$, and $^7Li$) yields \footnote{?, ?}

$$4.7 \times 10^{-10} < \eta < 6.2 \times 10^{-10}, \quad 0.017 < \Omega_b h^2 < 0.023.$$  

Despite the uncertainties there appears to be a general concordance between theoretical BBN predictions and observations, which is now being bolstered by the CMB data from several different experiments. The results from the ground based DASI experiment indicates $\Omega_b h^2 = 0.022^{+0.004}_{-0.003}$ \footnote{?, ?}, while the results from the BOOMERANG balloon-borne experiment imply $\Omega_b h^2 = 0.021^{+0.004}_{-0.003}$ \footnote{?, ?}. MAXIMA, another balloon experiment, quotes a somewhat larger value $\Omega_b h^2 = 0.0325 \pm 0.006$ \footnote{?, ?}.

### 2.3 Requirements for baryogenesis

As pointed out by Sakharov \footnote{?, ?}, baryogenesis requires three ingredients: (1) baryon number non-conservation, (2) $C$ and $CP$ violation, and (3) out of equilibrium condition. All these three conditions are believed to be met in the very early Universe\footnote{1}.

#### 2.3.1 Non-conservation of baryonic charge

In the SM, baryon number $B$ is violated by non-perturbative instanton processes \footnote{?, ?}. At the quantum level both baryon number current $J_B^\mu$ and the lepton number current $J_L^\mu$ are not conserved because of chiral anomalies \footnote{?, ?}. However, the anomalous diver-
gences of $J_B^\mu$ and $J_L^\mu$ come with an equal amplitude and an opposite sign. Therefore $B - L$ remains conserved, while $B + L$ may change via processes which interpolate between the multiple non-Abelian vacua of $SU(2)$. The probability for the $B + L$ violating transition is however exponentially suppressed [?, ?]. As was first pointed out by Manton [?], at high temperatures the situation is different, so that when $T \gg M_W$, baryon violating transitions are in fact copious (see Sect. 2.4.2).

In addition to baryogenesis, $B$ violation also leads to proton decay in GUTs. For instance, the dimension 6 operator $(QQQL)/\Lambda$ generates observable proton decay unless $\Lambda \geq 10^{15}$ GeV. In the MSSM the bound is $\Lambda \geq 10^{26}$ GeV because the decay can take place via a dimension 5 operator. In the MSSM superpotential there are also terms which can lead to $\Delta L = 1$ and $\Delta B = 1$. Similarly, there are other processes such as neutron-anti-neutron oscillations in SM and in supersymmetric theories which lead to $\Delta B = 2$ and $\Delta B = 1$ transitions [?]. These operators are constrained by the measurements of the proton lifetime, which yield the bound $\tau_p \geq 10^{33}$ years [?].

2.3.2 $C$ and $CP$ violation

Weak interactions ensures maximum $C$ violation while neutral Kaon is an example of $CP$ violation in the quark sector which has a relative strength $\sim 10^{-3}$ [?]. $CP$ violation could also expected to be found in the neutrino sector. Beyond the SM there are many sources for $CP$ violation. An example is the axion proposed for solving the strong $CP$ problem in QCD [?]. Quantum fluctuations of light scalars in the early Universe, in particular during inflation, can create different domains of various $C$ and $CP$ phases. $C$ and $CP$ can also be spontaneously broken during a phase transition, so that domains of broken phases form with different $CP$-charges [?].

2.3.3 Departure from thermal equilibrium

Departure from a thermal equilibrium cannot be achieved by mere particle physics considerations but is coupled to the dynamical evolution of the Universe. If $B$-violation processes are in thermal equilibrium, the inverse processes will wash out the pre-existing asymmetry $(\Delta n_b)_0$ [?]. This is a consequence of $S$-matrix unitarity and $CPT$-
theorem [?]. However, there are several ways of obtaining an out-of-equilibrium process in the early Universe.

- **Out-of-equilibrium decay or scattering:**
  The Universe in a thermal equilibrium can not produce any asymmetry, rather it tries to equilibrate any pre-existing asymmetry. If the scattering rate $\Gamma < H$, the process can take place out of equilibrium. Such a situation is appropriate for e.g. GUT baryogenesis [?, ?].

- **Phase transitions:**
  Phase transitions are ubiquitous in the early Universe. The transition could be of first, or of second (or of still higher) order. First order transitions proceed by barrier penetration and subsequent bubble nucleation resulting in a temporary departure from equilibrium. Second order phase transitions have no barrier between the symmetric and the broken phase. They are continuous and equilibrium is maintained throughout the transition.

  Prime examples of first order phase transitions in the early Universe are the QCD and electroweak phase transitions. The nature and details of QCD phase transition is still very much an open debate [?, ?, ?], and although a mechanism for baryogenesis during QCD phase transition has been proposed [?], much more effort has been devoted to the electroweak phase transition [?, ?] (see Sect. 2.4.2).

- **Non-adiabatic motion of a scalar field:**
  Any complex scalar field carries $C$ and $CP$, but the symmetries can be broken by terms in the scalar potential. This can lead to a non-trivial trajectory of a complex scalar field in the phase space. If a coherent scalar field is trapped in a local minimum of the potential and if the shape of the potential changes to become a maximum, then the field may not have enough time to readjust with the potential and may experience completely non-adiabatic motion. This is similar to a second order phase transition but it is the non-adiabatic classical motion which prevails over the quantum fluctuations, and therefore, departure from equilibrium can be achieved. If the field condensate carries a global charge
such as the baryon number, the motion can charge up the condensate. This is the basis for the Affleck-Dine baryogenesis [?](see Sect. 2.5).

\subsection{2.3.4 Sphalerons}

In the SM $B + L$ is very weakly violated in the vacuum [?]. At finite temperatures violation is large [?, ?, ?, ?, ?] by virtue of the sphaleron configurations, which mediate transitions between degenerate gauge vacua with different Chern-Simons numbers related to the net change of $B + L$. Thermal scattering produces sphalerons which in effect decay in $B + L$ non-conserving ways below $10^{12}$ GeV [?], and thus can exponentially wash away $B + L$ asymmetry. Sphalerons and associated electroweak baryogenesis has been reviewed in [?, ?, ?, ?, ?]. Let us here just give a brief summary of the main ingredients.

- **Chiral anomalies**

An anomaly means that a classical current conservation no longer holds at the quantum level; an example is the chiral anomaly [?]. In the SM there is classical conservation of the baryon and lepton number currents $J_B^\mu$ and $J_L^\mu$, but because of chiral anomaly the currents are not conserved. Instead [?],

\begin{align}
\partial_\mu J_B^\mu &= -\frac{\alpha_2^2}{8\pi} N_g W_i^{\mu\nu} \tilde{W}_{i\mu\nu} + \frac{\alpha_1^2}{8\pi} N_g \left( \frac{4}{9} + \frac{1}{9} - \frac{2}{36} \right) F^{\alpha\beta} \tilde{F}_{\alpha\beta}, \\
\partial_\mu J_L^\mu &= -\frac{\alpha_2^2}{8\pi} N_g W_i^{\mu\nu} \tilde{W}_{i\mu\nu} + \frac{\alpha_1^2}{8\pi} N_g \left( 1 - \frac{1}{2} \right) F^{\alpha\beta} \tilde{F}_{\alpha\beta},
\end{align}

where $N_g$ is the number of generations, $\alpha_2$ and $\alpha_1$ ($W_{i\mu\nu}$ and $F_{\mu\nu}$) are respectively the $SU(2)$ and $U(1)$ gauge couplings (field strengths), and the various numbers inside the brackets correspond to the squares of the hypercharges multiplied by the number of states. Note that while at the quantum level $B + L$ is violated, $B - L$ is still conserved.

- **Gauge theory vacua**

The vacuum structure of the gauge theories is very rich [?, ?]. In case of $SU(2)$, the vacua are classified by their homotopy class $\{\Omega_n(r)\}$, characterized by the
winding number \(n\) which labels the so-called \(\theta\)-vacua [?, ?]. A gauge invariant quantity is the difference in the winding number (Chern-Simons number)

\[
N_{CS} \equiv n_+ - n_- = \frac{\alpha_2^2}{32\pi^2} \int d^4x W^\mu_{\alpha} \tilde{W}_{\alpha\mu
u}.
\] (32)

In the electroweak sector the field density \(W\tilde{W}\) is related to the divergence of \(B + L\) current. Therefore, a change in \(B + L\) reflects a change in the vacuum configuration and is determined by the difference in the winding number

\[
\Delta(B + L) = \int d^4x \partial_{\mu} J_{B+L}^{\mu} = \frac{\alpha_2^2}{16\pi^2} N_g \int d^4x W^\mu_{\alpha} \tilde{W}_{\alpha\mu
u} = -2N_g N_{CS}.
\] (33)

For three generations of SM leptons and quarks the minimal violation is \(\Delta(B + L) = 6\). Note that the proton decay \(p \rightarrow e^+\pi^0\) requires \(\Delta(B + L) = 2\), so that despite \(B\)-violation, proton decay is completely forbidden in the SM.

The probability amplitude for tunneling from an \(n\) vacuum at \(t \rightarrow -\infty\) to an \(n + N_{CS}\) vacuum at \(t \rightarrow +\infty\) can be estimated by the WKB method [?]

\[
P(N_{CS})_{B+L} \sim \exp \left( \frac{-4\pi N_{CS}}{\alpha_2(M_Z)} \right) \sim 10^{-162N_{CS}}.
\] (34)

Therefore, as advertised, the baryon number violation rate is totally negligible in the SM at zero temperature, but as argued in a seminal paper by Kuzmin, Rubakov and Shaposhnikov [?], at finite temperatures the situation is completely different.

- **Thermal tunneling**

The sphaleron is a field configuration sitting at the top of the potential barrier between two vacua with different Chern-Simons numbers and can be reached simply because of thermal fluctuations [?]. Neglecting \(U(1)_Y\), the zero temperature sphaleron solution was first found by Manton and Klinkhamer [?, ?]. At finite temperature the energy obeys an approximate scaling law [?, ?] \(E_{sph}(T) = E_{sph}(0)\langle\Phi(T)\rangle/\langle\Phi(0)\rangle\):

\[
E_{sph}(T) = \frac{2m_W(T)}{\alpha_2} B(\lambda/g_2),
\] (35)

where \(m_W(T) = (1/2)g_2\langle\Phi(T)\rangle\) is the mass of the \(W\)-boson and the function \(B\) has a weak dependence on \(\lambda/g_2\), where \(\lambda\) is the quartic self coupling of the
Higgs. Below the critical temperature of the electroweak phase transition, the sphaleron rate is exponentially suppressed [?]:

\[ \Gamma \sim 2.8 \times 10^5 \kappa T^4 \left( \frac{\alpha_2}{4\pi} \right)^4 \left( \frac{E_{sph}(T)}{B(\lambda^2/g)} \right)^7 e^{-E_{sph}/T}. \]  

(36)

where \( \kappa \) is the functional determinant which can take the values \( 10^{-4} \leq \kappa \leq 10^{-1} \) [?]. Above the critical temperature the rate is however unsuppressed. Requiring that the Chern-Simons number changes at most by \( \Delta N_{CS} \sim 1 \), one can estimate from Eq. (32) that \( \Delta N_{CS} \sim g_{sph}^3 W_i^3 \sim 1 \rightarrow W_i \sim \frac{1}{g_{sph}^2} \). Therefore, a typical energy of the sphaleron configuration is given by

\[ E_{sph} \sim l_{sph}^4 (\partial W_i)^2 \sim \frac{1}{g_{sph}^2 l_{sph}}. \]  

(37)

At temperatures greater than the critical temperature there is no Boltzmann suppression, so that the thermal energy \( \propto T \geq E_{sph} \). This determines the size of the sphaleron as

\[ l_{sph} \geq \frac{1}{g_{sph}^2 T}. \]  

(38)

This is exactly the infrared cut-off generated by the magnetic mass of order \( \sim g_{sph}^2 T \). Therefore, based on this coherence length scale one can estimate the baryon number violation per volume \( \sim l_{sph}^3 \), and per unit time \( \sim l_{sph} \). On dimensional grounds the transition probability would then be given by

\[ \Gamma_{sph} \sim \frac{1}{l_{sph}^3 t} \sim \kappa(\alpha_2 T)^4. \]  

(39)

where \( \kappa \) is a constant which incorporates various uncertainties. However, the process is inherently non-perturbative, and it has been argued that damping of the magnetic field in a plasma suppresses the sphaleron rate by an extra power of \( \alpha_2 \) [?], with the consequence that \( \Gamma_{sph} \sim \alpha_2^5 T^4 \). Lattice simulations with hard thermal loops also give \( \Gamma_{sph} \sim \mathcal{O}(10)\alpha_2^5 T^4 \) [?]

### 2.3.5 Washing out \( B + L \)

In the early Universe the transitions \( \Delta N_{CS} = +1 \) and \( \Delta N_{CS} = -1 \) are equally probable. The ratio of rates for the two transitions is given by

\[ \frac{\Gamma_{sph}^+}{\Gamma_{sph}^-} = e^{-\Delta F/T}, \]  

(40)
where $\Delta F$ is the free energy difference between the two vacua. Because of a finite $B + L$ density, there is a net chemical potential $\mu_{B+L}$. Therefore,

$$\Delta F \sim \mu_{B+L}^2 T^2 + \mathcal{O}(T^4) \equiv \frac{n_{B+L}^2}{T^2} + \mathcal{O}(T^4). \quad (41)$$

One then obtains [7]

$$\frac{dn_{B+L}}{dt} = \Gamma_{sph} + - \Gamma_{sph} - \sim N_g \frac{\Gamma_{sph}}{T^2} n_{B+L}. \quad (42)$$

It then follows that an exponential depletion of $n_{B+L}$ due to sphaleron transitions remains active as long as

$$\frac{\Gamma_{sph}}{T^3} \geq H \Rightarrow T \leq \alpha_X^{1/2} \frac{M_Z}{g_*^{1/2}} \sim 10^{12} \text{ GeV}. \quad (43)$$

This result is important because it suggests that below $T = 10^{12}$ GeV, the sphaleron transitions can wash out any $B + L$ asymmetry being produced earlier in a time scale $\tau \sim N_g T^3 / \Gamma_{sph}$. This seems to wreck GUT baryogenesis based on $B - L$ conserving groups such as the minimal $SU(5)$.

### 2.4 Alternatives for baryogenesis

There are several scenarios for baryogenesis (for reviews on baryogenesis, see [? , ?]), the main contenders being GUT baryogenesis, electroweak baryogenesis, leptogenesis, and baryogenesis through the decay of a field condensate, or Affleck-Dine baryogenesis. Here we give a brief description of these various alternatives.

#### 2.4.1 GUT-baryogenesis

This was the first concrete attempt of model building on baryogenesis which incorporates out-of-equilibrium decays of heavy GUT gauge bosons $X, Y \rightarrow qq$, and $X, Y \rightarrow \bar{q}\bar{l}$ (see e.g., [? , ? , ? , ?]). The decay rate of the gauge boson goes as $\Gamma_X \sim \alpha_X M_X$, where $M_X$ is the mass of the gauge boson and $\alpha_X^{1/2}$ is the GUT gauge coupling. Assuming that the Universe was in thermal equilibrium at the GUT scale, the decay temperature is given by

$$T_D \approx g_*^{-1/4} \alpha_X^{1/2} (M_X M_P)^{1/2}, \quad (44)$$
which is smaller than the gauge boson mass. Thus, at $T \approx T_D$, one expects $n_X \approx n_\gamma$, and hence the net baryon density is proportional to the photon number density $n_B = \Delta B n_\gamma$. However, below $T_D$ the gauge boson abundances decrease and eventually they go out of equilibrium. The net entropy generated due to their decay heats up the Universe with a temperature which we denote here by $T_{rh}$. Let us naively assume that the energy density of the Universe at $T_D$ is dominated by the $X$ bosons with $\rho_X \approx M_X n_X$, and their decay products lead to radiation with an energy density $\rho = (\pi^2/30) g_* T_{rh}^4$, where $g_* \sim \mathcal{O}(100)$ for $T \geq M_{GUT}$. Equating the expressions for the two energy densities one obtains

$$n_X \approx \frac{\pi^2}{30} g_* T_{rh}^4 \frac{T_{rh}}{M_X}.$$  \hspace{1cm} (45)

Therefore, the net baryon number comes out to be

$$B \equiv \frac{n_B}{s} \approx \frac{\Delta B n_X}{g_* n_\gamma} \approx 3 \frac{T_{rh}}{4 M_X} \Delta B.$$  \hspace{1cm} (46)

$T_{rh}$ is determined from the relation $\Gamma_X^2 \approx H^2(T_D) \sim (\pi^2/90) g_* T_{rh}^4 / M_P^2$. Thus,

$$B \approx \left( \frac{g_*^{-1/2} \Gamma_X M_P}{M_X^2} \right)^{1/2} \Delta B \equiv \left( \frac{g_*^{-1/2} \alpha_X M_P}{M_X} \right)^{1/2} \Delta B.$$  \hspace{1cm} (47)

Uncertainties in $C$ and $CP$ violation are now hidden in $\Delta B$, but can be tuned to yield total $B \sim 10^{-10}$ in many models.

Above we have tacitly assumed that the Universe is in thermal equilibrium when $T \geq M_X$. This might not be true, since for $2 \leftrightarrow 2$ processes the scattering rate is given by $\Gamma \sim \alpha^2 T$, which becomes smaller than $H$ at sufficiently high temperatures. Elastic $2 \leftrightarrow 2$ processes maintain thermal contact typically only up to a maximum temperature $\sim 10^{14}$ GeV, while chemical equilibrium is lost already at $T \sim 10^{12}$ GeV [?, ?]. It has been argued that QCD gas, which becomes asymptotically free at high temperatures, never reaches a chemical equilibrium above $\sim 10^{14}$ GeV [?]. In supergravity the maximum temperature of the thermal bath should not exceed $10^{10}$ GeV [?] (see Sect. 3.6.3).
2.4.2 Electroweak baryogenesis

A popular baryogenesis candidate is based on the electroweak phase transition, during which one can in principle meet all the Sakharov conditions. There is the sphaleron-induced baryon number violation above the critical temperature, various sources of $CP$ violation, and an out of equilibrium environment if the phase transition is of the first order. In that case bubbles of broken $SU(2) \times U(1)_{Y}$ are nucleated into a symmetric background with a Higgs field profile that changes through the bubble wall [?, ?, ?].

There are two possible mechanisms which work in a different regime; local and non-local baryogenesis. In the local case both $CP$ violation and baryon number violation takes place near the bubble wall. This requires the velocity of the bubble wall to be greater than the speed of the sound in the plasma [?, ?], and the electroweak phase transition to be strongly first order with thin bubble walls.

The second alternative, where the bubble wall velocity speed is small compared to the sound speed in the plasma, appears to be more realistic. In this mechanism the fermions, mainly the top quark and the tau-lepton, undergo $CP$ violating interactions with the bubble wall, which results in a difference in the reflection and the transmission probabilities for the left and right chiral fermions. The net outcome is an overall chiral flux into the unbroken phase from the broken phase. The flux is then converted into baryons via sphaleron transitions inside the unbroken phase. The interactions are taking place in a thermal equilibrium except for the sphaleron transitions, the rate of which is slower than the rate at which the bubble sweeps the space.

One great difficulty with the electroweak baryogenesis is the smallness of $CP$ violation in the SM. It has been pointed out that an additional Higgs doublet [?, ?, ?, ?] would provide an extra source for $CP$ violation in the Higgs sector. However, the situation is much improved in the MSSM where there are two Higgs doublets $H_{u}$ and $H_{d}$, and two important sources of $CP$ violation [?]. The Higgses couple to the charginos and neutralinos at one loop level leading to a $CP$ violating contribution. There is also a new source of $CP$ violation in the mass matrix of the top squarks which can give rise to considerable $CP$ violation [?].

Bubble nucleation depends on the thermal tunneling rate, and the expansion rate
of the Universe. The tunneling rate has to overcome the expansion rate in order to have a successful phase transition via bubble nucleation at a given critical temperature $T_c > T_t > T_0$. The actual value of the baryon asymmetry produced at the electroweak baryogenesis is still an open debate [?, ?, ?, ?], but in general it is hard to generate a large baryon asymmetry. For $T_c \sim 100$ GeV, $N = 3$, $\alpha_2 = 0.033$, and $B(\lambda/g_2) \sim 1.87$, one obtains the condition [?, ?, ?, ?]

$$\frac{E_{\text{sph}}(T_c)}{T_c} \geq 7 \log \left( \frac{E_{\text{sph}}(T_c)}{T_c} \right) + 9 \log(10) + \log(\kappa).$$

which implies [?]

$$\frac{E_{\text{sph}}(T_c)}{T_c} \geq 45 \quad \text{for} \quad \kappa = 10^{-1},$$

$$\geq 37 \quad \text{for} \quad \kappa = 10^{-4}. \quad (49)$$

The standard bound is often taken to be that of Eq. (49). In terms of the Higgs field value at $T_c$, one then obtains from Eq. (35)

$$\Phi(T_c) \sim \frac{g_2}{4\pi B(\lambda/g_2)} \frac{E_{\text{sph}}(T_c)}{T_c} \sim \frac{1}{36} \frac{E_{\text{sph}}(T_c)}{T_c},$$

for the above values of $\alpha_2, B$. Then the bounds in Eqs. (49,50) translate to

$$\frac{\Phi(T_c)}{T_c} \geq 1.3 \quad (1),$$

where the number in parenthesis is for Eq. (50). Eq. (52) respectively, implies that the phase transition should be strongly first order in order that sphalerons do not wash away all the produced baryon asymmetry. This result is the main constraint on electroweak baryogenesis.

Lattice studies suggest that in the SM the phase transition is strongly first order only below Higgs mass $m_H \sim 72$ GeV [?, ?, ?, ?]. Above this scale the transition is just a cross-over. Such a Higgs mass is clearly excluded by the LEP measurements [?], thus excluding electroweak baryogenesis within the SM.

### 2.4.3 Electroweak baryogenesis in MSSM

In the MSSM the ratio $\Phi(T_c)/T_c$ can increase considerably. The MSSM Higgs sector at finite temperature has been considered in [?, ?, ?, ?], for lattice studies see [?, ?, ?].

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In the MSSM the right handed stop $\tilde{t}_R$ couples to the Higgs with a large Yukawa coupling. This leads to a strong first order phase transition \[?, ?, ?\]. The LEP precision tests then indicate that the lightest left handed stop should be heavy heavy with $m_Q \geq 500$ GeV. This implies the for lightest right handed stop mass

$$m_{\tilde{t}}^2 \approx m_U^2 + 0.15 M_Z^2 \cos(2\beta) + m_{\tilde{t}}^2 \left(1 - \frac{\tilde{A}_t^2}{m_Q^2}\right),$$

(53)

where $\tilde{A}_t = A_t - \mu / \tan(\beta)$ is the stop mixing parameter, and $\mu$ is the soft-SUSY breaking mass parameter for the right-handed stop. The coefficient $\beta$ of the cubic term $\beta H^3$ in the effective potential reads

$$\beta_{\text{MSSM}} \approx \beta_{\text{SM}} + \frac{h_t^3 \sin^3(\beta)}{4\sqrt{2}\pi} \left(1 - \frac{\tilde{A}_t^2}{m_Q^2}\right)^{3/2},$$

(54)

and can be at least one order of magnitude larger than $\beta_{\text{SM}}$. In principle this modification can give rise to a strong enough first order phase transition.

The sphaleron bound implies Higgs and stop masses in the range \[?, ?\]

$$110 \text{ GeV} \leq m_H \leq 115 \text{ GeV}, \quad \text{and} \quad 105 \text{ GeV} \leq m_{\tilde{t}} \leq 165 \text{ GeV}.$$ \hspace{1cm} (55)

The present LEP constraint on the Higgs mass is $m_H \geq 115$ GeV \[?\]. Hence, even an MSSM-based electroweak baryogenesis may be at the verge of being ruled out. The definitive test of the MSSM based electroweak baryogenesis will obviously come from the Higgs and the stop searches at the LHC and the Tevatron \[?, ?, ?, ?, ?\].

### 2.4.4 Leptogenesis

Even if $B+L$ is completely erased by the sphaleron transitions, a net baryon asymmetry in the Universe can still be generated from a non-vanishing $B-L$ \[?\], even if there were no baryon number violating interactions. Lepton number violation alone can produce baryon asymmetry $B \sim -L$ \[?\], a process which is known as leptogenesis (for a recent review \[?\], and references therein). For lepton number violation one however has to go beyond the SM.

A popular example is $SO(10)$ GUT model, which can either be broken into $SU(5)$ and then subsequently to the SM, or into the SM gauge group directly. The most
attractive aspect of $SO(10)$ is that it is left-right symmetric (for details, see [? , ?]), and has a natural foundation for the see-saw mechanism [? , ?] as it incorporates a singlet right-handed neutrino $N_R$ with a mass $M_R$. A lepton number violation appears when the Majorana right handed neutrino decays into the SM lepton doublet and Higgs doublet, and their $CP$ conjugate state through

$$N_R \to \Phi + l, \quad N_R \to \Phi + \bar{l},$$

There also exist $\Delta L = 0$, and $\Delta L = 2$ processes mediated by the right handed neutrino through

$$\left(\frac{l \Phi}{M_R}\right), \quad \left(\frac{l l \Phi \Phi}{M_R}\right),$$

which are dimension 5 operators [? , ?]. (There are other processes involving t-quarks which may also be important [? , ?]). $CP$ asymmetry is generated through the interference between tree level and one-loop diagrams.

The total baryon asymmetry and total lepton asymmetry can be found in terms of the chemical potentials as [?]

$$B = \sum_i (2\mu_{q_i} + \mu_{u_{q_i}} + \mu_{d_{q_i}}), \quad L = \sum_i (2\mu_{l_i} + \mu_{e_{l_i}}),$$

where $i$ denotes three leptonic generations. The Yukawa interactions establish an equilibrium between the different generations ($\mu_{l_i} = \mu_l$ and $\mu_{q_i} = \mu_q$, etc.), and one obtains expressions for $B$ and $L$ in terms of the number of colors $N = 3$, and the number of charged Higgs fields $N_H$

$$B = -\frac{4N}{3} \mu_l, \quad L = \frac{14N^2 + 9NN_H}{6N + 3N_H} \mu_l,$$

together with a relationship between $B$ and $B - L$ [?]

$$B = \left(\frac{8N + 4N_H}{22N + 13N_H}\right) (B - L).$$

A similar expression has also been found in [? , ? , ?], although there seems to be small of order one differences. The baryon asymmetry based on the decays of right handed neutrinos in a thermal bath has been computed in [? , ? , ? , ?]. In a recent analysis [?], it was pointed out that the baryogenesis scale is tightly constrained together with
with the heavy right handed neutrino mass $T_B \sim M_{1,R} = \mathcal{O}(10^{10})$ GeV, with an upper bound on the light neutrino masses $\sum_i m_i < \sqrt{3}$ eV. The current bound on the right handed neutrino mass is around $M_R \sim \mathcal{O}(10^{11})$ GeV for light neutrino masses $m_{1\nu} \approx m_{2\nu} \approx m_{3\nu} \sim \mathcal{O}(0.1)$ eV.

High scale leptogenesis is ruled out in a supersymmetric theory because of the gravitino problem (see Sect. 3.7.1). However, if the masses of the right handed neutrinos are such that the mass splitting is comparable to their decay widths, it is possible to obtain an enhancement in the $CP$ phase of order one [?], and possibly a low scale thermal leptogenesis [?]. Otherwise, one could resort to non-thermal leptogenesis [?, ?, ?, ?], or, to the scattering process discussed in [?], or to sneutrino driven leptogenesis [?, ?].

2.4.5 Baryogenesis through field condensate decay

Scalar condensates may have formed in the course of the evolution of the early Universe. In particular, during inflation all scalar fields are subject to fluctuations driven by the non-zero inflaton energy density so that fields with very shallow potentials may easily take non-zero values. An example is the MSSM, where for the squark and slepton fields there are several directions in the field space where the potential vanishes completely [?, ?]. These directions are called (perturbatively) flat. Field fluctuations along such flat directions will soon be smoothed out by inflation, which effectively stretches out any gradients, and only the zero mode, or the scalar condensate, remains. This mechanism is quite general and applicable to any order parameter with flat enough potential. Baryogenesis can then be achieved by the decay of a condensate that carries baryonic charge, as was first pointed out by Affleck and Dine [?]. As we will discuss, the flat direction condensate can get dynamically charged with a large $B$ and/or $L$ by virtue of $CP$-violating self-couplings.

Baryogenesis from MSSM flat directions has the virtue that it only requires two already quite popular paradigms: supersymmetry and inflation. In the old version [?] baryons were produced by a direct decay of the condensate, to be discussed in Sect. 2.5.2. It was however pointed out first by Kusenko and Shaposhnikov [?] in the case of gauge mediated supersymmetry breaking, and then by Enqvist and McDonald?
in [?] in the case of gravity mediated supersymmetry breaking, that the MSSM flat
direction condensate most often is not stable but fragments and eventually forms non-
topological solitons called $Q$-balls [?]. These issues will be dealt in Sects. 6 and 7.

2.5 Old Affleck-Dine baryogenesis

2.5.1 Classical motion of the order parameter

In the original Affleck-Dine baryogenesis [?] it was assumed that the order parameter
along the flat direction is displaced from the origin because of inflationary fluctuations.
Because of inflation, only the long wave-length model of the order parameter will
survive so that a spatially constant condensate field is formed along the flat direction.
This we shall sometimes call the Affleck-Dine (AD) field. In an expanding Universe
the coherent AD field $\phi$ obeys the usual equation of motion,

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0, \quad (61)$$

where $H$ is the Hubble parameter.

To follow the time evolution of the AD field, let us consider a toy model with the
potential

$$V(\phi) = m^2|\phi|^2 + \lambda(\phi^4 + \phi^{*4}) + \frac{|\phi|^6}{M^2} + \ldots. \quad (62)$$

Although this potential is unrealistic in that it does not take correctly into account
of supersymmetry breaking induced by the non-zero cosmological constant of the in-
flationary era, it nevertheless captures the main features of the initial cosmological
evolution of the AD field.

The theory Eq. (62) has a partially conserved current $j_\mu = i\phi^*\partial_\mu \phi$, with

$$\partial_\mu j^\mu = \partial_\mu (i\phi^*\partial^\mu \phi - i\partial^\mu \phi^* \phi) = i\lambda(\phi^{*4} - \phi^4). \quad (63)$$

The current is conserved for small $\phi$. The role of the higher order term $|\phi|^6$ is just
to stabilize the potential. In the toy model Eq. (62), we identify the baryon number
density $n_B$ with $j_0$. The model also has a $CP$ invariance under which $\phi \leftrightarrow \phi^*$ but
which is violated by the initial conditions, which are taken to be

$$\phi = i\phi_0, \quad \dot{\phi} = 0, \quad (64)$$

where $\phi_0$ is real. Writing $\phi = \phi_R + i\phi_I$ one finds the coupled equations of motion (see e.g., [?])

$$\ddot{\phi}_I + 3H\dot{\phi}_I + \left[m^2 + 12\lambda\phi_R\phi_I + \frac{3|\phi|^4}{M^2}\right]\phi_I = 4\lambda\phi_R^3,$$

$$\ddot{\phi}_R + 3H\dot{\phi}_R + \left[m^2 + \frac{3|\phi|^4}{M^2}\right]\phi_R = 4\lambda(3\phi_I\phi_R^2 - \phi_I^3). \quad (65)$$

In a matter dominated Universe $H = 2/(3t)$, so that for large times $t \gg m^{-1}$ the motion is damped and Eq. (65) has then oscillatory solutions of the form

$$\phi_k = \frac{A_k}{mt} \sin(mt + \delta_k), \quad k = I, R, \quad (66)$$

where the amplitudes $A_k$ and the phases $\delta_k$ depend on the parameters $m$, $\lambda$, $M$, and the initial conditions Eq. (64). For large times the baryon number is then found to be

$$n_B = 2(\phi_I\dot{\phi}_R - \phi_R\dot{\phi}_I) = \frac{2A_I A_R}{mt^2} \sin(\delta_i - \delta_R). \quad (67)$$

If $\phi_0^2 \ll mM$, as was tacitly assumed by Affleck and Dine [?], one may disregard the higher-order terms. In that case one obtains [?] $A_I = \phi_0$ and $A_R = a_R\lambda\phi_0^3/m^2$, where $a_R = 0.85$ is determined numerically. Likewise, numerically one finds that $\delta_I - \delta_R = 1.54$. Thus,

$$n_B = \frac{1.7\lambda\phi_0^4}{m^3t^2}, \quad (68)$$

and the generated baryon number per particle is

$$R = \frac{mn_B}{\rho_\phi} = \frac{1.7\lambda\phi_0^2}{m^2}. \quad (69)$$

Eq. (69) is true for matter dominated Universe; for radiation dominated Universe one obtains a similar result, but the numerical prefactor 1.7 should be replaced by $-1.3$. If $\phi_0^2 > mM$, one finds $A_I = a_l(MM)^{1/2}$ and $A_R = a_R(M^3/m)^{1/2}$ with $a_l = 0.94$, $a_R = -2.86$, $\delta_I = 011$, and $\delta_R = -0.41$. It then follows from Eq. (67), that

$$n_B = -\frac{2.7\lambda M^2}{mt^2}. \quad (70)$$
Thus, the baryon generation mechanism is remarkably robust. The initial conditions do not matter, nor the actual expansion rate of the Universe. The baryon number generated per $\phi$-particle is always large and, with $\lambda \sim m^2/\langle\phi\rangle^2$, typically $n_B \gg 1$. Although these conclusions were derived in a toy model, similar results hold true also for the MSSM flat directions.

Thus, to summarize, along a flat direction where squarks and sleptons have non-zero expectation values, evolution of the AD field condensate, starting from a $CP$ violating initial value, will dynamically generate large baryon number density and charge the condensate with $B$ and/or $L$.

### 2.5.2 Condensate decay

To provide the Universe with the observed baryon to entropy ratio, $n_B/s \sim 10^{-10}$, the AD condensate must eventually transform itself into ordinary quarks. Originally [?], it was thought that this could happen via the decay of the AD field components (squarks and sleptons) to ordinary quarks and leptons. The AD condensate can be thought of as a coherent state of $\phi$-particles where $\phi = \phi_0 e^{imt}$ and $|\phi_0| \gg m$. When supersymmetry breaking is switched on, the AD field starts to oscillate about the old vacuum $\langle\phi\rangle \gg m$. Writing $\phi = \langle\phi\rangle + \phi'$, one observes that all the fields to which the excitations $\phi'$ couple are heavy with masses $\mathcal{O}(\langle\phi\rangle)$. The field $\phi'$ itself has a mass of the order of supersymmetry breaking, $\mathcal{O}(m)$. Therefore, $\phi'$ can decay to light fields only through loop diagrams involving heavy fields, with an effective coupling of the type $(g^2/\langle\phi\rangle)\phi'\psi\overline{\psi}$, where $\psi$ is a light fermion and $g$ some coupling constant. The decay rate is thus, [?]

$$
\Gamma \sim \frac{g^4 m^3}{\langle\phi\rangle^2}.
$$

Because of the oscillations of the AD field, the Universe will eventually become dominated by the energy density in the oscillations, $\rho_\phi \simeq m^2 \langle\phi\rangle^2$, so that $H \sim \rho_\phi^{1/2}/M_P$. The AD field will decay when $\Gamma \simeq H$, or $\phi \simeq (m^2 M_P)^{1/3}$. This implies a reheating temperature $T_{rh} \simeq s^{1/3} \simeq \rho_\phi^{1/4}$ while the baryon number density is $n_B = R\rho_\phi/m$, 

28
where $R$ is given in Eq. (69). Therefore one finally obtains

$$\frac{n_B}{s} \simeq \frac{\lambda \phi_0^2}{m^2} \left( \frac{M}{m} \right)^{1/6}. \quad (72)$$

Depending on $\lambda$, and the size of the initial fluctuation $\phi_0$ of the AD condensate, $n_B/s$ can be either small or large. Therefore, determining the initial value is of utmost importance [?]. This requires us to consider theories of inflation in more detail, which will be done in the next Section. Following that, we shall discuss the disappearance of the AD condensate by fragmentation into (quasi)stable lumps of condensate matter, whose state of lowest energy is a spherical non-topological soliton, a $Q$-ball [?].
3 Field fluctuations during inflation

Apart from explaining the initial condition for the hot Big Bang model, the flatness problem, and the horizon problem, cosmological inflation \[^{22}\text{, }^{22}\text{, }^{22}\] is one of the most favored candidate for the origin of structure in the Universe (for reviews on inflation, see \[^{22}\text{, }^{22}\]). There are many models of inflation, but by far the simplest is one in which inflation is generated by the large energy density of a scalar field. The scalar field driven inflation not only explains the homogeneity and the flatness problems but also the observed scale invariance of the density perturbations.

Inflation based on a scalar field theory is described by the following Lagrangian:

\[
\mathcal{L} = \frac{M_p^2}{2} R + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi),
\]

where \( R \) is the curvature scalar. The energy-momentum tensor reads

\[
T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} \partial_\rho \phi \partial^\rho \phi - g_{\mu\nu} V(\phi)
\]

so that the energy density and the pressure are given by

\[
\rho \equiv T_{00} = \frac{1}{2} \dot{\phi}^2 + \frac{1}{2a^2(t)} (\nabla \phi)^2 + V(\phi),
\]

\[
p \equiv \frac{T_{ii}}{a^2(t)} = \frac{1}{2} \dot{\phi}^2 - \frac{1}{2a^2(t)} (\nabla \phi)^2 - V(\phi).
\]

One of the initial conditions for inflation is that there must be a homogeneous patch of the Universe which is bigger than the size of the Hubble horizon \[^{22}\] (also supported by numerical studies, see \[^{22}\]). However, such a stringent condition can be evaded in a chaotic inflation beginning at the Planck scale \[^{22}\text{, }^{22}\text{, }^{22}\text{, }^{22}\]. More complicated situation can be obtained if there are several fields that participate in inflation; the classic example is assisted inflation \[^{22}\text{, }^{22}\].

3.1 Fluctuation spectrum in de Sitter space

The plane wave solution of a massive scalar field \( \phi(x, t) \) in a spatially flat Robertson-Walker metric can be decomposed into Fourier modes by

\[
\phi = \frac{1}{(2\pi)^{3/2}} \int d^3k \left( \phi_k(t) e^{ik \cdot x} + \text{h.c.} \right).
\]
Solving the Klein-Gordon equation for the scalar field in a conformal metric: $ds^2 = g_{\mu\nu}dx^\mu dx^\nu = a^2(\tau, x)(d\tau^2 - dx^2)$, the mode function can be given by [? , ? , ? , ?]

$$\phi_k(\eta) = \left(\frac{\pi}{4}\right)^{1/2} H|\eta|^{3/2} \left(c_1 H^{(1)}_\nu(k\eta) + c_2 H^{(2)}_\nu(k\eta)\right),$$

$$\eta = -H^{-1}e^{-Ht}, \text{ and } \nu^2 = \frac{9}{4} - \frac{m^2}{H^2};$$

(78)

where $m$ is the mass of the scalar field, $H^{(1)}_\nu$ and $H^{(2)}_\nu$ are the Hankel functions and $c_1, c_2$ are constants. The readers might be tempted to take the limit $\eta \ll 0$, in order to match the above solution with the plane wave solution in a Minkowski background. However, this leads to a quasi static de Sitter solution [?]. More technically, it has been shown that using a point splitting de Sitter solution [?]. More technically, it has been shown that using a point splitting regularization scheme, it is possible to obtain a Bunch-Davies vacuum for a de Sitter background which actually corresponds to taking $c_1 = 0$, and $c_2 = 1$. A simple but intuitive way has been developed in [?], where it has been argued that during a de Sitter phase, the main contribution to the two point correlation function comes from the long wavelength modes; $k\eta \ll 1$ or $k \ll H \exp(Ht)$. Therefore, the two point function is defined by an infrared cutoff which is determined by the Hubble expansion [?]

$$\langle \phi^2 \rangle \approx \frac{1}{(2\pi)^3} \int_H^{e^{Ht}} d^3k |\phi_k|^2.$$  

(79)

The result of the integration yields [? , ? , ? , ?] an indefinite increase in the variance with time

$$\langle \phi^2 \rangle \approx \frac{H^3}{4\pi^2} t.$$  

(80)

This result can also be obtained by considering the Brownian motion of the scalar field [?].

For a massive field with $m \ll H$, and $\nu \neq 3/2$, one does not obtain an indefinite growth of the variance of the long wavelength fluctuations, but [? , ? , ? , ?]

$$\langle \phi^2 \rangle = \frac{3H^4}{8\pi^2 m^2} \left(1 - e^{-(2m^2/3H^2)t}\right).$$  

(81)

In the limiting case when $m \to H$, the variance goes as $\langle \phi^2 \rangle \approx H^2$. In the limit $m \gg H$, the variance goes as $\langle \phi^2 \rangle \approx (H^3/12\pi^2 m)$ [?]. Only in a massless case $\langle \phi^2 \rangle$ can be treated as a homogeneous background field with a long wavelength mode. This
result plays an important role for the rest of this review as it implies that in a de Sitter phase any scalar field, including the AD condensate, are subject to quantum fluctuations.

### 3.2 Slow roll inflation

A completely flat potential can render inflation eternal, provided the energy density stored in the flat direction dominates. The inflaton direction is however not completely flat but has a potential $V(\phi)$ with some slope. An inflationary phase is obtained while

\[
H^2 \approx \frac{1}{3M_P^2} V(\phi), \quad (82)
\]

\[
3H \dot{\phi} \approx -V'(\phi), \quad (83)
\]

where prime denotes derivative with respect to $\phi$. In the above the approximations are: $\dot{\phi}^2 < V(\phi)$, and $\ddot{\phi} < V'(\phi)$, which lead to the slow roll conditions (see e.g. [?])

\[
\epsilon(\phi) = \frac{M_P^2}{2} \left( \frac{V'}{V} \right)^2 \ll 1, \quad (84)
\]

\[
|\eta(\phi)| = M_P^2 \frac{V''}{V} \ll 1. \quad (85)
\]

Note that $\epsilon$ is positive by definition.

These conditions are necessary but not sufficient for inflation. They only constrain the shape of the potential but not the velocity of the field $\dot{\phi}$. Therefore, a tacit assumption behind the success of the slow roll conditions is that the inflaton field should not have a large initial velocity.

Inflation comes to an end when the slow roll conditions are violated, $\epsilon \sim 1$, and $\eta \sim 1$. However, there are certain models where this need not be true, for instance in hybrid inflation models [?], where inflation comes to an end via a phase transition, or in oscillatory models of inflation where slow roll conditions are satisfied only on average [?].

One of the salient features of the slow roll inflation is that there exists a late time attractor behavior. This means that during inflation the evolution of a scalar field at a given field value has to be independent of the initial conditions. Therefore, slow roll
inflation should provide an attractor behavior which at late times leads to an identical field evolution in the phase space irrespective of the initial conditions \[\cdots\]. In fact the slow roll solution does not give an exact attractor solution to the full equation of motion but is nevertheless a fairly good approximation \[\cdots\]. A similar statement has been proven for multi-field exponential potentials without slow roll conditions (i.e., assisted inflation) \[\cdots\]. The attractor behavior of the inflaton leads to powerful predictions which can be distinguished from other candidates of galaxy formation \[\cdots\].

The standard definition of the number of e-foldings is given by

\[
N \equiv \ln \frac{a(t_{\text{end}})}{a(t)} = \int_t^{t_{\text{end}}} H dt \approx \frac{1}{M_p^2} \int_{\phi_{\text{end}}}^{\phi} \frac{V}{V'} d\phi,
\]

where \(\phi_{\text{end}}\) is defined by \(\epsilon(\phi_{\text{end}}) \sim 1\), provided inflation comes to an end via a violation of the slow roll conditions. The number of e-foldings can be related to the Hubble crossing mode \(k = a_k H_k\) by comparing with the present Hubble length \(a_0 H_0\). The final result is \[\cdots\]

\[
N(k) = 62 - \ln \frac{k}{a_0 H_0} - \ln 10^{16} \text{GeV} + \ln \frac{V_{k}^{1/4}}{V_{\text{end}}^{1/4}} - \frac{1}{3} \ln \frac{V_{\text{end}}^{1/4}}{\rho_{\text{rh}}} \tag{87}
\]

where the subscripts end (rh) refer to the end of inflation (onset of reheating). The details of the thermal history of the Universe determine the precise number of e-foldings, but for most practical purposes it is sufficient to assume that \(N(k) \approx 50\), keeping all the uncertainties such as the scale of inflation and the end of inflation within a margin of 10 e-foldings. A significant modification can take place only if there is an epoch of late inflation such as thermal inflation \[\cdots\], or in theories with a low quantum gravity scale \[\cdots\].

### 3.3 Primordial density perturbations

Initially, the theory of cosmological perturbations has been developed in the context of FRW cosmology \[\cdots\], and for models of inflation in \[\cdots\]. For a complete review on this topic, see \[\cdots\]. For a real single scalar field there arise only adiabatic density perturbations. In case of several fluctuating fields there will in general also be isocurvature perturbations. We briefly describe the two perturbations and their observational differences.
3.3.1 Adiabatic perturbations and the Sachs-Wolfe effect

Let us consider small inhomogeneities \( \phi(x, t) = \phi(t) + \delta \phi(x, t) \) such that \( \delta \phi \ll \phi \). Perturbations in matter densities automatically induce perturbations in the background metric, but the separation between the background metric and a perturbed one is not unique. One needs to choose a gauge. A simple choice would be to fix the observer to the unperturbed matter particles, where the observer will detect a velocity of matter field falling under gravity; this is known as the Newtonian or the longitudinal gauge because the observer in the Newtonian gravity limit measures the gravitational potential well where matter is falling in and clumping. The induced metric can be written as

\[
 ds^2 = (1 + 2 \Phi) dt^2 - (1 - 2 \Psi) \delta_{ik} dx^i dx^k ,
\]

where \( \Phi \) has a complete analogue of Newtonian gravitational potential. In the case when the spatial part of the energy momentum tensor is diagonal, i.e. \( \delta T^i_j = \delta^i_j \), it follows that \( \Phi = \Psi \). Right at the time of horizon crossing one finds a solution for \( \delta \phi \) as

\[
 \langle |\delta \phi_k|^2 \rangle = \frac{H(t_*)^2}{2k^3} ,
\]

where \( t_* \) denotes the instance of horizon crossing. Correspondingly, we can also define a power spectrum

\[
 P_\phi(k) = \frac{k^3}{2\pi^2} \langle |\delta \phi_k|^2 \rangle = \left[ \frac{H(t_*)}{2\pi} \right]^2 \equiv \left[ \frac{H}{2\pi} \right]^2 _{k=aH} .
\]

Note that the phase of \( \delta \phi_k \) can be arbitrary, and therefore, inflation has generated a Gaussian perturbation.

In the limit \( k \to 0 \), one can find an exact solution for the long wavelength inhomogeneities \( k \ll aH \), which reads

\[
 \Phi_k \approx c_1 \left( \frac{1}{a} \int_0^t a \, dt' \right) + c_2 \frac{H}{a} ,
\]

\[
 \frac{\delta \phi_k}{\phi} = \frac{1}{a} \left( c_1 \int_0^t a \, dt' - c_2 \right) ,
\]

where the dot denotes time derivative. The growing solutions are proportional to \( c_1 \), the decaying proportional to \( c_2 \). Concentrating upon the growing solution, it is
possible to obtain a leading order term in an expansion with the help of the slow roll conditions:

\[ \Phi_k \approx -c_1 \frac{\dot{H}}{H^2}, \quad (93) \]

\[ \frac{\delta \phi_k}{\dot{\phi}} \approx \frac{c_1}{H}, \quad (94) \]

Note that at the end of inflation, which is indicated by \( \ddot{a} = 0 \), or equivalently by \( \dot{H} = -H^2 \), one obtains a constant Newtonian potential \( \Phi_k \approx c_1 \). This is perhaps the most significant result for a single field perturbation.

In a long wavelength limit one obtains a constant of motion \( \zeta \) defined as

\[ \zeta = \frac{2}{3} \frac{H^{-1} \dot{\Phi}_k + \Phi_k}{1 + w} + \Phi_k, \quad w = \frac{p}{\rho}. \quad (95) \]

If the equation of state for matter remains constant there is a simple relationship which connects the metric perturbations at two different times \( \Phi_k(t_f) \approx (3/5)c_1 \). Substituting the value of \( c_1 \) from Eq. (94), we obtain

\[ \Phi_k(t_f) \approx \frac{3}{5} \frac{H \delta \phi_k}{\dot{\phi}} \bigg|_{k=aH}. \quad (98) \]

In a similar way it is also possible to show that the comoving curvature perturbations is given by

\[ \mathcal{R}_k \approx \frac{H}{\dot{\phi}} \delta \phi \bigg|_{k=aH}, \quad (99) \]

where \( \delta \phi \) denotes the field perturbation on a spatially flat hypersurfaces, because on a comoving hypersurface \( \delta \phi = 0 \), by definition. Therefore, on flat hypersurfaces

\[ \delta \phi_k = \dot{\phi} \delta t, \quad (100) \]
where $\delta t$ is the time displacement going from flat to comoving hypersurfaces [?, ?]. As a result
\[ \mathcal{R}_k \equiv H \delta t. \] (101)
Note that during matter dominated era the curvature perturbation and the metric perturbations are related to each other
\[ \Phi_k = -\frac{3}{5} \mathcal{R}_k. \] (102)
In the matter dominated era the photon sees this potential well created by the primordial fluctuation and the redshift in the emitted photon is given by
\[ \frac{\Delta T_k}{T} = -\Phi_k. \] (103)
At the same time, the proper time scale inside the fluctuation becomes slower by an amount $\delta t/t = \Phi_k$. Therefore, for the scale factor $a \propto t^{2/3}$, decoupling occurs earlier with
\[ \frac{\delta a}{a} = \frac{2 \delta t}{3 t} = \frac{2}{3} \Phi_k. \] (104)
By virtue of $T \propto a^{-1}$ this results in a temperature which is hotter by
\[ \frac{\Delta T_k}{T} = -\Phi_k + \frac{2}{3} \Phi_k = -\frac{\Phi_k}{3}. \] (105)
This is the celebrated Sachs-Wolfe effect [?], which we shall revisit when discussing isocurvature fluctuations.

### 3.3.2 Spectrum of adiabatic perturbations

Now, one can immediately calculate the spectrum of the metric perturbations. For a critical density Universe
\[ \delta_k \equiv \delta \rho \left|_k \right. = -\frac{2}{3} \left( \frac{k}{aH} \right)^2 \Phi_k, \] (106)
where $\nabla^2 \to -k^2$, in the Fourier domain. Therefore, with the help of Eqs. (90,98), one obtains
\[ \delta_k^2 \equiv \frac{4}{9} \mathcal{P}_\Phi(k) = \frac{4}{9} \frac{9}{25} \left( \frac{H}{\phi} \right)^2 \left( \frac{H}{2\pi} \right)^2, \] (107)
where the right hand side can be evaluated at the time of horizon exit \( k = aH \). In fact the above expression can also be expressed in terms of curvature perturbations \( \delta_k \)

\[
\delta_k = \frac{2}{5} \left( \frac{k}{aH} \right)^2 R_k ,
\]

(108)
and following Eq. (97), we obtain \( \delta_k^2 = 4/25 \mathcal{P}_R(k) = (4/25)(H/\dot{\phi})^2(H/2\pi)^2 \), exactly the same expression as in Eq. (107). With the help of the slow roll equation \( 3H\dot{\phi} = -V' \), and the critical density formula \( 3H^2M_P = V \), one obtains

\[
\delta_k^2 \approx \frac{1}{75\pi^2M_P^3V^2} = \frac{1}{150\pi^2M_P^3} \frac{V}{\epsilon},
\]

(109)
where we have used the slow roll parameter \( \epsilon \equiv (M_P^2/2)(V'/V)^2 \). The COBE satellite measured the CMB anisotropy and fixes the normalization of \( \delta_\Phi(k) \) on a very large scale. For a critical density Universe, if we assume that the primordial spectrum can be approximated by a power law and ignoring gravitational waves:

\[
\delta_\Phi(k) = 1.91 \times 10^{-5} \left( \frac{k}{k_{\text{pivot}}} \right)^{(n-1)/2},
\]

(110)
where \( n \) is the spectral index and \( k_{\text{pivot}} = 7.5a_0H_0 \) is the scale at which the normalization is independent of the spectral index.

The spectral index \( n(k) \) is defined as

\[
n(k) - 1 \equiv \frac{d\ln \mathcal{P}_\Phi}{d\ln k}.
\]

(111)
This definition is equivalent to the power law behavior if \( n(k) \) is fairly a constant quantity over a range of \( k \) of interest. The power spectrum can then be written as

\[
\mathcal{P}_\Phi(k) \propto k^{n-1}.
\]

(112)
If \( n = 1 \), the spectrum is flat and known as Harrison-Zeldovich spectrum \([?]\). For \( n \neq 1 \), the spectrum is tilted and \( n > 1 \) is known as blue spectrum. In terms of the slow roll parameters, one can write \([?]\)

\[
\frac{d\epsilon}{d\ln k} = 2\epsilon\eta - 4\epsilon^2, \quad \frac{d\eta}{d\ln k} = -2\epsilon\eta + \xi^2 \quad \frac{d\xi^2}{d\ln k} = -2\epsilon\xi^2 + \eta\xi^2 + \sigma^3,
\]

(113)
where

\[
\xi^2 \equiv M_P^4V'(d^3V/d\phi^3)/V^2, \quad \sigma^3 \equiv M_P^6V'^2(d^4V/d\phi^4)/V^3.
\]

(114)
Thus one finds\[\,\]
\[n - 1 = -6\epsilon + 2\eta.\] (115)

Slow roll inflation requires that \(\epsilon \ll 1, |\eta| \ll 1,\) and therefore naturally predicts small variation in the spectral index within \(\Delta \ln k \approx 1.\) The recent Boomerang data suggest \[\,\]
\[|n - 1| \leq 0.1.\] (116)

The rate of change in \(\eta\) is also very small, and can be estimated in a similar way \[\,\]
\[\frac{dn}{d\ln k} = -16\epsilon\eta + 24\epsilon^2 + 2\xi^2.\] (117)

It is possible to extend the calculation of metric perturbation beyond the slow roll approximation basing on a formalism similar to that developed in \[\,\]
\[?, ?, ?, ?\].

### 3.3.3 Gravitational waves

Gravitational waves are linearized tensor perturbations of the metric and do not couple to the energy momentum tensor. Therefore, they do not give rise a gravitational instability, but carry the underlying geometric structure of the space-time. The first calculation of the gravitational wave production was made in \[\,\]
\[?,\] and the topic has been considered by many authors \[\,\]. For reviews on gravitational waves, see \[\,\], \[\,\].

The gravitational wave perturbations are described by a line element \(ds^2 + \delta ds^2,\)

\[ds^2 = a^2(\eta)(d\eta^2 - dx^i dx_i), \quad \delta ds^2 = -a^2(\eta)h_{ij} dx^i dx^j.\] (118)

The gauge invariant and conformally invariant 3-tensor \(h_{ij}\) is symmetric, traceless \(\delta^{ij} h_{ij} = 0,\) and divergenceless \(\nabla_i h_{ij} = 0\) (\(\nabla_i\) is a covariant derivative). Massless spin 2 gravitons have two degrees of freedom and as a result are also transverse. This means that in a Fourier domain the gravitational wave has a form

\[h_{ij} = h_+ e^+_{ij} + h_\times e^{\times}_{ij}.\] (119)

For the Einstein gravity, the gravitational wave equation of motion follows that of a massless Klein Gordon equation \[\,\]. Especially, for a flat Universe

\[\ddot{h}_j^i + 3H \dot{h}_j^i + \left(\frac{k^2}{a^2}\right) h_j^i = 0,\] (120)
As any massless field, the gravitational waves also feel the quantum fluctuations in an expanding background. The spectrum mimics that of Eq. (90)

\[ P_{\text{grav}}(k) = \frac{2}{M_F^2} \left( \frac{H}{2\pi} \right)^2 \bigg|_{k=aH} . \]  

(121)

Note that the spectrum has a Planck mass suppression, which suggests that the amplitude of the gravitational waves is smaller compared to that of the adiabatic perturbations. Therefore, it is usually assumed that their contribution to the CMB anisotropy is small. The corresponding spectral index can be calculated as

\[ n_{\text{grav}} = \frac{d \ln P_{\text{grav}}(k)}{d \ln k} \equiv -2\epsilon . \]

(122)

Note that the spectral index is negative.

### 3.4 Multi-field perturbations

In multi-field inflation models contributions to the density perturbations come from all the fields. However, unlike in a single scalar case, in the multi-field case there might not be a unique late time trajectory corresponding to all the fields. This is true in particular for those fields that are effectively massless during inflation, such as the MSSM flat direction fields. Therefore, in these cases scalar perturbations will depend on the field trajectories and thus on the choice of initial conditions, with an ensuing loss of predictivity. In a very few cases it is possible to obtain a late time attractor behavior of all the fields; an example is assisted inflation \[？？？\]. Let us here nevertheless assume that there is an underlying unique late time trajectory resulting in a simple expression for the amplitude of the density perturbations and the spectral index \[？？？？？\].

#### 3.4.1 Adiabatic and isocurvature conditions

There are only two kinds of perturbations that can be generated. The first one is the adiabatic perturbation discussed previously; it is a perturbation along the late time classical trajectories of the scalar fields during inflation. When the primordial perturbations enter our horizon they perturb the matter density with a generic \textit{adiabatic condition}, which is satisfied when the density contrast of the individual species
is related to the total density contrast $\delta_k$

$$\frac{1}{3} \delta_{kk} = \frac{1}{3} \delta_{kc} = \frac{1}{4} \delta_{k\nu} = \frac{1}{4} \delta_{k\gamma} = \frac{1}{4} \delta_k,$$

(123)

where $b$ stands for baryons, $c$ for cold dark matter, $\gamma$ for photons and $\nu$ for neutrinos.

The other type is the isocurvature perturbation. During inflation this can be viewed as a perturbation orthogonal to the unique late time classical trajectory. Therefore, if there were $N$ fluctuating scalar fields during inflation, there would be $N - 1$ degrees of freedom which would contribute to the isocurvature perturbation.

The *isocurvature condition* is known as $\delta \rho = 0$: the sum total of all the energy contrasts must be zero. The most general density perturbations is then given by a linear combination of an adiabatic and an isocurvature density perturbations.

### 3.4.2 Adiabatic perturbations due to multi-field

In a comoving gauge Eq. (97) with $\mathcal{R} = -H \delta \phi / \dot{\phi}$ holds good even for multi-field inflation models, provided we identify each field component of $\phi$ along the slow roll direction. There also exists a relationship between the comoving curvature perturbations and the number of e-foldings $N$ [?, ?, ?, ?]

$$\mathcal{R} = \delta N = \frac{\partial N}{\partial \phi_a} \delta \phi_a,$$

(124)

where $N$ is measured by a comoving observer while passing from flat hypersurface (which defines $\delta \phi$) to the comoving hypersurface (which determines $\mathcal{R}$) [?, ?]. The repeated indices are summed over and the subscript $a$ denotes a component of the inflaton. A more intuitive derivation has been given in [?, ?].

If again one assumes that the perturbations in $\delta \phi_a$ have random phases with an amplitude $(H/2\pi)^2$, one obtains

$$\delta_k^2 = \frac{V}{75\pi^2} \frac{\partial N}{\partial \phi_a} \frac{\partial N}{\partial \phi_a}.$$

(125)

For a single component $\partial N / \partial \phi \equiv (M_p^{-2}V/V')$, and then Eq. (125) reduces to Eq. (109). By using slow roll equations we can again define the spectral index

$$n - 1 = -\frac{M_p^2 V_a V_a}{V^2} - \frac{2}{M_p^2 N_a N_a} + 2 \frac{M_p^2 N_a N_b N_c N_c V_{ab}}{V N_c N_c},$$

(126)
where \( V_a = \partial V / \partial \phi_a \), and similarly \( N_a = \partial N / \partial \phi_a \). For a single component we recover Eq. (115) from Eq. (126). These results prove useful in constraining the AD potential by cosmological density perturbations, as will be discussed in Sect. 5.3.

### 3.4.3 Isocurvature perturbations and CMB

One may of course simply assume a purely isocurvature initial condition. For any species the entropy perturbation is defined by

\[
S_i = \frac{\delta n_i}{n_i} - \frac{\delta n_\gamma}{n_\gamma},
\]

(127)

Thus, if initially there is a radiation bath with a common radiation density contrast \( \delta_r \), a baryon-density contrast \( \delta_b = 3\delta_r / 4 \), and a CDM density contrast \( \delta_c \), then

\[
S = \delta_c - \frac{3}{4} \delta_r = \frac{\rho_r \delta \rho_c - (3/4) \rho_c \delta \rho_r}{\rho_r \rho_c} = \frac{\rho_r + (3/4) \rho_c}{\rho_r \rho_c} \delta \rho_c \approx \delta_c,
\]

(128)

where we have used the isocurvature condition \( \delta \rho_r + \delta \rho_c = 0 \), and the last equality holds in a radiation dominated Universe.

However, a pure isocurvature perturbation gives five times larger contribution to the Sachs-Wolfe effect compared to the adiabatic case \([?, ?, ?]\). This result can be derived very easily in a matter dominated era with an isocurvature condition \( \delta \rho_c = -\delta \rho_r \), which gives a contribution \( R_k = (1/3) S_k \). Therefore from Eqs. (102,105), we obtain \( \Delta T_k / T = -S_k / 15 \). There is an additional contribution from radiation because we are in a matter dominated era, see Eq. (128), \( S \approx \delta_c \equiv -(3/4) \delta_r \). The sum total isocurvature perturbation \( \Delta T_k / T = -S / 15 - S / 3 = -6S / 15 \), where \( S \) is measured on the last scattering surface. The Sachs-Wolfe effect for isocurvature perturbations fixes the slope of the perturbations, rather than the amplitude \([?, ?, ?]\). Present CMB data rules out pure isocurvature perturbation spectrum \([?, ?]\), although a mixture of adiabatic and isocurvature perturbations remains a possibility \([?, ?, ?, ?]\). In the latter case it has been argued that the adiabatic and isocurvature perturbations might naturally turn out to be correlated \([?, ?]\). The most general power spectrum is not a single function but a \( 5 \times 5 \) matrix, which contains all possible adiabatic and isocurvature perturbations together with their cross-correlations. As discussed in \([?]\), resolving the
perturbation spectrum in all its generality would be an observational challenge that probably would have to wait for the determination of the polarization spectrum by the Planck Surveyor Mission.

It is sometimes useful to consider the ratio $\alpha$ of the total power spectra, defined as $P_{\text{tot}} = P_{\text{ad}} + P_{\text{iso}}$, where $\alpha$ is defined as

$$\alpha = \frac{16 P_{\text{iso}}}{25 P_{\text{ad}}} \Bigg|_{k=aH} = \frac{\delta_\gamma^i}{\delta_\gamma^a}.$$  \hspace{1cm} (129)

where $\delta_\gamma^i$ is the perturbation in the photon energy density due to isocurvature perturbations and $\delta_\gamma^a$ is the perturbation due to adiabatic perturbations.

### 3.5 Inflation models

A detailed account on inflation model building can be found in many reviews [??, ??, ??, ??]. Here we briefly recall some of the popular models with a particular emphasis on supersymmetric inflation. First we recapitulate some aspects of non-supersymmetric models.

#### 3.5.1 Non-supersymmetric inflation

The very first attempt to build an inflation model was made in [??], where one loop quantum correction to the energy momentum tensor due to the space-time curvature were taken into account, resulting in terms of higher order in curvature invariants. Such corrections to the Einstein equation admit a de Sitter solution [??], which was presented in [??, ??]. Inflation in Einstein gravity with an additional $R^2$ term was considered in [??] (for a discussion of inflation in pure $R^2$ gravity, see [??]). Such a theory is conformally equivalent to a theory with a canonical gravity [??] with a scalar field having a potential term. A similar situation arises in theories with a variable Planck mass, i.e., in scalar tensor theories [??]. Inflation in these models has been studied extensively [??].

The simplest single field inflation model is arguably chaotic inflation [??, ??] with a generic potential

$$V = \frac{\lambda}{M_P^{4-\alpha}} \phi^\alpha,$$  \hspace{1cm} (130)
where $\alpha$ is a positive even integer. In chaotic inflation slow roll takes place for $\phi \gg M_P$, and the two slow roll parameters are given by [?] 

$$\epsilon \equiv \frac{\alpha^2 M_P^2}{2 \phi^2}, \quad \eta = \alpha(\alpha - 1) \frac{M_P^2}{\phi^2}. \quad (131)$$

Inflation ends when $\epsilon \equiv 1$, or, $\phi \approx \alpha M_P$. The cosmological scales leave the horizon when $\phi = \sqrt{2N} \alpha M_P$, and the spectral indices for scalar and tensor perturbations turn out to be [?]

$$n = 1 - \frac{2 + \alpha}{2N}, \quad r = \frac{3.1 \alpha}{N}. \quad (132)$$

The amplitude of the density perturbations, if normalized at the COBE scale, yields the constraint $\lambda \simeq 4 \times 10^{-14}$.

An exponential potential, such as might arise in string theories and theories with extra dimensions,

$$V(\phi) = V_0 \exp \left( - \sqrt{2} \frac{\phi}{p M_P} \right). \quad (133)$$

would give rise to a power law $a(t) \propto t^p$ for the scale factor, so that inflation occurs when $p > 1$. Multiple exponentials with differing slopes give rise to what has been dubbed as assisted inflation [?].

### 3.5.2 F-term inflation

In four dimensions the $N = 1$ supersymmetric potential receives two contributions: one from the F-term, which is related to the chiral supermultiplets, and the second from the D-term, which contains the gauge interactions. For a detailed discussion of supersymmetric inflation we refer to the review by Lyth and Riotto [?]. Here we give a brief resume of the two types of inflation.

Historically, supersymmetric inflation was first introduced to cure some of the problems associated with the fine tuning of new inflation [?], but since then utilizing supersymmetry as a tool for inflation has gained in popularity (we describe supersymmetry in Sect. 4.2, and for supergravity, see Sect. 4.5.2.). The F-term potential can be derived from the superpotential $W$

$$V(\phi, \phi^*) = F^{*i} F_i, \quad F_i = - \left( \frac{\partial W}{\partial \phi_i} \right)^*, \quad (134)$$
where for renormalizable interactions $W$ has a mass dimension three.

Supersymmetry is broken whenever $|F|^2 \neq 0$. A simple working example is to consider the superpotential

$$W = \lambda S(\phi^2 - \phi_0^2)$$

which is invariant under a global $R$ symmetry with the superfields $S$ and $N$ carrying respectively the $R$ charges 1 and 0. The scalar components of these superfields can be written in the form

$$S = \frac{\sigma}{\sqrt{2}}, \quad \phi = \frac{\phi_1 + i\phi_2}{\sqrt{2}},$$

where we have used $R$-transformation in order to make $S$ real. The potential follows from Eq. (134):

$$V = \lambda^2 \phi_0^4 - \lambda^2 \phi_0^2(\phi_1^2 - \phi_2^2) + \frac{\lambda^2}{4}(\phi_1^2 + \phi_2^2)^2 + \lambda^2 \sigma^2(\phi_1^2 + \phi_2^2).$$

The supersymmetric vacuum is located at $\sigma = 0$, $\phi_1 = \phi_0$, and $\phi_2 = 0$. Note that the potential has a flat direction along $\sigma$-axis when $\sigma > \sigma_{\text{inst}} = \phi_0$. When $\sigma < \sigma_{\text{inst}}$, the mass squared of $\phi_1$ becomes negative and suggests a phase transition along the $\phi_1$ direction. When this happens $\sigma$, $\phi_1$, and $\phi_2$ begin to oscillate around their supersymmetry preserving vacua. If $\phi_1 = \phi_2 = 0$, the height of the potential is given by $V = \lambda^2 \phi_0^4$, and as a consequence one obtains a period of inflation. This is the simplest example of a flat direction giving rise to an inflation potential, and it is known as the hybrid model, first described in a non-supersymmetric context in [?, ?] and in a supersymmetric context in [?].

In order to have a graceful exit from inflation one requires a slope for the flat direction such that $\sigma$ can roll down and approach $\sigma_{\text{inst}}$. The flatness of the potential can be lifted in two ways: by radiative corrections [?], or by the low energy soft supersymmetry breaking effects.

Due to supersymmetry breaking the fermions obtain a mass of the order $(\partial^2 W/\partial \phi^2) = \lambda S$, while the two complex scalars receive a mass squared $\lambda^2 S^2 \pm \lambda^2 \phi_0^2$. The one-loop radiative correction to the potential is given by [?]

$$\delta V = \frac{1}{64\pi^2} \sum_i (-)^f_i M_i^4 \ln \frac{M_i^2}{M^2},$$

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where \( f_i \) denotes the number of fermions, \( M_i^2 \) is the fermion mass squared, and \( M \) the cut-off or the renormalization scale. The summation should be taken over all helicity states \( i \). In the present example the effective potential along the flat direction is given by

\[
V = \lambda^2 \phi_0^4 \left( 1 + \frac{C\lambda^2}{8\pi^2} \ln \frac{\sigma}{\sqrt{2}M} \right),
\]

(139)

where \( C \) is a constant essentially counting the states running in the loops. If the loop correction dominates over the tree level potential, there is a period of inflation which typically ends when

\[
\sigma = \lambda \sqrt{\frac{CN}{4\pi^2} M_P},
\]

(140)

The COBE normalization sets the scale of inflation to

\[
V^{1/4} \sim \left( \frac{50}{N} \right)^{1/4} C^{1/4} \lambda \times 10^{15} \text{ GeV}
\]

(141)

while the spectral index is given by

\[
n = 1 - \frac{1}{N} \left( 1 + \frac{3C\lambda^2}{16\pi^2} \right).
\]

(142)

Therefore, depending on the coupling constant \( \lambda \) and the number of e-foldings \( N \), it is possible to have a wide range of inflation energy scales which all provide a spectral index \( n \sim 0.96 - 0.98 \).

Soft supersymmetry breaking contributions induce \( m_\sigma \sim \mathcal{O}(\text{TeV}) \). One could also imagine that the mass of \( \sigma \) appears dynamically if \( \sigma \) has couplings to bosons and fermions; these may induce a typical running mass \( \propto \sigma^2 \ln(\sigma/M) \) [?, ?, ?].

### 3.5.3 D-term inflation

In the above discussion we have neglected the gauge contribution. The D-term \( D^a = -g_a(\phi^* T^a \phi) \) gives rise to a scalar potential (see [?, ?])

\[
V(\phi, \phi^*) = \frac{1}{2} \sum D^a D_a
\]

(143)

where \((T^a)_i^j\) satisfy \([T^a, T^b] = i f^{abc} T^c\) (\( f^{abc} \) is the structure constant).

The simplest realization of D-term inflation reproduces the hybrid potential with three chiral superfields, \( S, \phi_+ \), and \( \phi_- \) with (non-anomalous) \( U(1) \) charges 0, +1, −1
The superpotential can be written as

$$W = \lambda S \phi_+ \phi_-.$$  \hfill (144)

The scalar potential then reads [7]

$$V = \lambda^2 |S|^2 \left( |\phi_+|^2 + |\phi_-|^2 \right) + \lambda^2 |\phi_+ \phi_-|^2 + \frac{g^2}{2} \left( |\phi_+|^2 - |\phi_-|^2 + \xi^2 \right)^2, \hfill (145)$$

where $g$ is the gauge coupling and $\xi$ is the Fayet-Iliopoulos D-term. Note that the potential allows unique supersymmetry preserving vacua with a broken gauge symmetry $S = \phi_+ = 0$, and $\phi_- = \xi$. By virtue of the coupling, when $|S| > S_{\text{inst}} = g \xi / \lambda$, the fields $\phi_+, \phi_- \to 0$, and therefore inflation occurs because of the Fayet-Iliopoulos D-term $V = g^2 \xi^4 / 2$. The slope along the inflaton direction $S$ can be generated by the one-loop contribution and reads

$$V = \frac{g^2 \xi^4}{2} \left( 1 + \frac{g^2}{16\pi^2} \ln \frac{\lambda^2 |S|^2}{M_P^2} \right). \hfill (146)$$

Inflation ends when slow roll condition breaks down for $S \sim (g/2\pi \sqrt{2}) M_P$, and the predictions for the inflationary parameters are similar to the previous discussion. D-term inflation based on an anomalous $U(1)$ symmetry (which could appear in string theory [7]) is no different.

Hybrid inflation is successful but has also problems that are related to the initial conditions. In [7], it was pointed out that hybrid inflation requires an extremely homogeneous field configuration for the fields orthogonal to the inflaton. In our example the orthogonal fields to the inflaton must be set to zero with a high accuracy over a region much larger than the initial size of the horizon. It is possible to solve this impasse by having a pre-inflationary matter dominated phase when the field orthogonal to the inflaton direction oscillates and decays into lighter degrees of freedom, gradually settling down to the bottom of its potential [7].

### 3.5.4 Supergravity corrections

When the field values are close to the Planck scale, supergravity (SUGRA) effects become important and may ruin the flatness of the inflaton potential. The soft breaking
mass of the scalar fields are typically $[^?,^?,^?,^?,^?]$

$$m_{soft}^2 \sim \frac{V}{3M_p^2} \sim \mathcal{O}(1)H^2.$$  \hspace{1cm} (147)

Once the inflaton gains a mass $\sim H$, the field simply rolls down to the minimum of the potential and inflation stops. Indeed, in SUGRA the slow roll parameter

$$|\eta| \equiv \frac{M_p^2 V''}{V} \sim \frac{m_{SUGRA}^2}{H^2} \sim \mathcal{O}(1),$$  \hspace{1cm} (148)

where $m_{SUGRA}^2 \approx m_{SUSY}^2 + (V_{SUSY}/3M_p^2) \sim m_{SUSY}^2 + \mathcal{O}(1)H^2$. Note that the latter contribution dominates in an expanding Universe and violates the slow roll condition. For field values smaller than Planck scale it is always possible to obtain $\epsilon \ll 1$, but in supergravity $\eta$ can never be made less than one for a single chiral field with a minimal kinetic term. This is known as the $\eta$ problem in SUGRA models of inflation $[^?]$.

When there are more than one chiral superfields, it might be possible to cancel the dominant $\mathcal{O}(1)H$ correction to the inflaton mass by choosing an appropriate Kähler term $[^?,^?]$ (see also discussion in $[^?]$). In hybrid inflation models derived from an F-term the dominant $\mathcal{O}(1)H$ correction in the mass term can be canceled if $|N| = 0$ exactly, which however seems to lead to an initial condition problem, as discussed above. The fact that the superpotential is linear in $S$ in Eqs. (135,144) guarantees the cancellation of the dominant contribution in the mass term for a minimal Kähler term $\sim |S|^2$. For non-minimal Kähler potential such as $K = |S|^2 + \beta |S|^4/M_p^2 + ...$, one obtains $(\partial^2 K/\partial S\partial S^*)^{-1} \sim 1 - 4\beta |S|^2/M_p^2$. These contributions again lead to a problematic $\beta \times \mathcal{O}(1)H$ contribution to the inflaton mass unless the value of the unknown constant $\beta$ is suppressed.

In $[^?]$, it was shown that the $\eta$ problem does not appear for D-term inflation even for the minimal Kähler potential because the main contribution to the inflation potential does not come from the vev of the inflaton field alone, but from the Fayet-Iliopoulos term which belongs to the D-sector of the potential. Based on this fact many D-term inflation models have been written down $[^?,^?]$. Therefore, hybrid inflation, whether realized as an effective potential coming from F-sector or from D-sector, appears to be among the most promising models for supersymmetric inflation.
3.6 Reheating of the Universe

3.6.1 Perturbative inflaton decay

Traditionally reheating has been assumed to be a consequence of the perturbative decay of the inflaton \([? , ? , ?]\). After the end of inflation, when \(H \leq m_\phi\), the inflaton field oscillates about the minimum of the potential. Averaging over one oscillation results in \([?]\) pressureless equation of state where \(\langle p \rangle = \langle \dot{\phi}^2 / 2 - V(\phi) \rangle\) vanishes\(^2\), so that the energy density redshifts as during matter domination with \(\rho_\phi = \rho_i(a_i / a)^3\) (subscript \(i\) denotes the quantities right after the end of inflation). If \(\Gamma_\phi\) represents the decay width of the inflaton to a pair of fermions, then the inflaton decays when \(H(a) = \sqrt{(1/3M_P^2)\rho_i(a_i / a)^{3/2}} \approx \Gamma_\phi\). When the inflaton decays, it releases its energy into the thermal bath of relativistic particles whose energy density is determined by the reheat temperature \(T_{rh}\), given by

\[
T_{rh} = \left( \frac{90}{\pi^2 g_*} \right)^{1/4} \sqrt{\Gamma_\phi M_P} = 0.3 \left( \frac{200}{g_*} \right)^{1/4} \sqrt{\Gamma_\phi M_P} . \tag{149}
\]

However, the inflaton might not decay instantaneously. In such a case there might already exist a thermal plasma of some relativistic species at a temperature higher than the reheat temperature already before the end of reheating \([?]\). If the inflaton decays with a rate \(\Gamma_\phi\), then the instantaneous plasma temperature is found to be \([?]\)

\[
T_{inst} \sim \left( g_*^{-1/2} H \Gamma_\phi M_P^2 \right)^{1/4} , \tag{150}
\]

where \(g_*\) denotes the effective relativistic degrees of freedom in the plasma. The temperature reaches its maximum \(T_{max}\) soon after the inflaton field starts oscillating around the minimum. Once the maximum temperature is reached, then \(\rho_\psi \sim a^{-3/2}\), and \(T \sim a^{-3/8}\) until reheating and thermalization is completely over \([? , ? , ? , ?]\).

The process of thermalization has two aspects; achieving kinetic equilibrium, and achieving chemical equilibrium. Kinetic equilibrium can be reached by \(2 \rightarrow 2\) scattering and annihilation. For chemical equilibrium one requires particle number changing interactions such as \(2 \rightarrow 3\) processes. In \([?]\), soft processes which allow for small

\(^2\)This will be discussed in a more detail in Sect. 5.8.
momentum transfer with a larger cross-section have been advocated for chemical equi-
libration, while in [?], hard processes have been invoked. Therefore, depending on
the interactions, thermalization time scale could be short, such as in the case of soft
scattering processes, or, it could be long compared to the Hubble time if only hard
processes are operative. Recently it has been argued [?, ?], that thermalization time
scale can be as long as the time it takes for the inflaton decay products with typical
energies $O(m_{\phi})$ to lose the energy $\sim (m_{\phi} - T_{rh})$. The main conclusion is that inelastic
scattering interactions $2 \rightarrow 3$ can thermalize the Universe faster compared to elastic
interaction $2 \rightarrow 2$. Inelastic interactions can achieve the kinetic and chemical equi-
librium both, and therefore, $\Gamma_{inel}^{-1}$ could be considered as the true thermalization time
scale.

3.6.2 Non-perturbative inflaton decay

Much effort has lately been devoted to non-perturbative effects which are essentially
non-thermal. These may lead to a rapid transfer of the inflaton energy to other degrees
of freedom by the process known as preheating. The requirement is that the inflaton
quanta couple to other (essentially massless) fields $\chi$ through e.g. terms like $\phi^2 \chi^2$.
The quantum modes of $\chi$ may then be excited during the inflaton oscillations via a
parametric resonance. Preheating has been treated both analytically [?, ?, ?, ?, ?, ?,
?, ?], and on lattice [?].

Like bosons, fermions can also be excited through preheating [?, ?, ?, ?]. In fact
it has been argued that fermionic preheating is perhaps more effective than bosonic
preheating [?, ?, ?]. However, note that supersymmetry is effectively broken during
the inflaton oscillations [?, ?, ?]. As a consequence, one naturally expects corrections
to the inflaton potential during the oscillations [?]. Therefore, in most supersymmetric
models of inflation preheating might not turn out to be very relevant.

3.6.3 Gravitino and inflatino problems

The reheat temperature should certainly be above the BBN temperature $T \geq O(1)$ MeV,
but there also exists an upper bound from gravitino overproduction. In supergravity
the superpartner of the graviton is a spin-3/2 gravitino, which gets a mass from the super-Higgs mechanism [?] when supersymmetry is spontaneously broken. Typically supergravity is broken in a hidden sector by some non-perturbative dynamics. Supersymmetry breaking is then mediated via gravitational (or possibly other) couplings to the observable sector in such a way that sfermions and gauginos get masses of order electroweak scale [?, ?]. In addition, the gravitino also gets a mass which in the simplest gravity mediated models is of order 1 TeV [?] (see Sect. 4.4).

If the gravitino is not the lightest supersymmetric particle (LSP), it will decay. Gravitino has two helicity states ±3/2 and ±1/2. The latter one is mainly the goldstino mode which is eaten by the super-Higgs mechanism. The goldstino coupling strength is inversely proportional to the momentum, so that at low energies the gravitino coupling is mainly dictated by the goldstino mode [?]. At temperatures much above the sparticle masses, it is the massless ±3/2 mode that governs the gravitino interactions. The helicity ±3/2 mode can decay into gauge bosons and gauginos through a dimension 5-operator with a lifetime

\[ \tau_{3/2 \rightarrow A_{\mu} \lambda} \approx \frac{4M_P^2}{m_{3/2}^2}. \]  

Typically \( \tau \sim 10^2 - 10^5 \) s for a gravitino mass in the range \( 10 \text{ TeV} \leq m_{3/2} \leq 100 \text{ GeV} \).

Although the gravitino interactions with matter are suppressed by the Planck mass, they can be generated in great abundances very close to the Planck scale [?]. Inflation would dilute their number density [?], but during reheating they would be regenerated though scattering of gauge and gaugino quanta, with adverse consequences [?, ?, ?, ?, ?, ?, ?]. The resulting gravitino abundance has been estimated to be [?]

\[ \frac{n_{3/2}}{s} \approx 2.4 \times 10^{-13} \left( \frac{T_{rh}}{10^9 \text{ GeV}} \right) \left[ 1 - 0.018 \ln \left( \frac{T_{rh}}{10^9 \text{ GeV}} \right) \right], \]  

where \( s \) defines the entropy density and \( T_{rh} \) denotes the reheating temperature of the Universe. The abundance Eq. (152) could well be increased by an order of magnitude if gravitino interactions with other chiral multiplets are included [?]. In [?] it was argued that at finite temperatures gravitino overproduction could be enhanced, but the calculation was criticized in [?, ?]; for a recent discussion on this topic, see [?].

Since the gravitino is a late decaying particle, BBN yields a restriction on the
reheat temperature [? , ?]. For instance, gravitino decay products can enhance the abundance of $D + ^3 He$ due to photo fission of $^4 He$ which implies [?]

$$\frac{m_{3/2}}{s} \leq (10^{-14} - 10^{-11}) \Rightarrow T_{rh} \leq (10^7 - 10^{10}) \text{ GeV, } 100 \text{ GeV} \leq m_{3/2} \leq 10 \text{ TeV}. \quad (153)$$

The constraint on the reheating temperature is [?]

$$T_{rh} \leq 2.5 \times 10^8 \left(\frac{m_{3/2}}{100 \text{ GeV}}\right)^{-1} \text{ GeV} , \quad (154)$$

for $m_{3/2} \leq 1.6 \text{ TeV}$.

In gauge mediated supersymmetry breaking scenarios, to be discussed in Sect. 4.4., the gravitino can have a very light mass $\sim 10^{-6} \text{ GeV}$ [?] and can be a hot dark matter candidate [?]. Small (or large) gravitino masses can also be obtained in SUGRA models with non-minimal Kähler terms, such as the no-scale model [?]. In anomaly mediation the gravitino mass is large with $m_{3/2} \sim m_{soft}/\alpha \gg m_{soft}$ [?]. In general, if the gravitino is not LSP and heavier than 10 TeV, it decays before nucleosynthesis and thus does not cause any cosmological problems [?].

Gravitinos could also be produced by non-perturbative processes, as was first described in [?], where the formalism for exciting the helicity $\pm 3/2$ component of the gravitino was developed. Later the production of the helicity $\pm 1/2$ state, which for a single chiral multiplet is the superpartner of the inflaton known as inflatino, has been studied by several authors [? , ? , ? , ? , ?]. The decay channels of the inflatino have been first discussed in [? , ?]. It has been suggested [?] and also explicitly shown [?] that in realistic models with several chiral multiplets, the helicity $\pm 1/2$ gravitino production is not a problem for nucleosynthesis as long as the inflationary scale is sufficiently higher than the scale of supersymmetry breaking in the hidden sector and the two sectors are gravitationally coupled. A very late decay of inflatino could however be possible, as argued in [? , ?]. In [?], it was argued that if the inflatino and gravitino were not LSP, then late off-shell inflatino and gravitino mediated decays of heavy relics could be significant.
4 Flat directions

4.1 Degenerate vacua

At the level of renormalizable terms, supersymmetric field theories generically have infinitely degenerate vacua. This is a consequence of the supersymmetry and the gauge symmetries (and discrete symmetries such as $R$-parity) of the Lagrangian, which allow for certain types of interaction terms only. Therefore, in general there are a number of directions in the space of scalar fields, collectively called the moduli space, where the scalar potential is identically zero. In low energy supersymmetric theories such classical degeneracy is accidental and is protected from perturbative quantum corrections by a non-renormalization theorem [?]. In principle the degeneracies could be lifted by non-perturbative effects. However, such effects are likely to be suppressed exponentially and thus unimportant because all the couplings of low energy theories are typically weak even at relatively large vevs. Therefore in the supersymmetric limit when $M_p \to \infty$, the potential for the flat direction always vanishes.

In the MSSM the moduli fields are quark, lepton and Higgs chiral fields. In string theories there are often additional moduli fields associated with the conformal field theory degrees of freedom and world sheet discreet $R$-symmetries [?]. The moduli space of string theory can also be lifted by a soft supersymmetry breaking masses of the order of the gravitino mass $m_{3/2}$. Since the moduli interactions with others fields are usually Planck mass suppressed, the string moduli are also a cause for worry because they may decay after nucleosynthesis. This problem has been dubbed as the moduli problem [?, ?]. However, the MSSM flat directions are made up of condensates of squarks, Higgses, and sleptons, and can evaporate much before nucleosynthesis.

However, there is an effective potential for the flat direction condensate fields which arises as a result of supersymmetry breaking terms and higher dimensional operators in the superpotential. In this sense the MSSM flat directions are only approximately flat at vevs larger than the supersymmetry breaking scale.
4.2 MSSM and its potential

Let us remind the reader that the matter fields of MSSM are chiral superfields $\Phi = \phi + \sqrt{2} \theta \bar{\psi} + \theta \bar{\theta} F$, which describe a scalar $\phi$, a fermion $\psi$ and a scalar auxiliary field $F$. In addition to the usual quark and lepton superfields, MSSM has two Higgs fields, $H_u$ and $H_d$. Two Higgses are needed because $H^\dagger$, which in the Standard Model gives masses to the $u$-quarks, is forbidden in the superpotential.

The superpotential for the MSSM is given by [?]

$$W_{MSSM} = \lambda_u Q H_u \bar{u} + \lambda_d Q H_d \bar{d} + \lambda_e L H_d \bar{e} + \mu H_u H_d,$$  \hspace{1cm} (155)

where $H_u, H_d, Q, L, \bar{u}, \bar{d}, \bar{e}$ in Eq. (155) are chiral superfields, and the dimensionless Yukawa couplings $\lambda_u, \lambda_d, \lambda_e$ are $3 \times 3$ matrices in the family space. We have suppressed the gauge and family indices. Unbarred fields are $SU(2)$ doublets, barred fields $SU(2)$ singlets. The last term is the $\mu$ term, which is a supersymmetric version of the SM Higgs boson mass. Terms proportional to $H_u^* H_u$ or $H_d^* H_d$ are forbidden in the superpotential, since $W_{MSSM}$ must be analytic in the chiral fields. $H_u$ and $H_d$ are required not only because they give masses to all the quarks and leptons, but also for the cancellation of gauge anomalies. The Yukawa matrices determine the masses and CKM mixing angles of the ordinary quarks and leptons through the neutral components of $H_u = (H_u^+, H_u^0)$ and $H_d = (H_d^0, H_d^-)$. Since the top quark, bottom quark and tau lepton are the heaviest fermions in the SM, we assume that only the $(3, 3)$ element of the matrices $\lambda_u, \lambda_d, \lambda_e$ are important. In this limit only the third family and the Higgs fields contribute to the MSSM superpotential.

The SUSY scalar potential $V$ is the sum of the F- and D-terms and reads

$$V = \sum_i |F_i|^2 + \frac{1}{2} \sum_a g_a^2 D^a D^a$$  \hspace{1cm} (156)

where

$$F_i \equiv \frac{\partial W_{MSSM}}{\partial \phi_i}, \quad D^a = \phi^\dagger T^a \phi.$$  \hspace{1cm} (157)

Here we have assumed that $\phi_i$ transforms under a gauge group $G$ with the generators of the Lie algebra given by $T^a$. 

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The \( \mu \) term provides masses to the Higgsinos
\[
L \supset -\mu (\tilde{H}_u^+ \tilde{H}_d^- - \tilde{H}_u^0 \tilde{H}_d^0) + c.c.,
\]
and contributes to the Higgs \((mass)^2\) terms in the scalar potential through
\[
-L \supset V \supset |\mu|^2 (|H_u^0|^2 + |H_u^+|^2 + |H_d^0|^2 + |H_d^-|^2) .
\]

Note that Eq. (159) is positive definite. Therefore, it cannot lead to electroweak symmetry breaking without including supersymmetry breaking \((mass)^2\) soft terms for the Higgs fields, which can be negative. Hence, \(|\mu|^2\) should almost cancel the negative soft \((mass)^2\) term in order to allow for a Higgs vev of order \(\sim 174\) GeV. That the two different sources of masses should be precisely of same order is a puzzle for which many solutions has been suggested [?, ?, ?, ?].

Note also that Eq. (155) is the minimal superpotential because we have not included terms which are gauge invariant and analytic in the chiral superfields but which violate either baryon number \(B\) or lepton number \(L\). The most general gauge invariant and renormalizable superpotential would not only include Eq. (155), but also the terms
\[
W_{\Delta L=1} = \frac{1}{2} \lambda^{ijk} L_i L_j \tilde{e}_k + \lambda'^{ijk} L_i Q_j \tilde{d}_k + \mu'^i L_i H_\mu,
\]
\[
W_{\Delta B=1} = \frac{1}{2} \lambda''^{ijk} \tilde{u}_i \tilde{d}_j \tilde{d}_k,
\]
where \(i = 1, 2, 3\) represents the family indices. The chiral supermultiplets carry baryon number assignments \(B = +1/3\) for \(Q_i\), \(B = -1/3\) for \(\tilde{u}_i, \tilde{d}_i\), and \(B = 0\) for all others. The total lepton number assignments are \(L = +1\) for \(L_i\), \(L = -1\) for \(\tilde{e}_i\), and \(L = 0\) for all the others. The terms in Eq. (160) violate lepton number by one unit, while those in Eq. (161) violate baryon number by one unit.

Unless \(\lambda'\) and \(\lambda''\) terms are very much suppressed, one would obtain rapid proton decay which violates both \(B\) and \(L\) by one unit. Many other processes also give rise to violation in baryon and lepton number (for a review, see [?]). Therefore, there must be a symmetry forbidding the terms in Eqs. (160,161), while allowing for the terms in Eq. (155). The symmetry is known as \(R\)-parity [?], which is a discrete parity defined for each particle as
\[
P_R = (-1)^{3(B-L)+2s}
\]
with $P_R = +1$ for the SM particles and the Higgs bosons, while $P_R = -1$ for all the sleptons, squarks, gauginos, and Higgsinos. Here $s$ is spin of the particle. Without the product $(-1)^{2s}$, the expression is known as matter parity [?], and denoted by $P_M = (-1)^{3(B-L)}$. The quantity $(-1)^{2s}$ is equal to 1 whenever conservation of angular momentum holds at a given vertex. In this case matter parity and $R$-parity are equivalent. If $R$-parity is conserved then there will be no mixing between the sparticles and the ones which have $P_R = +1$. This completely forbids potentially dangerous terms in Eqs. (160,161).

Matter parity is actually a discrete subgroup of the continuous $U(1)_{B-L}$ group. Therefore, if a gauged $U(1)_{B-L}$ is broken by scalar vevs which carry even integer values of $3(B - L)$, then $P_M$ survives as an exactly conserved discrete remnant [?]. Besides forbidding $B$ and $L$ violation from the renormalizable interactions, $R$-parity has interesting phenomenological and cosmological consequences. The lightest sparticle with $P_R = -1$, the LSP, must be absolutely stable. If electrically neutral, the LSP is a natural candidate for non-baryonic dark matter [?, ?]. It may be possible to produce LSPs in a next generation collider experiments.

### 4.2.1 F-and D-renormalizable flat directions of MSSM

For a general supersymmetric model with $N$ chiral superfields $X_i$, it is possible to find out the directions where the potential Eq. (156) vanishes identically by solving simultaneously

$$D^a \equiv X^\dagger T^a X = 0, \quad F_{X_i} \equiv \frac{\partial W}{\partial X_i} = 0.$$  \hfill (163)

Field configurations obeying Eq. (163) are called respectively D-flat and F-flat.

D-flat directions are parameterized by gauge invariant monomials of the chiral superfields. A powerful tool for finding the flat directions has been developed in [?, ?, ?, ?, ?, ?], where the correspondence between gauge invariance and flat directions has been employed. The configuration space of the scalar fields of the MSSM contains 49 complex dimensions (18 for $Q_i$, 9 each for $\bar{u}_i$ and $\bar{d}_i$, 6 for $L_i$, 3 for $\bar{e}_i$, and 2 each for $H_u$ and $H_d$), out of which there are 12 real D-term constraints (8 for $SU(3)_C$, 3 for $SU(2)_L$, and 1 for $U(1)_Y$), which leaves a total of 37 complex dimensions [?, ?].
The trick is to construct gauge invariant monomials forming $SU(3)_C$ singlets and then using them as building blocks to generate $SU(3)_C \times SU(2)_L$, and subsequently the whole $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariant polynomials [?, ?]. However, these invariant monomials give only the D-flat directions. For F-flat directions, one must solve explicitly the constraint equations $F_{X_i} = 0$.

A single flat direction necessarily carries a global $U(1)$ quantum number, which corresponds to an invariance of the effective Lagrangian for the order parameter $\phi$ under phase rotation $\phi \to e^{i\theta}\phi$. In the MSSM the global $U(1)$ symmetry is $B - L$. For example, the $LH_u$-direction (see below) has $B - L = -1$.

A flat direction can be represented by a composite gauge invariant operator, $X_m$, formed from the product of $k$ chiral superfields $\Phi_i$ making up the flat direction: $X_m = \Phi_1\Phi_2 \cdots \Phi_m$. The scalar component of the superfield $X_m$ is related to the order parameter $\phi$ through $X_m = c\phi^m$.

### 4.2.2 An example of F-and D-flat direction

The flat directions in the MSSM are tabulated in Table 1. An example of a D-and F-flat direction is provided by

$$ H_u = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi \end{pmatrix}, \quad L = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi \\ 0 \end{pmatrix}, $$

where $\phi$ is a complex field parameterizing the flat direction, or the order parameter, or the AD field. All the other fields are set to zero. In terms of the composite gauge invariant operators, we would write $X_m = LH_u$ ($m = 2$).

From Eq. (164) one clearly obtains $F_{H_u}^* = \lambda_u Q\bar{u} + \mu H_d = F_L^* = \lambda_d H_d\bar{e} \equiv 0$ for all $\phi$. However, there exists a non-zero F-component given by $F_{H_d}^* = \mu H_u$. Since $\mu$ can not be much larger than the electroweak scale $M_W \sim \mathcal{O}(1)$ TeV, this contribution is of the same order as the soft supersymmetry breaking masses, which are going to lift the degeneracy. Therefore, following [?], one may nevertheless consider $LH_u$ to correspond to a F-flat direction.

The relevant D-terms read

$$ D_{SU(2)}^g = H_u^\dagger \tau_3 H_u + L^\dagger \tau_3 L = \frac{1}{2}|\phi|^2 - \frac{1}{2}|\phi|^2 \equiv 0. $$

(165)
Therefore the \( LH_u \) direction is also D-flat.

The only other direction involving the Higgs fields and thus soft terms of the order of \( \mu \) is \( H_u H_d \). The rest are purely leptonic, such as \( LL \bar{e} \), or baryonic, such as \( \bar{u} \bar{d} \bar{d} \), or mixtures of leptons and baryons, such as \( QQ \bar{u} \bar{e} \). These combinations give rise to several independent flat directions that can be obtained by permuting the flavor indices. For instance, \( LL \bar{e} \) contains the directions \( L_1 L_2 \bar{e}_3 \), \( L_2 L_3 \bar{e}_1 \), and \( L_1 L_3 \bar{e}_2 \).

Along a flat direction gauge symmetries get broken, with the gauge supermultiplets gaining mass by super-Higgs mechanism with \( m_g = g \langle \phi \rangle \). Several chiral supermultiplets typically become massive by virtue of Yukawa couplings in the superpotential; for example, in the \( LH_u \) direction one finds the mass terms \( W_{\text{mass}} = \lambda_u \langle \phi \rangle Q \bar{u} + \lambda_e \langle \phi \rangle H_d \bar{e} \).

Of course, there may simultaneously exist several flat directions. For the purpose of AD mechanism it is the lowest dimensional operator which determines the baryonic charge of the eventual condensate. In what follows we will therefore mostly consider a single flat direction.

### Table 1: Renormalizable F and D flat directions in the MSSM

<table>
<thead>
<tr>
<th>( \text{H} ) ( \text{D} )</th>
<th>( B - L )</th>
<th>( \text{F} ) ( \text{D} )</th>
<th>( B - L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_u H_d )</td>
<td>0</td>
<td>( LH_u )</td>
<td>-1</td>
</tr>
<tr>
<td>( \bar{u} \bar{d} \bar{d} )</td>
<td>-1</td>
<td>( QL \bar{d} )</td>
<td>-1</td>
</tr>
<tr>
<td>( LL \bar{e} )</td>
<td>-1</td>
<td>( QQ \bar{u} \bar{d} )</td>
<td>0</td>
</tr>
<tr>
<td>( QQQL )</td>
<td>0</td>
<td>( QL \bar{u} \bar{e} )</td>
<td>0</td>
</tr>
<tr>
<td>( \bar{u} \bar{u} \bar{e} )</td>
<td>0</td>
<td>( QQQQ \bar{u} )</td>
<td>1</td>
</tr>
<tr>
<td>( QQ \bar{u} \bar{u} \bar{e} )</td>
<td>1</td>
<td>( LL \bar{d} \bar{d} )</td>
<td>-3</td>
</tr>
<tr>
<td>( \bar{u} \bar{u} \bar{u} \bar{e} )</td>
<td>1</td>
<td>( QLQL \bar{d} \bar{d} )</td>
<td>-2</td>
</tr>
<tr>
<td>( QQLL \bar{d} \bar{d} )</td>
<td>-2</td>
<td>( \bar{u} \bar{u} \bar{d} \bar{d} )</td>
<td>-2</td>
</tr>
<tr>
<td>( QQQQ \bar{d} LL )</td>
<td>-1</td>
<td>( QLQL \bar{L} \bar{e} )</td>
<td>-1</td>
</tr>
<tr>
<td>( QL \bar{u} QQ \bar{d} )</td>
<td>-1</td>
<td>( \bar{u} \bar{u} \bar{d} \bar{d} \bar{e} )</td>
<td>-1</td>
</tr>
</tbody>
</table>
4.3 Lifting the flat direction

Vacuum degeneracy along a flat direction can be broken in two ways: by supersymmetry breaking, or by higher order non-renormalizable operators appearing in the effective low energy theory. Let us first consider the latter option. Supersymmetry breaking will then be discussed in more detail in Sects. 4.4 and 4.5.

4.3.1 Lifting by non-renormalizable operators

Non-renormalizable superpotential terms in the MSSM can be viewed as effective terms that arise after one integrates out fields with very large mass scales appearing in a more fundamental (say, string) theory. Here we do not concern ourselves with the possible restrictions on the effective terms due to discrete symmetries present in the fundamental theory, but assume that all operators consistent with symmetries may arise. Thus in terms of the invariant operators \( X_m \), one can have terms of the type [? , ?]

\[
W = \frac{h}{dM^{d-3}}X^k_m = \frac{h}{dM^{d-3}}\phi^d,
\]

where the dimensionality of the effective scalar operator \( d = mk \), and \( h \) is a coupling constant which could be complex with \( |h| \sim O(1) \). Here \( M \) is some large mass, typically of the order of the Planck mass or the string scale (in the heterotic case \( M \sim M_{GUT} \)). The lowest value of \( k \) is 1 or 2, depending on whether the flat direction is even or odd under \( R \)-parity.

A second type of term lifting the flat direction would be of the form [? , ?]

\[
W = \frac{h'}{M^{d-3}}\psi \phi^{d-1},
\]

where \( \psi \) is not contained in \( X_m \). The superpotential term Eq. (167) spoils \( F \)-flatness through \( F_{\psi} \neq 0 \). An example is provided by the direction \( \bar{u}_1 \bar{u}_2 \bar{u}_3 \bar{e}_1 \bar{e}_2 \), which is lifted by the non-renormalizable term \( W = (h'/M)\bar{u}_1 \bar{u}_2 \bar{d}_2 \bar{e}_1 \). This superpotential term gives a non-zero contribution \( F^*_{d_2} = (h'/M)\bar{u}_1 \bar{u}_2 \bar{e}_1 \sim \phi^3 \) along the flat direction.

Assuming minimal kinetic terms, both types discussed above in Eqs. (166,167) yield a generic non-renormalizable potential contribution that can be written as

\[
V(\phi) = \frac{|\lambda|^2}{M^{2d-6}}(\phi^* \phi)^{d-1},
\]
where we have defined the coupling $|\lambda|^2 \equiv |h|^2 + |h'|^2$. By virtue of an accidental $R$-symmetry under which $\phi$ has a charge $R = 2/d$, the potential Eq. (168) conserves the $U(1)$ symmetry carried by the flat direction, in spite of the fact that at the superpotential level it is violated, see Eqs. (166,167). The symmetry can be violated if there are multiple flat directions, or by higher order operator contributions. However, it turns out [?] that the $B – L$ violating terms are always subdominant. This is of importance for baryogenesis considerations, where the necessary $B – L$ violation should therefore arise from other sources.

The process of finding all the possible non-renormalizable superpotential contributions lifting a particular flat direction is similar to finding the D-flat directions discussed in Sect. 4.2.1. All the non-renormalizable operators can be generated from SM gauge monomials with $R$-parity constraint which allows only even number of odd matter parity fields ($Q, L, \bar{u}, \bar{d}, \bar{e}$) to be present in each superpotential term. At each dimension $d$, the various $F = 0$ constraints are separately imposed in order to construct the basis for monomials.

As an example, consider flat directions involving the Higgs fields such as $H_u H_d$ and $LH_u$ directions. Even though they are already lifted by the $\mu$ term, since $\mu$ is of the order of supersymmetry breaking scale, for cosmological purposes they can be considered flat, as was discussed in Sect. 4.2.2. At the $d = 4$ level the superpotential reads

$$W_4 \supset \frac{\lambda}{M} (H_u H_d)^2 + \frac{\lambda_{ij}}{M} (L_i H_u) (L_j H_u).$$

Let us assume $\lambda, \lambda_{ij} \neq 0$. Note that $F_{H_d} = 0$ constraint implies $\lambda H_u^a (H_u H_d) = 0$, which acts as a basis for the monomials. An additional constraint can be obtained by contracting $F_{H_d} = 0$ by $\epsilon_{\alpha\beta\gamma\delta} H^\beta_\gamma H^\delta_\delta$, which forms the polynomial $H_u H_d = 0$ in the same monomial basis. Similarly, the constraint $F_{H_u} = 0$, along with the contraction yields $\lambda^{ij} (L_i H_u) (L_j H_u) = 0$. This implies that $L_i H_u = 0$ for all $i$. Therefore, the two monomials $LH_u$ and $H_u H_d$ can be lifted by $d = 4$ terms in the superpotential Eq. (169).

The other renormalizable flat directions are $LLE, \bar{u}\bar{d}L, QQQL, Q\bar{u}Q\bar{d}, \bar{u}\bar{d}e$ and $Q\bar{u}L\bar{e}, \bar{d}\bar{d}LL, \bar{u}\bar{u}\bar{e}e, Q\bar{u}Q\bar{u}e, QQQQ\bar{u}, \bar{u}\bar{d}QdQ\bar{d}$, and $(QQQ)_4 LLL\bar{e}$. These are
lifted primarily by the superpotential terms which involve either $H_u$ or $H_d$ if $d$ is odd, or those which contain neither $H_u$ nor $H_d$. The complete list of superpotential terms which lift the flat directions can be found in [?].

4.3.2 Lifting by soft supersymmetry breaking

Vacuum degeneracy will also be lifted by supersymmetry breaking, as will be discussed in more detail in Sects. 4.4 and 4.5. It is induced by the soft terms, which in the simplest case read

$$V(\phi) = m_0^2|\phi|^2 + \left[ \frac{\lambda A \phi^d}{d M^{d-3}} + \text{h.c.} \right],$$

(170)

where the supersymmetry breaking mass $m_0$ and $A$ are typically of the order of the gravitino mass $m_{3/2}$. An additional soft source for supersymmetry breaking are the gaugino masses $m_g$. The $A$-term in Eq. (170) violates the $U(1)$ carried by the flat direction and thus provides the necessary source for $B-L$ violation in AD baryogenesis. In general, the coupling $\lambda$ is complex and has an associated phase $\theta_\lambda$. Writing $\phi = |\phi| \exp(i\theta)$, one obtains a potential proportional to $\cos(\theta_\lambda + n\theta)$ in the angular direction. This has $n$ discrete minima for the phase of $\phi$, at each of which $U(1)$ is broken.

4.4 Supersymmetry breaking in the MSSM

In the MSSM there are several proposals for supersymmetry breaking, which we shall discuss below. However, most of the time it is not important to know the exact mechanism of low energy supersymmetry breaking. This ignorance of the origin of supersymmetry breaking can always be hidden by simply writing down explicitly the soft breaking terms with arbitrary couplings.

4.4.1 Soft supersymmetry breaking Lagrangian

The most general soft supersymmetry breaking terms in the MSSM Lagrangian can be written as (see e.g. [?])

$$\mathcal{L}_{\text{soft}} = -\frac{1}{2} (M_\lambda \lambda^a \lambda^a + \text{c.c.}) - (m_2^2)_{ij} \phi^i \phi^j - \left( \frac{1}{2} b_{ij} \phi_i \phi_j + \frac{1}{6} a^{ijk} \phi_i \phi_j \phi_k + \text{c.c.} \right),$$

(171)
where $M_\lambda$ is the common gaugino mass $(m^2)_j$ are $3 \times 3$ matrices determining the masses for squarks and sleptons, denoted as $m^2_Q, m^2_\tilde{u}, m^2_\tilde{d}, m^2_L, m^2_\tilde{e}$; $b_{ij}$ is the mass term for the combination $H_u H_d$; and finally, $a^{ijk}$ are complex $3 \times 3$ matrices in the family space which yield the $A$-terms $a_u, a_d, a_e$. There are a total of 105 new entries in the MSSM Lagrangian which have no counterpart in the SM. However, the arbitrariness in the parameters can be partly removed by the experimental constraints on flavor changing neutral currents (FCNC) and $CP$ violation \([?]\). In order to avoid FCNC and excessive $CP$ violation, the squark and slepton (mass)$^2$ matrices are often taken to be flavor blind, so that the squark and slepton mixing angles can be rotated away. Similarly, one may assume that the $\phi^3$ couplings are proportional to the Yukawa coupling matrix, so that $a_u = A_{u0} \lambda_u, a_d = A_{d0} \lambda_d$, and $a_e = A_{e0} \lambda_e$. Large $CP$ violating effects can be avoided if the soft parameters do not involve new $CP$ phases in addition to the SM CKM phases. One can also fix $\mu$ parameter and $b$ to be real by an appropriate phase rotation of $H_u$ and $H_d$.

There are a number of possibilities for the origin of supersymmetry breaking. Fayet-Iliopoulos mechanism \([?]\) provides supersymmetry breaking by virtue of a non-zero D-term but requires a $U(1)$ symmetry. However, this mechanism does not work in the MSSM because some of the squarks and sleptons will get non-zero vevs which may break color, electromagnetism, and/or lepton number without breaking supersymmetry. Therefore, the contribution from the Fayet-Iliopoulos term should be negligible at low scales.

There are models of supersymmetry breaking by F-terms, known as O’Raifeartaigh models \([?]\), where the idea is to pick a set of chiral supermultiplets $\Phi_i \supset (\phi_i, \psi_i F_i)$ and a superpotential $W$ in such a way that $F_i = -\delta W/\delta \phi_i^* = 0$ have no simultaneous solution. The model requires a linear gauge singlet superfield in the superpotential. Such singlet chiral supermultiplet is not present in the MSSM. The scale of supersymmetry breaking has to be set by hand.

The only mechanism of supersymmetry breaking where the breaking scale is not introduced either at the level of superpotential or in the gauge sector is through dynamical supersymmetry breaking \([?, ?]\). In these models a small supersymmetry breaking
scale arises by dimensional transmutation. It is customary to treat the supersymmetry breaking sector as a hidden sector which has no direct couplings to the visible sector represented by the chiral supermultiplets of the MSSM. The only allowed interactions are those which mediate the supersymmetry breaking in the hidden sector to the visible sector.

The main contenders are gravity mediated supersymmetry breaking, which is associated with new physics which includes gravity at the string scale or at the Planck scale [??], and gauge mediated supersymmetry breaking, which is transmitted to the visible sector by the ordinary electroweak and QCD gauge interactions [??]. There are other variants of supersymmetry breaking based upon ideas on gravity and gauge mediation with some extensions, such as dynamical supersymmetry breaking (see [??], and references therein), and anomaly mediation (see [??], and references therein), which we do not consider here.

4.4.2 Gravity mediated supersymmetry breaking

Let us assume that supersymmetry is broken by the vev \( \langle F \rangle \neq 0 \) and is communicated to the MSSM by gravity. On dimensional grounds, the soft terms in the visible sector should then be of the order
\[
\langle F \rangle \left( \frac{m_{\text{soft}}}{M_p} \right)^2 \sim \mathcal{O}(100) \text{ GeV},
\]
In order to obtain a phenomenologically acceptable soft supersymmetry mass \( m_{\text{soft}} \sim \mathcal{O}(100) \text{ GeV} \), one therefore requires the scale of supersymmetry breaking in the hidden sector to be \( \sqrt{\langle F \rangle} \sim 10^{10} - 10^{11} \text{ GeV} \).

Another possibility is that the supersymmetry is broken via gaugino condensate \( \langle 0|\lambda^a \lambda^b|0 \rangle = \delta^{ab} \Lambda^3 \neq 0 \), where \( \Lambda \) is the condensation scale [??]. If the composite field \( \lambda^a \lambda^b \) belongs to the \( \langle F \rangle / M_p \)-term, then again on dimensional grounds one would expect the soft supersymmetry mass contribution to be
\[
m_{\text{soft}} \sim \frac{\Lambda^3}{M_p^2}.
\]
In this case the nature of supersymmetry breaking is dynamical and the scale is given by \( \Lambda \sim 10^{13} \text{ GeV} \).
The supergravity Lagrangian must contain the non-renormalizable terms which communicate between the hidden and the observable sectors. For the cases where the kinetic terms for the chiral and gauge fields are minimal, one obtains the following soft terms

\[ m_{1/2} \sim \frac{\langle F \rangle}{M_P}, \quad m_0^2 \sim \frac{|\langle F \rangle|^2}{M_P^2}, \quad A_0 \sim \frac{\langle F \rangle}{M_P}, \quad B_0 \sim \frac{\langle F \rangle}{M_P}. \] (174)

The gauginos get a common mass \( M_1 = M_2 = M_3 = m_{1/2} \), the squark and slepton masses are \( m_Q^2 = m_{\bar{u}}^2 = m_d^2 = m_L^2 = m_{\bar{e}}^2 = m_0^2 \), and for the Higgses \( m_{H_u}^2 = m_{H_d}^2 = m_0^2 \). The \( A \)-terms are proportional to the Yukawa couplings while \( b = B_0 \mu \).

Some particular models of gravity mediated supersymmetry breaking give more detailed estimates of the soft supersymmetry terms. They include: Dilaton dominated models [?], which arise in a particular limit of superstring theories, which have \( m_0^2 = m_{3/2}^2 \), and \( m_{1/2} = -A_0 = \sqrt{3}m_{3/2} \); Polonyi models [?], where \( m_0^2 = m_{3/2}^2 \), \( A_0 = (3 - \sqrt{3})m_{3/2} \), and \( m_{1/2} = \mathcal{O}(m_{3/2}) \); and No-scale models [?], which also arise in the low energy limit of superstrings and in which the gravitino mass is undetermined at the tree level while the at the string scale \( m_{1/2} \gg m_0, A_0, m_{3/2} \).

The predictions for the mass spectrum and other observable can be found renormalization group (RG) equations; these will be described in connection with the dynamical evolution of the AD field. Therefore, a generic flat direction in gravity mediated supersymmetry breaking has two important components: the soft supersymmetry breaking terms, and the RG induced logarithmic dependence of the vev.

### 4.4.3 Gauge mediated supersymmetry breaking

In gauge mediated supersymmetry breaking one employs a heavy messenger sector which couples directly to the supersymmetry breaking sector but indirectly to the observable sector via standard model gauge interactions only [?, ?]. As a result the soft terms in the MSSM arise through ordinary gauge interactions. There will still be gravitational communication, but it is a weak effect.

The simplest example is a messenger sector with a pair of \( SU(2) \) doublet chiral fields \( \ell, \bar{\ell} \) and a pair of \( SU(3) \) triplet fields \( q, \bar{q} \), which couple to a singlet field \( z \) with
Yukawa couplings $\lambda_2$, $\lambda_3$, respectively. The superpotential is given by

$$W_{\text{mess}} = \lambda_2 z \bar{l} + \lambda_3 z \bar{q}q.$$  \hspace{1cm} (175)

The singlet acquires a non-zero vev and a non-zero F-term $\langle F_z \rangle$. This can be accomplished either substituting $z$ into an O’Raifeartaigh type model [?, ?], or by a dynamical mechanism [?, ?, ?]. One may parameterize supersymmetry breaking in a superpotential $W_{\text{break}}$ by $\langle \partial W_{\text{break}}/\partial z \rangle = -\langle F_z^* \rangle$. As a consequence, the messenger fermions acquire masses

$$\mathcal{L} = - (\lambda_2 \langle z \rangle \psi \bar{\psi} + \lambda_3 \langle z \rangle \psi_q \bar{\psi}_q + \text{c.c}) ,$$  \hspace{1cm} (176)

while the scalar messenger partners have a scalar potential given by

$$V = |\lambda_2 \langle z \rangle|^2 \left(|l|^2 + |\bar{l}|^2\right) + |\lambda_3 \langle z \rangle|^2 \left(|q|^2 + |\bar{q}|^2\right) - (\lambda_2 \langle F_z \rangle \bar{l} + \lambda_3 \langle F_z \rangle q\bar{q} + \text{c.c})$$

$$+ \text{quartic terms},$$  \hspace{1cm} (177)

where we have used $\langle \partial W_{\text{mess}}/\partial z \rangle = 0$, and we have replaced $z$ and $F_z$ by their vevs. It is easy to read off the eigenvalues of the squared scalar masses and the fermionic and bosonic spectrum of the messenger sector; for $(l, \bar{l})$, $m^2_{\text{fermions}} = |\lambda_2 \langle z \rangle|^2$, and $m^2_{\text{scalars}} = |\lambda_2 \langle z \rangle|^2 \pm |\lambda_2 \langle F_z \rangle|$; for $(q, \bar{q})$, $m^2_{\text{fermions}} = |\lambda_3 \langle z \rangle|^2$, and $m^2_{\text{scalars}} = |\lambda_3 \langle z \rangle|^2 \pm |\lambda_3 \langle F_z \rangle|$.

Supersymmetry breaking is then mediated to the observable fields by one-loop corrections, which generate masses for the MSSM gauginos [?]. The $q, \bar{q}$ messenger loop diagrams provide masses to the gluino and the bino, while $l, \bar{l}$ messenger loop diagrams provide masses to the wino and the bino, i.e., $M_{a=1,2,3} = (\alpha_a/4\pi)\Lambda$, where $\Lambda = \langle F_z \rangle/\langle z \rangle$.

For squarks and sleptons the leading term comes from two-loop diagrams, therefore $m^2_\phi \propto \alpha^4$. The $A$-terms get negligible contribution at two-loop order compared to the gaugino masses, therefore $a_u = a_d = a_e = 0$ is a good approximation. The Yukawa couplings at the electroweak scale are generated by evolving the RG equations.

One can estimate [?] the soft supersymmetry breaking masses to be of order

$$m_{\text{soft}} \sim \frac{\alpha_a \langle F \rangle}{4\pi M_a}.$$  \hspace{1cm} (178)

If $M_s$ and $\sqrt{\langle F \rangle}$ are comparable mass scales, then the supersymmetry breaking can take place at about $\sqrt{\langle F \rangle} \sim 10^4 - 10^6$ GeV.
4.5 Supersymmetry breaking in the early Universe

Non-zero inflationary potential gives rise to supersymmetry breaking, the scale of which is given by the Hubble parameter. At early times this breaking is dominant over breaking from the hidden sector. After the end of inflation, in most models the inflaton oscillates and its finite energy density still dominates and breaks supersymmetry in the visible sector. Supersymmetry is broken also by quantum mechanical effects but these are negligible compared to the classical supersymmetry breaking from the non-zero energy density of the Universe.

4.5.1 Inflaton-induced terms

The early Universe supersymmetry breaking can be transmitted to the MSSM flat directions either by renormalizable or non-renormalizable interactions \[ \text{(179)} \]. However, at least for a single flat direction, renormalizable interactions do not lift the MSSM flat directions. In contrast, the effective potential generated by non-renormalizable interactions can induce a mass for the flat direction which is independent of the field values as long as they are below the Planck scale.

At tree level \( N = 1 \) SUGRA potential in four dimensions is given by the sum of F and D-terms \[ \text{(180)} \]

\[
V = e^K M_P^2 \left[ (K^{-1})_i^j F_i F_j - 3 \frac{|W|^2}{M_P^2} \right] + \frac{g^2}{2} \text{Re} f_{ab} \hat{D}^a \hat{D}^b, \tag{179}
\]

where

\[
F^i = W^i + K_i \frac{W}{M_P^2}, \quad \hat{D}^a = -K^i (T^a)_i^j \phi_j + \xi^a. \tag{180}
\]

where we have added the Fayet-Iliopoulos contribution \( \xi^a \) to the D-term. Here \( K \) is the Kähler potential, which is a function of the fields \( \phi_i \), and \( K_i \equiv \partial K / \partial \phi_i \).

A particular class of non-renormalizable interaction terms induced by the inflaton arise if the Kähler potential has a form \[ \text{(181)} \]

\[
K = \int d^4 \theta \frac{1}{M_P^2} (I^I)(\phi^\dagger \phi), \tag{181}
\]

where \( I \) is the inflaton whose energy density \( \rho \approx \langle I^I \rangle \) dominates during inflation, and \( \phi \) is the flat direction. The interaction Eq. (181) will generate an effective mass
term in the Lagrangian in the global supersymmetric limit, given by

\[ \mathcal{L} = \frac{\rho_I}{M_P^2} \phi^\dagger \phi = 3H_I^2 \phi^\dagger \phi, \]  

(182)

where \( H_I \) is the Hubble parameter during inflation.

### 4.5.2 Supergravity corrections

In addition, there are also inflaton-induced supergravity corrections to the flat direction. By inspecting the supergravity potential, one finds the following terms

\[ \left(e^{K(\phi^\dagger \phi)/M_P^2} V(I)\right), \quad \left(K_\phi K^{\phi^\dagger} K_\phi^\dagger \frac{|W(I)|^2}{M_P^4}\right), \]

and \( \left(K_\phi K^{\phi^\dagger} D_I \frac{W^*(I)W(I)}{M_P^4} + \text{h.c.}\right). \)  

(183)

Above \( D_I \equiv \partial/\partial I + K_I W/M_P^2 \). All these terms provide a general contribution to the flat direction potential which is of the form [?]

\[ V(\phi) = H^2 M_P^2 f \left( \frac{\phi}{M_P} \right), \]  

(184)

where \( f \) is some function. Note that this contribution exists also when the flat direction is lifted by non-renormalizable superpotential terms.

For a minimal choice of flat direction Kähler potential \( K(\phi^\dagger, \phi) = \phi^\dagger \phi \), during inflation the effective mass for the flat direction is found to be [?]

\[ m_\phi^2 = \left(2 + \frac{F_I^* F_I}{V(I)}\right) H^2. \]  

(185)

Here it has been assumed that the main contribution to the inflaton potential comes from the F-term. If there were D-term contributions \( V_D(I) \) to the inflationary potential, then a correction of order \( V_F(I)/(V_F(I) + V_D(I)) \) must be taken into account. In purely D-term inflation there is no Hubble induced mass correction to the flat direction during inflation because \( F_I = 0 \). However, when D-term inflation ends, the energy density stored in the D-term is converted to an F-term and to kinetic energy of the inflaton. Thus again a mass term \( m_\phi^2 = \pm \mathcal{O}(1) H^2 \) appears naturally, however the overall sign is undetermined [?].

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There are additional inflationary contributions to the potential if the flat direction is lifted by the non-renormalizable operators discussed earlier in this Section. These new terms come explicitly from the superpotential part of the flat direction

\[
\left( W_{\phi}K^{\phi\phi}K_{\phi} \frac{W^*(I)}{M_P^2} + \text{h.c.} \right), \quad \left( W_{\phi}K^{\phi I}D_I W^*(I) + \text{h.c.} \right),
\]

and

\[
\left( \frac{1}{M_P^2}K_I K^{II}K_I - 3 \right) \left( \frac{W(\phi)^*W(I)}{M_P^2} + \text{h.c.} \right).
\] (186)

The first one comes from the cross term between the derivative of the flat direction superpotential and the inflaton superpotential, the second is due to the Kähler potential coupling between the flat direction and the inflaton, and the third term is a cross term between the two superpotentials. All these terms give a generalized contribution equivalent to an $A$-term of the MSSM:

\[
V(\phi) = H M_P^3 f \left( \frac{\phi^d}{M_P^2} \right),
\] (187)

where $d$ is the dimensionality at which the flat direction is lifted. The induced $A$-term has an important role to play during the evolution of the flat direction. A possible $A$-term can also be generated from the expansion of the Kähler potential for field values $I, \phi < M_P$, which is of the form [?]

\[
\frac{1}{M_P} \int d^4 \theta I \phi^d \phi + \text{h.c.} \sim \frac{F_I^\dagger F_{\phi} \phi}{M_P} + \text{h.c.},
\]

\[
\frac{1}{M_P} \int d^2 \theta I W_i + \text{h.c.} \sim \frac{F_I}{M_P} W_i + \text{h.c.}
\] (188)

Note that the $A$-terms arise only from terms with a linear coupling of the inflaton superfield to a gauge invariant-operator $\phi_i$. If $I$ were a composite field rather than a singlet, then such a term will not arise and an $A$-term will not be generated. Also, in the case of D-term inflation, the inflaton cannot induce an $A$-term because $F_I = 0$. More generally, if there is a symmetry preventing a linear coupling of the inflaton, then order $H$ $A$-terms can be eliminated also in F-term inflation. As long as the thermal bath of the inflaton decay products dominates over the low energy supersymmetry breaking scale, we should have Hubble induced corrections to $m^2_0, m_{1/2}, A$.

If there is a non-minimal dependence of the gauge superfield kinetic terms on the inflaton field, a Hubble-induced gaugino mass can also be produced. Generally the
gauge superfield kinetic terms must depend on the field(s) of the hidden sector in order to obtain gaugino masses of roughly the same order as (or larger than) scalar masses, as required by phenomenology. Having $m_{1/2} \sim H$ thus appears to be quite natural unless an $R$-symmetry forbids terms which are linear in the inflaton superfield [?].

Since the $\mu$-term does not break supersymmetry, there is a priori no reason to assume that a $\mu$-term of order $H$ will be created. (For a discussion, see [?]). In what follows we will treat $\mu$ as a free parameter.

So far we have not discussed the sign of the Hubble induced mass correction. In fact with a general Kähler term either sign is possible. Depending on the sign, the dynamical behavior of the AD field is completely different and therefore the predictions depend crucially upon the sign. There are however certain cases where the Hubble-induced terms might not occur at all. An $R$-symmetry [?, ?] or a special choice of the Kähler potential could forbids the AD field getting the Hubble-induced mass correction [?, ?].

### 4.6 The potential for flat direction

#### 4.6.1 F-term inflation

Let us collect all the terms which contribute to the flat direction potential, which in the case of F-term inflation can be written as [?, ?]

$$V(\phi) = -C_I H_I^2 |\phi|^2 + \left( a \lambda_d H - \frac{\phi^d}{dM^{d-3}} + \text{h.c.} \right) + m_3^2 |\phi|^2 + \left( A \phi \lambda_d \frac{\phi^d}{dM^{d-3}} + \text{h.c.} \right) + |\lambda|^2 |\phi|^{2d-2} \frac{1}{M^{2d-6}}.$$  \hspace{1cm} (189)

The first and the third terms are the Hubble-induced and low-energy soft mass terms, respectively, while the second and the fourth terms are the Hubble-induced and low-energy $A$ terms. The last term is the contribution from the non-renormalizable superpotential. The coefficients $|C_I|$, $a \sim \mathcal{O}(1)$, and the coupling $\lambda \approx 1/(d-1)!$.

Note here the importance of the relative sign of the coefficient $C_I$. At large field values the first term dictates the dynamics of the AD field. If $C_I < 0$, the absolute minimum of the potential is $\phi = 0$ and during inflation the AD field will settle down
to the bottom of the potential roughly in one Hubble time. In such case the AD field will not have any interesting classical dynamics. Its presence would nevertheless be felt because of quantum fluctuations. These would be chi-squared in nature since then the classical energy density of the AD field would be due to its own fluctuations.

If \( C_I \ll 1 \), the AD field takes some time to reach the bottom of the potential, and if it has a non-zero amplitude after the end of inflation, its dynamics is non-trivial.

However, the most interesting scenario occurs when \( C_I > 0 \). In this case the absolute value of the AD field settles during inflation to the minimum given by

\[
|\phi| \simeq \left( \frac{C_I}{(d-1)\lambda_d} H_I M^{d-3} \right)^{1/(d-2)}.
\]

Here we have ignored the potential term \( \propto a \); if \( C_I > 0 \), the \( a \)-term will not change the vev qualitatively. On the other hand, even for \( C_I < 0 \) the potential Eq. (189) will have a minimum with a non vanishing vev if \(|a|^2 > 4(d-1)C_I \). However, the origin will also be a minimum in this case. The dynamics then depends on which minimum the AD field will choose during inflation.

The \( a \)-term in Eq. (189) violates the global \( U(1) \) symmetry carried by \( \phi \). If \(|a|\) is \( \mathcal{O}(1) \), the phase \( \theta \) of \( \langle \phi \rangle \) is related to the phase of \( a \) through \( n\theta + \theta_a = \pi \); otherwise \( \theta \) will take some random value, which will generally be of \( \mathcal{O}(1) \). This is the initial \( CP \)-violation which is required for baryogenesis/leptogenesis. In practice, the superpotential term lifting the flat direction is also the \( B \) and \( CP \) violating operator responsible for AD baryogenesis, inducing a baryon asymmetry in the coherently oscillating \( \phi \) condensate.

### 4.6.2 D-term inflation

In D-term inflation one does not get the Hubble induced mass correction to the flat direction so that \( C_I = 0 \). Also the Hubble induced \( a \)-term is absent. However, the Hubble induced mass correction eventually dominates once D-term induced inflation comes to an end. The potential for a generic flat direction during D-term inflation is given by

\[
V(\phi) = m_\phi^2 |\phi|^2 + \left( A_\phi \lambda_d \frac{\phi^d}{d! M^{d-3}} + \text{h.c.} \right) + |\lambda|^2 \frac{|\phi|^{2d-2}}{M^{2d-6}},
\]
and after the end of inflation the flat direction potential is given by \[ V(\phi) = \left( m_\phi^2 - CH^2 \right) |\phi|^2 + \left( A_\phi \lambda_d \frac{\phi^d}{dM^{d-3}} + \text{h.c.} \right) + |\lambda|^2 \frac{|\phi|^{2d-2}}{M^{2d-6}}, \] (192)

where \( C \sim \mathcal{O}(1) \). For \( C \) positive, the flat direction settles down to one of its minima given by Eq. (190) provided \( \phi \geq \sqrt{m_\phi M/\lambda} \), otherwise

\[ |\phi| \simeq \left( \frac{2C}{\lambda_d A_\lambda (d-1)} H(t)^2 M^{d-3} \right)^{1/d-2}, \] (193)

Note that in this case that the \( A \)-term is also responsible for \( B \) and/or \( L \), and \( CP \) violation. Another generic point to remember is that in \( R \)-parity conserving models the \( B \) and/or \( L \) violating operators must have even dimensions, so that \( d = 4 \) yields the minimal operator for AD baryogenesis.
5 Dynamics of flat directions

After the end of inflation $\langle \phi \rangle$ continues first to track the instantaneous local minimum of the scalar potential, obtained by replacing $H_I$ with $H(t)$ in Eq.(190) or by following Eq. (193) in the D-term inflation case. Once $H \simeq m_0 \sim m_{3/2}$, the low-energy soft terms take over. Then $m_\phi^2$ becomes positive and $\langle \phi \rangle$ starts to move in a non-adiabatic way (the phase of $\langle \phi \rangle$ differs from the phase of $A$-term during inflation). As a result $\langle \phi \rangle$ begins a spiral motion in a complex plane, which charges up the flat direction condensate, and eventually leads to generation of a net baryon and/or lepton asymmetry [?].

For baryogenesis purposes it is essential that the AD condensate obtains a non-zero vev during the inflationary epoch. In Sect. 4, we pointed out that a non-zero vev of the flat direction condensate is acquired only when the negative (mass)$^2$ contribution dominates the potential. The MSSM flat directions which are made up of squarks and sleptons have Yukawa and gauge interactions. The couplings render the evolution of a particular flat direction non-trivially, especially when the flat direction has a time varying mass due to the Hubble expansion [?, ?, ?, ?]. Moreover, if thermalization is not instantaneous, thermal effects from reheating can be substantial and might trigger the motion of the flat direction at an earlier time, there by changing the evolution of the flat direction condensate in a significant way [?, ?].

5.1 Running of the couplings

5.1.1 Running of gravitational coupling

Any flat direction has two kinds of interactions: renormalizable gauge or Yukawa interactions, and a non-trivial coupling to the curvature. Both types of interactions contribute to the logarithmic running of (mass)$^2$ of the flat direction condensate. The coupling to the curvature is generic because in principle any scalar field in an expanding background receives a contribution from the curvature by virtue of the Lagrangian term $\xi R \phi^2$, where $\xi$ is a coupling constant. Note that $R \propto H^2$ in an expanding background. Any scalar field always gets an additional positive Hubble induced mass
correction, provided $\xi$ is positive. The fundamental theory might have a conformal invariance, in which case the coupling strength $\xi = 1/6$ [?]? but it is known that conformal invariance is not protected by any symmetry, and that quantum corrections always break conformal invariance. Especially for the flat direction condensate, spontaneously broken supersymmetry induces soft supersymmetry breaking terms which break conformal invariance, and the value of $\xi$ remains undetermined.

It is of course possible to simply set $\xi = 0$. If initially $\xi = 0$ at some high scale, renormalization effects due to scalar field self-interaction will nevertheless generate a non-zero $\xi$ at lower scales. In an expanding Universe the value of $\xi$ also changes under the influence of a varying curvature (see [?] , and references therein). In the simplest case of a single scalar field with a quartic self-interaction strength $\lambda$ leads (at one-loop level) to a logarithmically running $\xi$ [?]

$$\xi_{\text{eff}} = \xi + \left( \xi - \frac{1}{6} \right) \frac{12}{4\pi^2} \lambda \ln \left( \left| \frac{m^2 + \left( \xi - \frac{1}{6} \right) R}{m^2} \right| \right). \quad (194)$$

It is obvious that $\xi = 1/6$ is a fixed point of the RG equation. If the theory has fermions and gauge fields, then obviously the coefficient in front of the logarithmic term in Eq. (194) will be modified [?].

As we have seen in Sect. 4.2, when supersymmetry is promoted to a local theory, a supergravity correction is induced to the flat direction which is proportional to the curvature, and supergravity theories also allow for $\xi R \phi^2$ (e.g. superconformal supergravity [?]).

In the context of MSSM flat directions we have implicitly assumed $\xi = 0$. This is justified from the very definition of F- and D-flat directions. The only leading order self coupling term in the flat direction potential is the Hubble induced A-term in Eq. (189). The overall self coupling constant is relatively large when the flat direction is lifted at $d = 4$, i.e. the suppression is proportional to $\mathcal{O}(1)(H/M_P)$, where we have replaced $M$ by $M_P$ in Eq. (189). In any inflation model the ratio $H_I/M_P \ll 1$, which in conjunction with Eq. (194), suggests that the effect of running on $\xi$ is minimal. For a running $\xi$ the curvature term in Eq. (194) dominates over the mass term. This might not be the case with the flat direction condensate because the condensate also
receives a field dependent mass while it is evolving. As long as the vev dependent mass is larger than the curvature induced mass, the running of any parameter in the theory will be dictated mainly by the renormalizable quantum effects. We therefore conclude that the running of $\xi$ can be neglected.

When the field dependent mass of the flat direction field becomes of order $m^2 \sim \mathcal{O}(H^2)$, it might be prudent to start worrying about the curvature induced term, especially during inflation. A simple inspection of Eq. (194) suggests that $\xi$ is always of order $\ln(\mathcal{O}(1))$, with virtually no alteration in $\xi_{\text{eff}}$. From now onwards we fix the non-minimal coupling to be $\xi = 0$.

### 5.1.2 Renormalization group equations in the MSSM

Let us consider the running of the flat direction (mass)$^2$ below $M_{\text{GUT}}$ by assuming that it is the scale where supersymmetry breaking is transmitted to the visible sector, in order to avoid uncertainties about physics between $M_{\text{GUT}}$ and $M_\text{P}$. The running of low-energy soft breaking masses has been studied in great detail in the context of MSSM phenomenology [?], in particular in connection with radiative electroweak symmetry breaking [?].

Let us recall some of the salient features of the MSSM one-loop RG equations. The ones relevant to flat directions involve the Higgs doublet $H_u$ which couples to the top quark, the right-handed stop $\tilde{u}_3$, the left-handed doublet of third generation squarks $\tilde{Q}_3$ and the $A-$parameter $A_t$ associated with the top Yukawa interaction. The RG equations read [?]

\[
\begin{align*}
\frac{d}{dq} m^2_{H_u} &= \frac{3h_t^2}{8\pi^2} \left( m^2_{H_u} + m^2_{Q_3} + m^2_{u_3} + |A_t|^2 \right) - \frac{1}{2\pi^2} \left( \frac{1}{4} g_1^2 |m_1|^2 + \frac{3}{4} g_2^2 |m_2|^2 \right), \\
\frac{d}{dq} m^2_{u_3} &= \frac{2h_t^2}{8\pi^2} \left( m^2_{H_u} + m^2_{Q_3} + m^2_{u_3} + |A_t|^2 \right) - \frac{1}{2\pi^2} \left( \frac{4}{9} g_1^2 |m_1|^2 + \frac{4}{3} g_3^2 |m_3|^2 \right), \\
\frac{d}{dq} m^2_{Q_3} &= \frac{h_t^2}{8\pi^2} \left( m^2_{H_u} + m^2_{Q_3} + m^2_{u_3} + |A_t|^2 \right) - \frac{1}{2\pi^2} \left( \frac{13}{36} g_1^2 |m_1|^2 + \frac{3}{4} g_2^2 |m_2|^2 + \frac{4}{3} g_3^2 |m_3|^2 \right), \\
\frac{d}{dq} A_t &= \frac{3h_t^2}{8\pi^2} A_t - \frac{1}{2\pi^2} \left( \frac{13}{36} g_1^2 m_1 + \frac{3}{4} g_2^2 m_2 + \frac{4}{3} g_3^2 m_3 \right).
\end{align*}
\]

(195)

Here $q$ denotes the logarithmic scale; this could be an external energy or momentum scale, but in the case at hand the relevant scale is set by the vev(s) of the fields
themselves. $h_t$ is the top Yukawa coupling, while $g_i$ and $m_i$ are respectively the gauge couplings and soft breaking gaugino masses of $U(1)_Y \times SU(2) \times SU(3)$. If $h_t$ is the only large Yukawa coupling (i.e. as long as tan $\beta$ is not very large), the beta functions for (mass)$^2$ of squarks of the first and second generations and sleptons only receive significant contributions from gauge/gaugino loops. A review of these effects can be found in [?]. Here we only mention the main results for the case of universal boundary conditions, where at $M_{\text{GUT}}$ all the scalar masses are $m_0^2$ and the gauginos have a common soft breaking mass $m_{1/2}$. For a low value of tan $\beta = 1.65$, \[ m_{H_u}^2 \simeq -\frac{1}{2}m_0^2 - 2m_{1/2}^2 \] (196) at the weak scale, while $m_{\tilde{u}_3}^2$ and $m_{\tilde{Q}_3}^2$ remain positive. The soft breaking (mass)$^2$ of the first and second generations of squarks is $\simeq m_0^2 + (5 - 7)m_{1/2}^2$, while for the right-handed and left-handed sleptons one gets $\simeq m_0^2 + 0.1m_{1/2}^2$ and $\simeq m_0^2 + 0.5m_{1/2}^2$, respectively. The important point is that the sum $m_{H_u}^2 + m_{\tilde{u}_3}^2$, which describes the mass along the $H_uL$ flat direction, is driven to negative values at the weak scale only for $m_{1/2} \gtrsim m_0$. This is intuitively understandable, since Eqs.(195) have a fixed point solution [?] $m_{H_u}^2 + m_{\tilde{u}_3}^2 + m_{\tilde{Q}_3}^2 = A_t = 0$ when $m_{1/2} = 0$.

### 5.2 Hubble induced radiative corrections

Here we describe radiative corrections in a cosmological set-up relevant for the AD mechanism [?]. When the Hubble induced supersymmetry breaking is dominant, i.e. for $H > \mathcal{O}(\text{TeV})$, the evolution of the soft terms is different from the vacuum RG equations given in Eq. (195). For the low-energy supersymmetry breaking case, constraints from the weak scale (e.g. realization of electroweak symmetry breaking, and experimental limits on the sparticle masses) give information about the soft breaking parameters $m_0^2$ and $m_{1/2}$. Together with fine tuning arguments, these constraints

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3This value corresponds to the case of maximal top Yukawa coupling, so called fixed point scenario [?, ?]. Such a low value of tan $\beta$ is excluded by Higgs searches at LEP [?], unless one allows stop masses well above 1 TeV. We nevertheless include this scenario in our discussion since it represents an extreme case.
imply that $m_0^2 > 0$ and that $m_0$, $m_{1/2}$ are $\mathcal{O}(\text{TeV})$. In the Hubble induced supersymmetry breaking case $m_0^2$ and $m_{1/2}$ are determined by the scale of inflation (and the form of the Kähler potential). At low scales the Hubble induced terms are completely negligible because at temperature $T \sim M_W$, $H \sim \mathcal{O}(1)$ eV; at present the Hubble parameter is tiny, $H_0 \sim \mathcal{O}(10^{-33})$ eV.

There exists an even more fundamental difference between the Hubble induced and ordinary radiative corrections. In Minkowski space the loop contributions to beta functions freeze at a scale of the order of the mass of the particles in the loop. In an expanding Universe the horizon radius $\propto H^{-1}$ defines an additional natural infrared cut-off for the theory. The masses of particles coupled to the flat direction receive contributions from two sources. There is a supersymmetry preserving part proportional to the vev $\langle \phi \rangle$, and the Hubble induced supersymmetry breaking part. The loop contributions to beta functions should thus be frozen at a scale given by the largest of $|\langle \phi \rangle|$ and $H$ (recall that $h_t$ and gauge couplings are close to one). In particular, if the squared mass of the flat direction condensate is positive at very large scales but turns negative at some intermediate scale $Q_c$, the origin of the flat direction potential will cease to be a minimum, provided the Hubble parameter is less than $Q_c$. On the other hand, if $m_\phi^2 < 0$ at the GUT scale, its running should already be terminated at the scale $|\langle \phi \rangle|$ determined by Eq.(190).\(^4\)

In the following two subsections we discuss separately the cases of positive and negative GUT-scale (mass)$^2$ for the flat direction condensate.

### 5.2.1 The case with $C_I \approx -1$

In this case all scalar fields roll towards the origin very rapidly and settle there during inflation, provided radiative corrections to their masses are negligible. A typical flat

\(^4\)Here we note that the Hubble cut-off usually plays no role in loop corrections to the inflaton potential. In most inflation models the masses of the fields which may run in the loop are larger than the Hubble expansion rate due to the presence of a finite coupling to the inflaton. This will happen if the (time varying) inflaton vev is large and the couplings are not very small. In those cases, which are somewhat similar to our case with $C_I > 0$, one can trust the usual loop calculation evaluated in a flat space time background [?].
Table 2: The scale $Q_c$ (in GeV) where the squared mass of the $H_uL$ flat direction changes sign, shown for $C_I = -1$ and several values for the ratios $A_t/H$ and $m_{1/2}/H$ as well as the top Yukawa coupling $h_t$, all taken at scale $M_{GUT} = 2 \cdot 10^{16}$ GeV, from [?]

direction condensate $\phi$ is a linear combination $\phi = \sum_{i=1}^{N} a_i \varphi_i$ of the MSSM scalars $\varphi_i$, implying that $m_\phi^2 = \sum_{i=1}^{N} |a_i|^2 m_\varphi^2$. In [?], it was noticed that with small values for the $\mu$ parameter, the running of $m_\phi^2$ crucially depends on $m_{1/2}$.

Let us consider sample cases with gaugino masses $m_{1/2} = (H; 3H; H/3)$, the $A$-term$^5$ $A_t(M_{GUT}) = (\pm H; \pm 3H; \pm H/3)$, top Yukawa $h_t(M_{GUT}) = (2, 0.5)$ and couplings $g_1(M_{GUT}) = g_2(M_{GUT}) = g_3(M_{GUT}) = 0.71$, and follow the running of scalar soft masses from $M_{GUT}$ down to $10^3$ GeV, where low-energy supersymmetry breaking becomes dominant. The main result is that only the $LH_u$ flat direction can acquire a negative (mass)$^2$ at low scales. In this case $m_\phi^2 = (m_{H_u}^2 + m_L^2 + \mu^2)/2$, where the last term is from the Hubble induced $\mu$ term. The results are summarized in Table 2, where it has been assumed that $\mu(M_{GUT}) \lesssim H/4$ so that the the $\mu$-term contribution to $m_\phi^2$ is negligible. In general $m_\phi^2$ changes sign at a higher scale for $h_t(M_{GUT}) = 2$.

$^5$The RG equations (195) for $A_t$ show that the relative sign between $A_t$ and $m_{1/2}$ matters, since it affects the running of $|A_t|$, and subsequently, scalar soft masses. Without loss of generality we take the common gaugino mass $m_{1/2}$ to be positive.
This is expected since a large Yukawa coupling naturally maximizes the running of $m_{H_u}^2$. Furthermore, the difference between $A_t/m_{1/2} < 0$ and $A_t/m_{1/2} > 0$ becomes more apparent as $|A_t/m_{1/2}|$ increases and $h_t$ decreases. The quasi fixed-point value of $A_t/m_{1/2}$ is positive [?, ?]. Positive input values of $A_t$ will thus lead to positive $A_t$ at all scales, but a negative $A_t(M_{\text{GUT}})$ implies that $A_t \simeq 0$ for some range of scales, which diminishes its effect in the RG equations, see Eq. (195). The sign of $A_t(M_{\text{GUT}})$ is more important for smaller $h_t$, since then $A_t/m_{1/2}$ will evolve less rapidly.

It was noticed in [?] that the squared mass of the $H_u L$ flat direction does not change sign when $m_{1/2} = H/3$, except for $A_t = \pm 3H$ and $h_t = 0.5$. This can be explained by the fact that for small $m_{1/2}$ and small or moderate $|A_t|$ we are generally close to the fixed point solution

$$m_{H_u}^2 \simeq -\frac{1}{2}H^2; \quad m_{u_3}^2 \simeq 0; \quad m_{Q_3}^2 \simeq \frac{1}{2}H^2.$$  \hspace{1cm} (197)

Nevertheless, even for $m_{1/2} \ll H$ the squared mass of the $L H_u$ flat direction as well as $m_{u_3}^2$ are $< 0.2H^2$ above 1 TeV, because of the fixed point behavior. This implies that the $L H_u$ flat direction can still be viable for baryogenesis, as pointed out by McDonald [?]. Flat directions built out of $\bar{u}_3$ will be marginal at best, since the decrease in $m_{u_3}^2$ will be counteracted by other contributions to $m_\phi^2$; e.g. for the $\bar{u}_3 \bar{d}_1 \bar{d}_2$ flat direction we find $m_\phi^2 > 2H^2/3$ at all scales.

The AD mechanism for baryogenesis should always work if $Q_c > H_I$, since in that case the global minimum of the potential during inflation is located at $|\langle \phi \rangle| \neq 0$. Note that in this case the vev $|\langle \phi \rangle|$ is usually determined by $Q_c$ rather than by Eq. (190).

For scales close to $Q_c$ the mass term in the scalar potential Eq. (189) can be written as $\beta_\phi H^2 |\phi|^2 \log(|\phi|/Q_c)$, where the coefficient $\beta_\phi$ can be obtained from the RG equations. If $\beta_\phi > 0$, which is true for the $H_u L$ flat direction for $C_I < 0$, this term will reach a minimum at $\log(|\phi|/Q_c) = -1$. If $Q_c < (H_I M_{\text{GUT}}^{d-3})^{1/d-2}$ the non-renormalizable contributions to the scalar potential are negligible for $|\phi| \sim Q_c$, so that the minimum of the quadratic term essentially coincides with the minimum of the complete potential given by Eq. (189). In models of high scale inflation (e.g. chaotic inflation models),

---

\footnote{For this choice of parameters, $A_t$ runs initially very slowly. It will therefore remain large for some time and helps $m_{H_u}^2$ to decrease quickly towards lower scales.}
the Hubble constant during inflation $H_I$ can be as large as $10^{13}$ GeV. This implies that $m_\phi^2$ for the $H_uL$ flat direction can only become negative during inflation if $m_{1/2}^2 \gg H^2$, which includes the “no-scale” scenario studied in [?].

The region of the parameter space safely allowing AD leptogenesis is much larger in models of intermediate and low scale inflation (e.g. some new inflation models [?]) where $H_I$ is substantially smaller. In such models one can easily have $H_I < Q_c$ at least for the $H_uL$ flat direction, unless $m_{1/2}^2 \ll H^2$ or $\mu^2 \gtrsim m_{1/2}^2$ [?].

If $Q_c < H_I$, the condensate $\phi$ settles at the origin during inflation and its post-inflationary dynamics will depend on the process of thermalization. If the inflaton decay products thermalize very slowly, $m_\phi^2$ is only subject to zero-temperature radiative corrections and $\langle \phi \rangle$ can move away from the origin once $H \lesssim Q_c$; a necessary condition for this scenario is that inflatons do not directly decay into fields that are charged under $SU(3) \times SU(2) \times U(1)$. If $Q_c \gg 1$ TeV, $\phi$ will readily settle at the new minimum and AD leptogenesis can work.

The situation will be completely different if inflatons directly decay into some matter fields. In such a case the plasma of inflaton decay products has a temperature $T \sim (\Gamma_d H M_{\text{Planck}}^2)^{1/4}$ [?]. $\Gamma_d$ is the inflaton decay rate. Fields which contribute to the running of $m_\phi^2$ are in thermal equilibrium (recall that the flat direction field is stuck at $\phi = 0$) and their back reaction results in thermal corrections of order $+T^2$ to $m_\phi^2$. For generic models of inflation $T > H$, implying that thermal effects exceed radiative corrections. Therefore, $\langle \phi \rangle$ remains at the origin at all times and AD leptogenesis will not work.

5.2.2 The case with $C_I \approx +1$

In this case all flat directions are viable for baryogenesis purposes provided the running of $m_\phi^2$ is negligible. Radiative corrections may change the sign (in this case to positive) at small vev(s), resulting in the entrapment of $\phi$ at the origin.

A quantitative study of the sample cases discussed above can be summarized as follows [?]. The squared mass of the $LH_u$ flat direction is always negative at small scales, unless $\mu^2 \gtrsim H^2/2$. For $m_{1/2} = 3H$, $m_\phi^2$ changes sign twice; it is positive for
scales $Q$ between roughly $10^{14}$ and $10^6$ GeV, the precise values depending on $h_t$ and $A_t$. Slepton masses only receive positive contributions from electroweak gauge/gaugino loops. As a result, the squared mass of the $LL\bar{e}$ flat direction remains negative down to 1 TeV, unless $m_{1/2} > 2H$; for $m_{1/2} \gtrsim 3H$, $Q_c \gtrsim 10^9$ GeV even for this flat direction. The squared masses of all squarks (except $\tilde{u}_3$) change sign at $Q_c > 1$ TeV unless $m_{1/2} \lesssim H/3$; we find $Q_c \simeq 10^{10}$ ($10^{15}$) GeV for $m_{1/2}/H = 1$ ($3$). This is due to the large positive contribution $\propto m_3^2$ to the squared squark masses at scales below $M_{\text{GUT}}$.

The corresponding values for the $\bar{u}_3\bar{d}_i\bar{d}_j$ and $LQ\bar{d}$ flat directions are usually somewhat smaller, due to the Yukawa terms in the $\beta$–function and the slower running of the slepton masses, respectively; however, the values of $Q_c$ listed in Table 2 are still a fair approximation.

The positive contribution to the scalar potential from the non-renormalizable superpotential term now dominates $-H^2$ (see Eq. (190)). If $Q_c > (H_I M_{\text{GUT}}^{n-3})^{1/n-2}$, $m_\phi^2$ is positive for all vev(s) and hence the flat direction will settle at the origin during inflation and remain there. In such a case the flat direction is not suitable for AD baryogenesis. This can easily happen for flat directions involving squarks in models with low scale inflation, but is not likely for high scale inflation models (unless $m_{1/2} \gtrsim 3H$). For $H_I < Q_c < (H_I M_{\text{GUT}}^{n-3})^{1/n-2}$, feasible for some flat directions in both intermediate/high scale and low scale models. During inflation the potential has two minima, at $\langle \phi \rangle = 0$ and at $|\langle \phi \rangle| \sim (H_I M_{\text{GUT}}^{d-3})^{1/d-2}$. Depending on the initial conditions, $\phi$ can roll down towards either of them and settle there but only the latter one will be useful for AD baryogenesis.

If $Q_c < H_I$, the flat direction condensate will settle at the value determined by Eq. (190) (the only minimum during inflation) and remain there. The appearance of another minimum at the origin after inflation, which is possible once $H < Q_c$, does not change the situation since these minima are separated by a barrier. In this case radiative corrections will not change the picture qualitatively; however, they will still modify the quantitative analysis, since $C_I$ in Eq.(190) will become scale-dependent.

In brief, the main conclusion is that among the flat directions $LH_u$ is the only robust one in the sense that it gives rise to AD leptogenesis independently of the sign
of $C_I$.

5.2.3 Running of the flat direction field in no-scale supergravity

So far we have dealt with a minimal choice of the Kähler potential. An alternative is non-flat Kähler potential; an example of this is provided by e.g. no-scale models, for which $K \sim \ln(z + z^* + \phi_i^* \phi_i)$, where $z$ belongs to supersymmetry breaking sector, and $\phi_i$ belongs to the matter sector (for a review, see [?]). In no-scale models there exists an enhanced symmetry known as the Heisenberg symmetry [?], which is defined on the chiral fields as $\delta z = \epsilon^* \phi_i$, $\delta \phi_i = \epsilon^i$, and $\delta y^i = 0$, where $y^i$ are the hidden sector fields, such that the combinations $\eta = z + z^* - \phi_i^* \phi_i$, and $y_i = 0$ are invariant. For a especial choice

$$K = f(\eta) + \ln|W(\phi)|^2 + g(y), \quad (198)$$

The $N = 1$ supergravity potential reads [?, ?]

$$V = e^{f(\eta) + g(y)} \left[ \left( \frac{f'^2}{f''} - 3 \right) |W|^2 - \frac{1}{f'^2} |W_i|^2 + g_a (g^{-1})^a_b g^b |W|^2 \right]. \quad (199)$$

Note that there is no cross term in the potential such as $|\phi_i^* W|^2$. As a consequence any tree level flat direction remains flat even during inflation [?] (in fact it is the Heisenberg symmetry which protects the flat directions from obtaining Hubble induced masses [?]). The symmetry is broken by gauge interactions or by coupling in the renormalizable part of the Kähler potential. Then the mass of the flat direction condensate arises from the running of the gauge couplings.

For $f(\eta) = -3 \ln \eta$, the one-loop corrected supergravity induced mass term has been calculated in [?, ?, ?], which gives an effective mass for the flat direction field in the presence of finite energy density stored in the inflaton sector. The typical mass of the flat direction has been computed and comes out to be $m^2_{\phi} \sim 10^{-2}H^2$ during inflation. The only constraint is that flat direction must not involve stops [?].

5.3 Post-inflationary running of the flat direction

Now we focus on the running of the flat direction after inflation. Here we must take into account the low energy supersymmetry breaking effects. In particular, the running of
the condensate mass will depend on how supersymmetry is transmitted to the visible sector.

5.3.1 Gravity mediated supersymmetry breaking

For gravity-mediated supersymmetry breaking the scalar potential along a flat direction has been evaluated as \[ U(\Phi) \approx m_\phi^2 \left( 1 + K \log \left( \frac{|\Phi|^2}{M^2} \right) \right) |\Phi|^2 + \frac{\lambda^2 |\Phi|^{2(d-1)}}{M_{\text{Pl}}^{2(d-3)}} + \left( \frac{A_\lambda \Phi^d}{d M_{\text{Pl}}^{d-3}} + \text{h.c.} \right) , \]

where \( m_\phi \) is the conventional gravity-mediated soft supersymmetry breaking scalar mass term (\( m_\phi \approx 100 \text{ GeV} \)), \( K \) is a parameter which depends on the flat direction, and the logarithmic contribution parameterizes the running of the flat direction potential with \( \mu = (M_{\text{Pl}}^{d-3} m_{3/2}/|\lambda|)^{(1/d-2)} \). In the gravity mediated case \( |A_\lambda| < d m_{3/2} \), for \( d = 4, 6 \).

\( K \) can be computed from the RG equations, which to one loop have the form

\[
\frac{\partial m^2_{\text{2S}}}{\partial t} = \sum_g a_{ig} m^2_g + \sum_a h^2_a \left( \sum_b b_{ij} m^2_j + A^2 \right) ,
\]

where \( a_{ig} \) and \( b_{ij} \) are constants, \( m_g \) is the gaugino mass, \( h_a \) the Yukawa coupling, \( A \) is the A-term, and \( t = \ln M_X/q \). The full RG equations have been listed in \([?]\). The potential along the flat direction is then characterized by the amount of stop mixture (where appropriate), the values of gluino mass and \( A \), and in the special case of the \( d = 4 \) \( H_u L \)-direction, on the \( H_u H_d \)-mixing mass parameter \( \mu_H \).

The mass of the AD scalar \( \phi \) is the sum of the masses of the squark and slepton fields \( \phi_i \) constituting the flat direction, \( m_\phi^2 = \sum_i \hat{p}_i^2 m_i^2 \), where \( \hat{p}_i \) is the projection of \( \phi \) along \( \phi_i \), and \( \sum \hat{p}_i^2 = 1 \). The parameter \( K \) is then given simply by

\[
K = \frac{1}{q^2} \frac{\partial m_\phi^2}{\partial t} \bigg|_{t=\log q} .
\]

To compute \( K \), one has to choose the scale \( q \). The appropriate scale is given by the value of the AD condensate amplitude when it first begins to oscillate at \( H \approx m_\phi \) or

\[
Q = |\phi_0| = \left[ \frac{m_\phi^2 M^{2(d-3)}}{(d-1) \lambda^2} \right]^{1/(d-2)} ,
\]

81
Figure 1: Contours of $K$ for two $d=4$ flat directions in the $(A, \xi \equiv m_g/m(t = 0))$-plane: (a) $K = 0$ (b) $K = -0.01$; (c) $K = -0.05$; (d) $K = -0.1$. The directions are (i) $Q_3 Q_3 Q L$; (ii) $Q Q Q L$, no stop; (iii) $\bar{u}_3 \bar{u} \bar{d} \bar{e}$; (iv) $\bar{u} \bar{u} \bar{d} \bar{e}$ with equal weight for all $\bar{u}$-squarks, from [?].

The RG running of the flat directions in the case of gravity mediated supersymmetry breaking was studied in [?, ?], where unification at $t = 0$ was assumed and all the other Yukawa couplings except the top Yukawa were neglected.

The contours of $K$ for the $d = 4$ $\bar{u} \bar{u} \bar{d} \bar{e}$ and $Q Q Q L$ directions are shown in Fig. (1) in the $(A, \xi)$-plane, where $\xi \equiv m_g/m(t = 0)$ (for $\tan \beta = 1$ and $\lambda = 1$). These are representative of all the other directions, too, except for $H_u L$. For $\xi \sim O(1)$, typical value for $K$ is found to be about $-0.05$. Similar contours can be obtained for the $d = 6$ $(\bar{u} \bar{d} \bar{e})^2$ and $(Q L d)^2$ directions, see [?]. For all the squark directions with no stop, as long as $h_b$ and $h_u$ can be neglected, $K$ is always negative, and the contours of equal $K$ do not depend on $A$. In the presence of stop mixing $K < 0$ is no longer automatic.
even in the purely squark directions. The more there is stop, the larger value of $\xi$ is required for $K < 0$. Even for pure stop directions, positive $K$ is typically obtained only for relatively light gaugino masses with $\xi \lesssim 0.5$.

In contrast to the squark directions, $K$ was found [?] to be always positive in the $H_u L$-direction. This is due to the fact $H_u L$ does not involve strong interactions which in other directions are mainly responsible for the decrease of the running scalar masses. Very roughly, instability is found when $m_\tilde{g} \gtrsim m_\tilde{t}$, although the exact condition should be checked case by case. In general, the sign of $K$ could be deduced from the observation of SUSY parameters such as $\tan \beta$, the gluino mass and the supersymmetry breaking parameter $m_\phi$ [?].

### 5.3.2 Gauge mediated supersymmetry breaking

A similar analysis can be made for the gauge mediated case, where supersymmetry breaking is transmitted to the observable sector below some relatively low messenger sector scale $\Lambda_S$, above which the potential is completely flat (see Sect. 4.4.3.). In the gravity mediated scenario the soft masses stay intact, modulo RG running, up to the Planck scale; in gauge mediation the masses simply disappear above $\Lambda_S$. For a large condensate vev, one can integrate out the gauge and chiral fields coupled to the flat direction in order to obtain an effective low energy theory. In such a case, as was first pointed out by Kusenko and Shaposhnikov [?], the potential along the flat direction obtains a logarithmic correction of the form [?, ?]

$$U(\Phi) = m_\phi^4 \log \left(1 + \frac{|\Phi|^2}{m_\phi^2}\right) + \frac{\lambda^2|\Phi|^{2(d-1)}}{M_P^{2(d-3)}} + \frac{A_\lambda \lambda \Phi^d}{dM_P^{d-3}} + \text{h.c.}$$

(204)

where $m_\phi \sim 1 - 100$ TeV. Because of the differences in the potential, the dynamical evolution of the condensate field will be markedly different from the gravity mediated case. Because the messenger sector is not constrained by experiments, one cannot provide a detailed description of the mass parameters. Here one should note that in order to have an AD condensate, the $A$-term is actually constrained. In the gauge mediated case $|A_\lambda| \leq (10^{-4} - 10^{-7})m_\phi$, for $d = 4, 6$.

Eqs. (200) and (204) are two book-keeping equations which are useful for the rest of this review.
5.4 Density perturbations from the flat direction condensate

The role of MSSM flat directions is not just limited to generating the lepton and/or baryon asymmetry in the Universe, but they also play an interesting role in the dynamics of density perturbations.

5.4.1 Energetics of flat direction and the inflaton field

Once the flatness of the flat direction potential is lifted by non-renormalizable terms, for large field values the condensate energy density can dominate over the inflaton potential. This could be disastrous: either inflationary expansion would come to a halt, or the flat direction condensate fluctuations might ruin the successful predictions for the angular power spectrum [? , ? , ? , ?].

In [?], the generation of adiabatic density perturbations was studied for both D- and F-term inflation models. Note that in the former case there is no Hubble induced mass correction to the flat direction condensate. The scalar potential for F-and D-flat direction of dimension \(d\) is given by (see Eq. (191) in Sect. 4.6.2.)

\[
V(\phi) \approx \frac{\lambda^2 |\Phi|^{2(d-1)}}{M^{2(d-3)}},
\]  

(205)

where only the dominant term from Eq. (191) corresponding to superpotential term of the form \(W = \lambda \Phi^n / n M^{d-3}\) has been kept. Throughout this discussion \(R\)-parity is conserved and therefore we deal only with even dimensions \(d = 4, 6, 8\).

For illustrative purposes let us assume that the D-term inflationary potential is given by (see Eq. (146) in Sect. 3.5.3.)

\[
V(S) = \frac{g^2 \xi^4}{2} + \frac{g^4 \xi^4}{32 \pi^2} \ln \left( \frac{S^2}{Q^2} \right),
\]  

(206)

where \(S\) is the inflaton component; \(Q\) is here the renormalization scale. For a large initial vev for \(\phi, S \sim \mathcal{O}(M_P)\), the dynamics is first dominated by \(V(\phi)\). For a sufficiently large vev of \(\phi\) the effective condensate mass squared \(V''(\phi)\), becomes larger than \(H^2\). This occurs if \(\phi > \phi_H\), where [?]

\[
\phi_H = \frac{2^{\frac{(d-1)}{2d}}}{(6(2d-2)(2d-3))^{\frac{1}{2d-2}}} \left( \frac{g}{\lambda} \right)^{\frac{1}{d-2}} \xi^{\frac{1}{d-2}} \frac{1}{2^{d-2}} M_P^{\frac{d-4}{d-2}}.
\]  

(207)
If initially $\phi_i > \phi_H$, then $\phi$ will first oscillate in its potential with a decreasing amplitude: $\phi(t) \propto a^{-3/d(t)}$ [?]. This period ends before the onset of inflaton domination. The system then enters a regime where both $\phi$ and $S$ are slowly rolling.

The slow rolling dynamics of the scalar fields is given by the solution of

$$3H\dot{\Psi}_a = -\frac{\partial V(\Psi_a)}{\partial \Psi_a}; \quad H = \left(\frac{\sum_a V(\Psi_a)}{3M_p^2}\right)^{1/2},$$

(208)

where $\Psi_a \equiv S, \phi$. By taking the ratio of the equations for $\phi$ and $S$, one obtains

$$\frac{\partial \phi}{\partial S} = \frac{16\pi^2(d-1)\lambda^2\phi(2d-3)S}{2^{d-2}g^4\xi^4M_p^{2(d-3)}},$$

(209)

which has a general solution of the form

$$\phi = \phi_i \left[1 + \alpha_d\phi_i^{2d-4} (S_i^2 - S^2)^{-1/(2d-4)}\right]^{-1/(2d-4)}; \quad \alpha_d = \frac{16\pi^2(d-2)(d-1)\lambda^2}{2^{d-2}M_p^{2(d-3)}g^4\xi^4},$$

(210)

where $\phi_i$ and $S_i$ are the initial values at the onset of inflation. There are two features about this solution. First, since $S_i$ is large compared with the value of $S$ at $N = 50$ e-foldings before inflation, we see that for sufficiently large $\phi_i$ the value of $\phi$ at late times is fixed by $S_i$,

$$\phi \equiv \phi_s \approx \left(\frac{1}{\alpha_d}\right)^{\frac{1}{2d-4}} \frac{1}{S_i^{1/(d-2)}}.$$

(211)

This is true if $\phi_i > \phi_s$, otherwise, $\phi$ simply remains at $\phi_i$. Second, we can relate $S_i$ to the total number of e-foldings during the $V(S)$ dominated period of inflation. In general, for sufficiently large $\phi_i$, we could have an initial period of $V(\phi)$ dominated inflation. During this period $S$ does not significantly change from $S_i$. The potential is dominated by $V(\phi)$ once $\phi > \phi_S$, where [?]

$$\phi_S = \sqrt{2M_p^{d-3} \lambda^{d-1}} \left(\frac{g^2\xi^4}{2}\right)^{\frac{1}{2(d-1)}}.$$  

(212)

$\phi_S$ is generally less than $\phi_H$ (see Eq. (207)), therefore $\phi$ will be slow rolling during $V(S)$ domination.

From Eq. (210), we find that the condition for $S$ to change significantly from $S_i$ at a given value of $\phi$ is given by

$$S_i < \frac{1}{\alpha_d^{1/2}} \left(\frac{1}{\phi}\right)^{d-2}.$$

(213)
The condition for $S$ to change significantly during $V(\phi)$ dominated inflation is given by Eq. (213) with $\phi = \phi_S$; one finds

$$S_i < S_{i,c} \approx \frac{2^{\frac{d-2}{2(d-1)}} g^{\frac{d}{2}} \xi^{\frac{d-2}{2(d-1)}} M_P^{\frac{d-3}{2(d-1)}}}{\lambda^{\frac{1}{d-1}}}.$$  \hspace{1cm} (214)$$

Since $S_{i,c}$ is small compared with $M_P$, whereas the value of $S$ required to generate 50 e-foldings of inflation, $S_{50} = g\sqrt{50}M_P/(2\pi)$ is close to $M_P$, it follows that $S_i (> S_{50})$ will generally be larger than $S_{i,c}$, and so the inflaton will remain at $S_i$ until the Universe becomes inflaton dominated.

In this case the total number of e-foldings during inflaton domination is given by $N_S$ where $S_i = (g/2\pi)N_S^{1/2}M_P$. If $\phi_i > \phi_*$, then $\phi$ at $N \approx 50$ e-foldings of inflation will be given by [?]

$$\phi_* \approx \left(1 \over \alpha_d\right)^{d-1} \left(2\pi \over gM_P N_S^{1/2}\right)^{d-1}.$$  \hspace{1cm} (215)$$

Note that the dependence on $N_S$ is quite weak; for the case of $d = 4$ ($d = 6$) AD baryogenesis, $\phi_* \propto N_S^{-1/4}$ ($N_S^{-1/8}$). If there is no large number of inflationary e-foldings one can essentially fix the value of $\phi_*$. In this case one can predict the magnitude of the baryonic isocurvature perturbation.

Imposing a chaotic-type initial condition $V(\phi_i) \approx M_P^4$ yields

$$\phi_i \approx \sqrt{2}M_P \over \lambda^{\frac{1}{d-1}}.$$  \hspace{1cm} (216)$$

By directly solving the slow roll equations for $\phi$ and $S$, we obtain the total number of e-foldings of inflation:

$$N_T = N_\phi + N_S \approx \frac{1}{4(d-1)M_P^2} \phi_i^2 + \frac{4\pi^2S_i^2}{g^2M_P^2},$$  \hspace{1cm} (217)$$

where $N_\phi$ is the number of e-foldings during $V(\phi)$ domination, provided $\phi_i > \phi_S$. $V(S)$ will dominate the total number of e-foldings only if

$$N_S \approx \frac{1}{2(d-1)\lambda^{2/(d-1)}}.$$  \hspace{1cm} (218)$$

Since $N_S > 50$, the above condition will be satisfied so long as $\lambda$ is not very small (for example, if $\lambda \approx 1/(d-1)!$). In this case the value of $\phi$ at the time when the
CMB perturbations are generated will be determined mainly by the total number of e-foldings of inflation, i.e. $N_T \approx N_S$.

### 5.4.2 Adiabatic perturbations during D-term inflation

The potential for the flat direction condensate is far from flat, and so if the magnitude of the flat direction condensate is large, it will cause a large deviation from scale-invariance to the adiabatic perturbation. This will impose an upper limit on the amplitude of the flat direction condensate at 50 e-foldings before the end of inflation. If we assume that the flat direction follows a late time attractor trajectory together with the inflaton, then following the analyses in [?], and Kawasaki and Takahashi [?], the flat direction induced adiabatic density perturbation can be estimated from Eq. (125).

For a potential of the form $V = V(S) + V(\phi)$, one obtains (with the help of Eqs. (84,85,115, 126)) [?]

$$
\eta = -\frac{M_P^2}{(V_S + V_\phi)V} \left[ V_S V''_S + V_\phi V''_\phi - \frac{2(V_S + V_\phi)(V''_S V'_S^2 + V''_\phi V'_\phi^2)}{(V'_S^2 + V'_\phi^2)} \right] \quad (219)
$$

and

$$
\epsilon = \frac{M_P^2}{(V_S + V_\phi)V} \left[ \frac{(V'_S + V'_\phi)(V''_S V'_S^2 + V''_\phi V'_\phi^2)}{2V} \right]. \quad (220)
$$

Particularly, for the case of D-term inflation, if $V'_\phi < V'_S$ and $V''_\phi < V''_S$, we obtain the conventional result, see Eq. (115). Here the isocurvature contribution to the spectral index has been neglected; we will discuss it in the next subsection.

Since the main contribution to the scale-dependence of the perturbations comes from $\eta$, let us estimate the deviation from scale-invariance due to the presence of the flat direction condensate. Note that when $V''_\phi > V''_S$, with $V'_\phi \ll V'_S$ and $V_\phi \ll V_S$ still satisfied, we can expand $\eta$ in order to obtain corrections to the conventional D-term inflation model [?]

$$
\eta \approx M_P^2 V''_S - M_P^2 V'_S V''_S. \quad (221)
$$

The condensate scalar induced deviation from the scale invariance in the spectral index is given by

$$
\Delta n_\phi \approx -\frac{2V''_\phi V'_S M_P^2}{V_S V''_S}. \quad (222)
$$
Requiring that $|\Delta n_\phi| < K$, the present CMB observations imply that $n = 1.2 \pm 0.3$; 
adoption $K < 0.2$ imposes an upper bound on $\phi$,

$$\phi < \phi_c = k_d \left( \frac{K}{\sqrt{N}} \right)^{\frac{1}{4d-7}} g^{\frac{5}{2d-7}} \lambda^{\frac{4}{4d-7}} \xi^{\frac{8}{4d-7}} M_P^{\frac{4d-15}{4d-7}},$$

(223)

where

$$k_d = \left( \frac{2^{2(d-1)}}{128 \pi (d-1)^2 (2d-3)} \right)^{\frac{1}{4d-7}}.$$  

(224)

For the case of minimal $d = 4$ AD baryogenesis, one obtains [?]

$$\phi_c = 0.53 \left( \frac{K}{\sqrt{N}} \right)^{\frac{1}{7}} \left( g^5 \lambda^{-4} \xi^8 M_P \right)^{\frac{1}{2}} \sim 10^{16} \text{ GeV},$$

(225)

while for $d = 6$ AD baryogenesis scenario, one gets

$$\phi_c = 0.77 \left( \frac{K}{\sqrt{N}} \right)^{\frac{1}{7}} \left( g^5 \lambda^{-4} \xi^8 M_P^9 \right)^{\frac{1}{17}} \sim 10^{17} \text{ GeV}.$$  

(226)

### 5.4.3 Adiabatic perturbations during F-term inflation

During F-term inflation, the dominant part of the flat direction potential is given by (see Eq. (189), Sect. 4.6.1.)

$$V_{total}(\phi) \approx \frac{C_I H^2 \phi^2}{2} + V(\phi),$$

(227)

where $V(\phi)$ is the usual part from the non-renormalizable superpotential term. Here we assume that $C_I \approx -O(1)$. In such a case the local minimum of the flat direction condensate is given by Eq. (190), denoted here by $\phi_m$.

Note that if $\phi$ is close to $\phi_m$ ($|\delta \phi| \equiv |\phi - \phi_m| \lesssim \phi_m$), then inflation will damp $\delta \phi$ to be close to zero. The equation of motion for perturbations around this local minimum is given by

$$\delta \ddot{\phi} + 3H \dot{\delta \phi} = -k H^2 \delta \phi \ ; \ k = (2d-4)C_I \gtrsim 1,$$

(228)

which has a solution of the form:

$$\delta \phi = \delta \phi_0 e^{\alpha H t} \ ; \ \alpha = \frac{1}{2}(-3 + \sqrt{9 - 4k}).$$

(229)
As long as $H t \gg 1$, i.e. there are a significant number of e-foldings, the amplitude of the flat direction condensate will be damped to be exponentially close to the minimum of its local minimum.

In general, it is likely that the initial value of $\phi$ will not be close to $\phi_m$. It has been shown that the deviation of the adiabatic perturbation from scale-invariance implies that the value of the potential at $N \approx 50$ cannot be very much larger than $\phi_m$ [7]. The deviation from scale-invariance due to the flat direction condensate is then

$$\Delta n_\phi = -\frac{2}{\xi} \frac{d \xi}{dN} = -\frac{3V'(\phi)}{V(\phi) + V(S)} \frac{\partial \phi}{\partial N}.$$  \hspace{1cm} (230)

For $\phi \gg \phi_m$, the $\phi$ field will be rapidly oscillating in its potential and the change in the amplitude of $\phi$ over an e-folding due to damping by expansion will be $\partial \phi/\partial N \sim -\phi$. Requiring that $|\Delta n_\phi| < K$ imposes an upper bound on $\phi$

$$\phi \lesssim \left( \frac{K d}{6(d-1)\lambda^2} \right)^{\frac{1}{2(d-1)}} \sqrt{2H \frac{1}{d-1}} M \frac{d-2}{d-1}.$$  \hspace{1cm} (231)

For $d = 4$, one finds [7]

$$\frac{\phi}{\phi_m} \lesssim \frac{0.8}{C_1^{1/4}} \left( \frac{\lambda M_P}{H} \right)^{\frac{1}{6}},$$  \hspace{1cm} (232)

while for $d = 6$

$$\frac{\phi}{\phi_m} \lesssim \frac{0.9}{C_1^{1/8}} \left( \frac{\lambda M_P}{H} \right)^{\frac{1}{20}},$$  \hspace{1cm} (233)

where we have used $K = 0.2$. For typical values of $H_I$, the scale-invariance of the density perturbations implies that $\phi$ at $N \approx 50$ e-foldings cannot be much more than an order of magnitude greater than $\phi_m$. Since there is no reason for $\phi$ to be close to this upper limit when $N \approx 50$, it is most likely that $\phi$ will be close to $\phi_m$ when the primordial perturbations responsible for large scale structure formation have left the horizon during inflation.

### 5.4.4 Isocurvature fluctuations in D-term inflation

The isocurvature perturbation in the baryon number arises from the AD scalar if the angular direction is effectively massless, i.e. mass is small compared with $H$ during
and after inflation \([?, ?, ?]\). The resulting perturbations will be unsuppressed until the baryon number of the Universe is generated. This in turn requires that there are no order \(H\) corrections to the supersymmetry-breaking \(A\)-terms.

The baryon number from AD baryogenesis is generated at \(H \approx m_{\text{susy}} \sim 100\ \text{GeV}\) when the \(A\)-term can introduce \(B\) and \(CP\) violation into the coherently oscillating AD scalar. If the phase of the AD scalar relative to the real direction (defined by the \(A\)-term) is \(\theta\), then the baryon number density given by (see Eq. (251), in Sect. 5.5.)

\[
(n_B) \approx m_{\text{susy}} \phi_0^2 \sin 2\theta ,
\]

where \(\phi_0\) is the amplitude of the coherent oscillations at \(H \approx m_{\text{susy}}\). One can then obtain fluctuation in the baryon number or an isocurvature perturbation as

\[
\frac{\delta n_B}{n_B} = \frac{2\delta \theta}{\tan(2\theta)} = \frac{H}{\pi \phi \tan(2\theta)} ,
\]

where \(\delta \theta \approx (H/2\pi \phi)\) is generated by quantum fluctuations of the AD field at the time when the perturbations cross the horizon. The magnitude of the CMB isocurvature perturbation relative to the adiabatic perturbation can be written as \([?, ?, ?, ?]\), (see Eq. (129), in Sect. 3.4.3.)

\[
\alpha = \left| \frac{\delta^i}{\delta_\gamma} \right| = \frac{\omega}{3} \left( \frac{2M^2 V'(S)}{V(S) \tan(2\theta)} \right) ,
\]

where \(V(S)\) is the inflaton potential (see Sect. 5.4.1). For purely baryonic isocurvature perturbations \(\omega = \Omega_B/\Omega_m\), where \(\Omega_B\) is the baryon density and \(\Omega_m\) is the total matter density. For the case of D-term inflation one obtains \([?]\)

\[
\alpha = \frac{1}{6\pi} \frac{g\omega M}{\phi N^{1/2} \tan(2\theta)} ,
\]

where \(N \approx 50\).

Requiring that the deviations from the spectral index due to the AD scalar are acceptably small, for \(d = 4\), one finds

\[
\alpha > \alpha_c = \frac{3.3 \omega (g\lambda)^{4/9}}{K^{4/9} \tan(2\theta)} ,
\]

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and for \(d = 6\)

\[
\alpha > \alpha_c = \frac{0.18 \omega (g^3 \lambda)^{4/17}}{K^{1/17} \tan(2\theta)}.
\]

(239)

The range of \(\Omega_B\) allowed by nucleosynthesis is \(0.006 \lesssim \Omega_B \lesssim 0.036\) (for \(0.6 \lesssim h \lesssim 0.87\)) [?]. For \(\Omega_m = 0.4, K = 0.2,\) and for \(d = 4,\) we obtain

\[
\alpha_c = (0.06 - 0.36) \frac{(g \lambda)^{4/9}}{\tan(2\theta)},
\]

(240)

and for \(d = 6\)

\[
\alpha_c = (3.0 \times 10^{-3} - 0.018) \frac{(g^3 \lambda)^{4/17}}{\tan(2\theta)}.
\]

(241)

(The lower limits should be multiplied by 0.4 for the case \(\Omega_m = 1\).) If, for example, \(g \sim \lambda \sim 0.1\) and \(\tan(2\theta) \lesssim 1,\) one would obtain a lower bound \(\alpha \gtrsim 10^{-2}\) for \(d = 4\) and \(\alpha \gtrsim 10^{-3}\) for \(d = 6.\) Such small isocurvature contamination could be detectable in future CMB experiments.

Present CMB and large-scale structure observations require that \(\alpha \lesssim 0.1\) [?, ?]. COBE normalization combined with the value of \(\sigma_8\) (the rms of the density field on a scale of 8 Mpc) as obtained from X-ray observations of the local cluster together with the shape parameter \(\Gamma \approx \Omega_m h = 0.25 \pm 0.05\) [?] from galaxy surveys, which is also consistent with the recent observations of high-redshift supernovae [?, ?]) yields the limit \(\alpha \lesssim 0.07.\) The limit may however rely too much on COBE normalization, which is just one experimental result among many.

Future CMB observations by MAP will be able to probe down to \(\alpha \approx 0.1,\) while PLANCK (with CMB polarization measurements) should be able to see isocurvature perturbations as small as 0.04 [?] (see also [?]). For the case of minimal \((d = 4)\) AD baryogenesis, there is a good chance that PLANCK will be able to observe isocurvature perturbations at least if inflation is driven by D-term. For higher dimension AD baryogenesis \((d \geq 6)\) the situation is less certain.

All this assumes that \(\phi\) can take any value. This is true if \(\phi_i < \phi_*\), in which case \(\phi\) remains at its initial value \(\phi_i.\) We have seen that the dynamics of the flat direction during D-term inflation implies that if \(\phi_i > \phi_*\) then \(\phi\) will equal \(\phi_*\) at \(N \approx 50.\) In this case we can fix the magnitude of the isocurvature perturbation. For \(d = 4, N \approx 50.\)
and $\Omega_m = 0.4$ it is given by \cite{??}

$$\alpha = \alpha_* \approx (0.17 - 1.03) \left( \frac{N_S}{50} \right)^{1/4} \left( \frac{g\lambda}{\tan(2\theta)} \right)^{1/2}.$$  \hspace{1cm} (242)

(For $\Omega_m = 1$ this should be multiplied by 0.4.) For $d = 6$ and $\Omega_m = 0.4$,

$$\alpha = \alpha_* \approx (4.4 \times 10^{-3} - 2.6 \times 10^{-2}) \left( \frac{N_S}{50} \right)^{1/8} g^{3/4} \lambda^{1/4} \frac{\tan(2\theta)}{\tan(2\theta)}.$$ \hspace{1cm} (243)

If $g, \lambda \gtrsim 0.1$ then for the $d = 4$ case one expects $\alpha_* \approx 0.01 - 0.1$. For the $d = 6$ case the isocurvature perturbation might just be at the observable level.

It is important that one can fix the isocurvature perturbation to be not much larger than the lower bound coming from adiabatic perturbations. This is because there is typically a very small range of values of $\phi$ over which the isocurvature perturbation is less than the present observational limit, $\alpha \lesssim 0.1$, but larger than the adiabatic perturbation lower bound, $\alpha \gtrsim 0.01$ for $d = 4$.

### 5.4.5 Isocurvature fluctuations in F-term inflation

If the flat direction condensate is stuck in a local minimum $\phi \approx \phi_m$ given by Eq. (190), the isocurvature perturbation is given by \cite{??, ??}

$$\alpha \approx \frac{2\omega}{3} \frac{H}{\tan(2\theta)\delta\rho\phi_m},$$ \hspace{1cm} (244)

where $\delta\rho = 3\delta T/T \approx 3 \times 10^{-5}$ is the density perturbation. Given $H, d$, and the value of $\phi_m$, the magnitude of the isocurvature perturbation is essentially fixed. For $d = 4$ and $\Omega_m = 0.4$, the isocurvature perturbation has been found to be

$$\alpha = (3.1 - 18.6) \times 10^2 \frac{\lambda^{1/2}}{C_1^{1/4} \tan(2\theta)} \left( \frac{H_I}{M_P} \right)^{1/2}.$$ \hspace{1cm} (245)

while for $d = 6$

$$\alpha = (2.9 - 17.4) \times 10^2 \frac{\lambda^{1/4}}{C_1^{1/8} \tan(2\theta)} \left( \frac{H_I}{M_P} \right)^{3/4}.$$ \hspace{1cm} (246)

If we require that $\alpha \lesssim 0.1$ we find the upper bounds $H_I/M_P \lesssim 10^{-7}/\lambda$ (for $d = 4$) and $H_I/M_P \lesssim 10^{-5}/\lambda^{1/3}$ (for $d = 6$). For typical values of $H$ the isocurvature perturbation in the F-term inflation can be close to the present observational limits.
5.5 Baryon number asymmetry

In both D- and F-term inflation the inflaton and other scalar fields begin to oscillate coherently about the minimum of their respective potential after the end of inflation, and the post-inflationary evolution of the flat direction condensate is no exception. If $C_I$ and $C$ are positive in Eqs. (189) and (192), the corresponding vevs are either given by Eqs. (190) or (193). In fact, in D-term inflation models for $|C|$ less than about 0.5, it is possible to have a positive $H^2$ correction and still generate the observed baryon asymmetry as shown by McDonald [7]. In addition, there has been attempts for AD baryogenesis in F-term inflation, basing on the low energy effective action of the heterotic string theory, where inflation is driven by $T$-moduli [8]. Flat directions beyond MSSM involving a triplet Higgs has also been considered [8]. Here we will mainly concentrate on the negative $H^2$ correction only.

After inflation, $\langle \phi \rangle$ initially continues to track the instantaneous local minimum of the scalar potential, which can be derived by replacing $H_I$ with $H(t)$ in Eq.(190). Once $H \approx m_0$, the low-energy soft terms take over. The condensate mass squared turns positive, and since the phase of $\langle \phi \rangle$ differs from the phase of $A$, $\langle \phi \rangle$ starts to change non-adiabatically.

In an absence of $H$ corrections to the $A$-terms, the initial phase $\theta$ of the AD field relative to the real direction is random and so typically $\approx 1$. As a result $\langle \phi \rangle$ starts a spiral motion in the complex plane (see Figs. (2), and forthcoming discussion on the condensate trajectory), which leads to a generation of a net baryon and/or lepton asymmetry [7, 9].

The baryon number density is related to the AD field as

$$n_{B,L} = \beta i (\dot{\phi}^\dagger \phi - \phi^\dagger \dot{\phi}),$$ (247)

where $\beta$ is corresponding baryon and/or lepton charge of the AD field. The equations of motion for the AD field are given by

$$\ddot{\phi} + 3H \dot{\phi} + \frac{\partial V(\phi)}{\partial \phi^*} = 0.$$ (248)

\footnote{There have been attempts for AD baryogenesis in local domains, see [?]}

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The above two equations lead to
\[ \dot{n}_{B,L} + 3Hn_{B,L} = 2\beta \Im \left[ \frac{\partial V(\phi)}{\partial \phi} \right], \]
\[ = 2\beta \frac{m_\phi}{dM^{d-3}} \Im (a_\phi \phi^d). \quad (249) \]

By integrating Eq. (249), we obtain the baryon and/or lepton number as
\[ a^3(t)n_{B,L}(t) = 2\beta |a_\phi| \frac{m_\phi}{M^{d-3}} \int_t^t a^3(t') |\phi(t')|^d \sin(\theta) \, dt'. \quad (250) \]

Note that \( a_\phi \) introduces an extra \( CP \) phase which we may parameterize as \( \sin(\delta) \). After a few expansion times, the amplitude of the oscillations will become damped by the expansion of the Universe and the \( A \)-term, which is proportional to a large power of \( \phi \), will become gradually negligible. The net baryon and/or lepton asymmetry is then given by [?]
\[ n_{B,L}(t_{osc}) = \beta \frac{2(2d-2)}{3(3d-3)} m_\phi \phi_0^2 \sin 2\theta \sin \delta, \]
\[ \approx \frac{2(2d-2)}{3(3d-3)} m_\phi (m_\phi M^{d-3})^{2/(d-2)} \sin 2\theta \sin \delta, \quad (251) \]

where \( \sin \delta \sim \sin 2\theta \approx \mathcal{O}(1) \).

When the inflaton decay products have completely thermalized with a reheat temperature \( T_{rh} \), the baryon and/or lepton asymmetry is given by [?]
\[ \frac{n_{B,L}}{s} = \frac{1}{4} \frac{T_{rh}}{M_P^2 H(t_{osc})^2} n_{B,L}(t_{osc}) = \frac{d-2}{6(d-3)} \beta \frac{T_{rh}}{M_P^2 m_\phi} (m_\phi M^{d-3})^{2/(d-2)} \sin 2\theta \sin \delta, \quad (252) \]

where we have used \( H(t_{osc}) \approx m_\phi \), and \( s \) is the entropy density of the Universe at the time of reheating. For \( d = 4 \), the baryon-to-entropy ratio turns out to be [?]
\[ \frac{n_{B,L}}{s} \approx 1 \times 10^{-10} \times \beta \left( \frac{m_{3/2}}{m_\phi} \right) \left( \frac{M}{M_P} \right) \left( \frac{T_{rh}}{10^9 \text{ GeV}} \right), \quad (253) \]
and for \( d = 6 \)
\[ \frac{n_{B,L}}{s} \approx 5 \times 10^{-10} \times \beta \left( \frac{m_{3/2}}{m_\phi} \right) \left( \frac{1 \text{ TeV}}{m_\phi} \right)^{1/2} \left( \frac{M}{M_P} \right)^{3/2} \left( \frac{T_{rh}}{100 \text{ GeV}} \right), \quad (254) \]

where we have taken the net \( CP \) phase to be \( \sim \mathcal{O}(1) \). The asymmetry remains frozen unless there is additional entropy production afterwards. Note that for \( d = 4 \), the
required reheat temperature of the Universe is below the gravitino overproduction bound (see Sect. 3.6.2). For higher dimensional non-renormalizable operators, a low reheat temperature is favorable, which is indeed a good news.

In this regard low scale inflation, which guarantees a low reheat temperature, has been given some consideration [?] (see also [?] where AD baryogenesis after a brief period of thermal inflation, required to solving the cosmological moduli problem, has been discussed). However, in [?] it was pointed out that in gauge mediated supersymmetry breaking it is hard to reconcile AD baryogenesis with a moduli problem.

Among the host of MSSM flat directions which are lifted by non-renormalizable operator and listed in Table 1, the $LH_u$ flat direction carrying the lepton number is the candidate for producing lepton asymmetry in the Universe (there has been some earlier attempts of direct baryogenesis via $\bar{u}\bar{d}\bar{d}$ directions, see e.g. [?]). The lepton asymmetry calculated above in Eqs. (253,254) can be transformed into baryon number asymmetry via sphalerons $n_B/s = (8/23)n_L/s$. AD leptogenesis has important implications in neutrino physics also, because in the MSSM, the $LH_u$ direction is lifted by the $d = 4$ non-renormalizable operator which also gives rise to neutrino masses [?]:

$$W = \frac{1}{2M_i}(L_i H_u)^2 = \frac{m_{\nu_i}}{2\langle H_u \rangle^2}(L_i H_u)^2,$$

where we have assumed the see-saw relation $m_{\nu_i} = \langle H_u \rangle^2/M_i$ with diagonal entries for the neutrinos $\nu_i$, $i = 1, 2, 3$. The final $n_B/s$ can be related to the lightest neutrino mass since the flat direction moves furthest along the eigenvector of $L_i L_j$ which corresponds to the smallest eigenvalue of the neutrino mass matrix [?].

$$\frac{n_L}{s} \approx 1 \times 10^{-10} \times \beta \left( \frac{m_{3/2}}{m_\phi} \right) \left( \frac{T_{rh}}{10^8 \text{ GeV}} \right) \left( \frac{10^{-6} \text{ eV}}{m_{\nu_l}} \right),$$

where $m_{\nu_l}$ denotes the lightest neutrino.

### 5.6 Thermal effects

In our discussion on the baryon/lepton asymmetry we have tacitly assumed that the asymmetry has been generated before the Universe has thermalized and reheated. This might not be the case if there were light degrees of freedom which have thermalized
with an instantaneous plasma temperature $T_{\text{inst}} \leq (H \Gamma_d M_P)^{1/4}$ before the inflaton has decayed. We remind that the bulk of energy density is still in the form of inflaton oscillations, and only a fraction of the energy density has gone into these light Standard Model degrees of freedom. If the MSSM flat direction couples with this thermal bath, there arises a modification in the flat direction potential [?, ?, ?].

5.6.1 Thermal corrections to the flat direction potential

Besides the D-term couplings of the form $g^2 \phi^2 \alpha^2$, where $\alpha$ is some field with gauge interactions, there are also F-term Yukawa couplings to fields $\chi$ which result in a term $h^2|\phi|^2|\chi|^2$ in the flat direction potential. $\chi$ and $\alpha$ obtain large masses due to the flat direction vev, and therefore do not feel the effect of temperature if the condensate amplitude $\phi$ is large. As pointed out by Allahverdi, Campbell and Ellis [?], the back-reaction effect induces a mass-squared term $h^2 T^2$ for the flat direction. If this exceeds the negative Hubble induced mass squared term, the flat direction oscillations starts earlier than otherwise expected. In order for thermal correction to play a significant role the couplings $h, g$ must have intermediate strength. Otherwise, a large coupling would induce a large vev dependent mass for $\alpha$, which would prevent its thermal excitations, and a very small coupling would not have significant thermal backreaction at all.

For the inflationary scale $H_I \sim 10^{13}$ GeV and $M = M_{\text{GUT}}$, it has been found [?] that a generic MSSM flat direction with a Yukawa coupling $h \sim 10^{-2}$ starts oscillating at $H \gg 10^2$ GeV for $4 \leq d \leq 8$. Since thermal effects induce early oscillations, baryon asymmetry is also produced much earlier, which could have interesting consequences.

Another thermal effect has been discussed by Anisimov and Dine [?, ?]. All the flat directions which are lifted at large $d$ give rise to a large mass for $\alpha$, and consequently one should account for their effect by integrating out the heavy modes. This would result in terms like

$$A \frac{16 \pi^2}{F_{\mu \nu}^2} \ln \frac{|\phi|^2}{M^2}. \quad (257)$$

In particular for the flat direction $LH_u$ the effective potential thus obtained has the
\[ V_{\text{eff}} = \alpha_s^2(T) a_g T^4 \ln \frac{\phi^2}{M^2}, \quad (258) \]

where \( \alpha_s \equiv g_s^2 / 4\pi \), and \( a_g = \frac{3 N_g}{288} \left( \frac{5 N_f}{4} + \frac{7 N_f}{2} \right) \) includes leading order contribution of the gluons, gluinos, and quarks to the free energy. The oscillations in the flat direction are induced when \( H_{\text{osc}}^2 = \partial^2 V_{\text{eff}} / \partial |\phi|^2 = \alpha_s^2 a_g T^4 / |\phi|^2 \) and one can check that \( 7 \) for \( d = 4 \), the thermal mass correction \( \sim h^2 T^2 \) wins over the logarithmic counterpart Eq. (258), but for \( d = 5 \) and/or 6, the logarithmic correction dominates and the oscillations start earlier than otherwise one would have expected.

There could also be a thermal enhancement of the \( A \)-term \( ? \), which can arise from the cross terms

\[ W \supset h \phi \bar{\alpha} \alpha + \lambda_d \frac{\phi^d}{d M^{d-3}}. \quad (259) \]

However a symmetry forbids such enhancement \( ? \), although the situation might change if one adds more terms in the superpotential such as \( ? \).

\[ W = \frac{1}{M} \left( a I + b \frac{I^2}{M} \right) \frac{\phi^d}{M^{d-3}}. \quad (260) \]

5.6.2 Thermal evaporation of the flat direction

It has been argued that in general the flat direction condensate decays as a result of scattering with the thermalized decay products of the inflaton \( ? \). Usually the scattering interactions preserve \( B \) and \( L \), and therefore the previously produced baryon and/or lepton asymmetry remains unchanged. In \( ? \) it was assumed that thermalized fermions scatter with the condensate with a rate

\[ \Gamma_{\text{scatt}} \simeq y g^2 T. \quad (261) \]

where \( y T \) corresponds to the mass of the condensate. A complete evaporation was found to occur only after reheating if \( T_{\text{rh}} \leq \left( y g^2 \right)^{2/3} M_p^{5/6} H^{1/6} \), which is usually satisfied for a reasonable range of reheat temperatures and Yukawa couplings.

5.7 Baryosynthesis and neutrino mass

As discussed in Sect. 5.4., the lepton asymmetry via \( LH_u \) direction leads to a prediction on the lightest neutrino mass. It is however pertinent to include also the finite
temperature effects [?]. At finite T, the flat direction potential potential for \( LH_u \) direction can be written as [?]

\[
V_{\text{total}} = \left( m_\phi^2 - C_I H^2 + \sum_{f_k | \phi | < T} c_k f_k^2 T^2 \right) |\phi|^2 + \frac{m_{3/2}}{8M} \left( a_m \phi^4 + \text{h.c.} \right) + \frac{H}{8M} \left( a_H \phi^4 + \text{h.c.} \right) + a_g \alpha_s^2 T^4 \ln \left( \frac{|\phi|^2}{T^2} \right) + \frac{|\phi|^6}{4M^2},
\]

(262)

where \( c_k = \) are real positive constants and couplings \( f_k = 1 - 10^{-5} \) in MSSM [?]. The mismatch in phases between \( a_m \) and \( a_H \) leads to the helical motion of the flat direction. Once the inflaton decay products generate a thermal plasma with a temperature \( T = \left( T_{rh} M_p H_I \right)^{1/4} \), thermal corrections take over the Hubble induced term

\[
H^2 \leq m_\phi^2 + \sum_{f_k | \phi | < T} c_k f_k^2 T^2 + a_g \alpha_s^2(T) \frac{T^4}{|\phi|^2}.
\]

(263)

The flat direction starts to oscillate when [?]

\[
H_{\text{osc}} \approx \max \left[ m_\phi, H_i, \alpha_s T_{rh} \left( \frac{a_g M_p}{M} \right)^{1/2} \right],
\]

(264)

where \( H_i \) is given by [?, ?]

\[
H_i \approx \min \left\{ \frac{1}{f_i^2} \frac{M_p T_{rh}^2}{M^2}, \left( c_i^2 f_i^4 M_p T_{rh}^2 \right)^{1/3} \right\}.
\]

(265)

The lepton asymmetry is then given by [?]

\[
a^3(t)n_L(t) \approx \frac{m_{3/2}}{2M} \int^t dt' a^3(t') \Im \left( a_m \phi^4 \right),
\]

(266)

The right hand side of Eq. (266) initially increases until \( H \approx H_{\text{osc}} \), after which the integrand is rapidly damped because \( a^3 \phi^4 \sim t^{-n} \) for \( n > 1 \). The final lepton asymmetry is determined approximately by the configuration at the time when the oscillations commence [?]

\[
n_L = \left. \frac{m_{3/2}}{2M} \Im \left( a_m \phi^4 \right) t \right|_{H = H_{\text{osc}}} = \frac{1}{3} m_{3/2} M H_{\text{osc}} \delta_{\text{eff}},
\]

(267)

where \( \delta_{\text{eff}} = \sin(4\arg(\phi) + \arg(a_m)) \) is the net \( CP \) phase. The final baryon to entropy ratio turns out to be [?]

\[
\frac{n_B}{s} = 10^{-11} \delta_{\text{eff}} \times \left( \frac{m_{\nu, 1}}{10^{-8} \text{eV}} \right)^{-3/2} \left( \frac{m_{3/2}}{1 \text{ TeV}} \right).
\]

(268)
Figure 2: Affleck-Dine condensate formation with \( x = \phi_1 \) and \( y = \phi_2 \), for (a) gravity mediated case with \( d = 4 \) (solid) and \( d = 6 \) (dashed), and (b) gauge mediated case with \( d = 4 \), \( m_\phi = 1 \) TeV (solid) and \( m_\phi = 10 \) TeV (dashed) with the initial condition \( \theta_i = -\pi/10 \), from [?].

Note that the final expression obtained does not depend upon the reheat temperature \( T_{rh} \), mainly due to the fact that the \( H_{osc} \) is determined by thermal correction \( \sim T^4 \ln(|\phi|^2) \). This is however true only for \( 10^8 \) GeV \( \leq T_{rh} \leq 10^{12} \) GeV as pointed out by Fujii, Hamaguchi, and Yanagida in [?]. For \( 10^5 \) GeV \( \leq T_{rh} \leq 10^8 \) GeV, the dependence on reheat temperature appears as \( n_B/s \propto T_{rh}^{1/3} \), because then the thermal mass term \( \sim T^2 |\phi|^2 \) dominates.

It is possible [?] to obtain the right amount of baryon asymmetry with the lightest neutrino mass \( m_{\nu l} \simeq (0.1 - 3) \times 10^{-9} \) eV and with a \( CP \) phase \( \delta_{eff} \simeq (0.1 - 1) \) for a fairly wide range of reheat temperature \( 10^5 \) GeV \( \leq T_{rh} \leq 10^{12} \) GeV.

5.8 Trajectory of a flat direction

Let us now turn our attention to the dynamical evolution of the flat direction after the end of inflation. Here we assume that the flat direction is tracking its minimum which is determined by Eq. (190). The trajectory of the flat direction depends upon the potentials Eqs. (200) and (204). Here we sketch the main differences between the gravity and gauge mediated cases.

Jokinen [?] has studied numerically the trajectories of the flat direction condensate in gravity and gauge mediated cases, following Eqs. (200) and (204). The rotation of the condensate depends on the low energy supersymmetry breaking mass terms. The
classical motion for the condensate $\phi = (\phi_1 + i\phi_2)/\sqrt{2}$ is illustrated in Fig. (2). In the gravity mediated case, Fig. (2a), we see that the orbit is a spiraling ellipse and in the gauge mediated case, Fig. (2b), a precessing trefoil. From Fig. (2), one can see that there is a twist on the orbit much before the rotation starts properly. This is the time of the phase transition, when the condensate $\phi$ starts to rotate in the pit of the symmetry breaking minimum. The rotation begins when the symmetry breaking minimum is the vacuum, and ends when it has become a false vacuum, and twists when the false vacuum has completely vanished forming a kink on the orbit. It is possible to produce a condensate through a second order phase transition, but the charge in that case will be small. It should also be noted that in the gravity mediated case condensate formation starts when $C_I H^2 \sim m_{3/2}^2$ for all values of $d$, $A$ and $a$. In the gauge mediated case the condensate formation starts at $C_I H^2 \sim m_\phi^4/|\phi|^2$, so that the formation happens earlier if the condensate mass $m_\phi$ is increased, as can be seen from the different positions of the kink in Fig. (2b).

5.9 Instability of the coherent condensate

5.9.1 Negative pressure

The effective equation of state of a coherent scalar field oscillating in a potential $U(\phi)$ with a frequency which is large compared with $H$ is obtained by averaging $p/\rho = (2|\dot{\phi}|^2/\rho) - 1$ over one oscillation cycle $T$. The result is [?]

$$p = (\gamma - 1)\rho ,$$

(269)

where

$$\gamma = \frac{2}{T} \int_{0}^{T} (1 - U(\phi)/\rho) \, dt .$$

(270)

For the case $U \sim \phi^2$, one finds $\gamma = 1$, so that one effectively obtains the usual case of pressureless, non-relativistic cold matter.

When the motion of the condensate field is not simply oscillatory, such as in the case for the condensate trajectory, one can generalize Eq. (270) by integrating over
the orbit $c$ of the AD field. In that case

$$\gamma = \frac{2 \int_c d|\phi| (1 - U(\phi)/\rho)^{1/2}}{\int_c d|\phi| (1 - U(\phi)/\rho)^{-1/2}}.$$  \hfill (271)

In practice the orbits are nearly elliptical. Then the arc length is given by

$$d|\phi| = \frac{d\phi_1}{\sqrt{2}} \left[ 1 + \frac{B^2\phi_1^2}{A^4(1 - \phi_1^2/A^2)} \right],$$  \hfill (272)

where $A$ and $B \leq A$ are respectively the semi-major and the semi-minor axis of the ellipse, and $\phi_1 = Re\phi/\sqrt{2}$. For a circular orbit $B = A$, whereas for pure oscillation (no charge in the condensate) $B = 0$.

It is therefore obvious that small corrections to a harmonic potential of a coherent condensate can easily generate a pressure. As we have seen, in the gravity mediated case quantum corrections typically modify the flat direction mass terms by

$$U(\phi) = \frac{1}{2}m_\phi^2\phi^2 + Km_\phi^2\phi^2 \log \left( \frac{\phi^2}{\mu^2} \right) + \ldots,$$  \hfill (273)

where $K$ is some constant. If one writes

$$U(\phi) = \frac{1}{2}m_\phi^2\phi^2 \left( \frac{\phi^2}{\mu^2} \right)^x$$  \hfill (274)

one finds that

$$\gamma = \frac{1 + x}{1 + \frac{x}{2}}, \quad p = \frac{x}{2 + x}. \hfill (275)$$

In the case of the logarithmic potential $x \simeq 2K$. There arises a negative pressure \( p = K\rho \) whenever $K < 0$ or whenever $x$ is small and negative. This is a sign of an instability of the condensate under arbitrarily small perturbations.

This is exactly the situation one finds in the MSSM with flat directions. The effective mass $m^2_{\text{eff}}(\phi) \equiv dU/d\phi^2$ decreases for a range in $\phi$, albeit for different reasons, both for gravity mediated and gauge mediated supersymmetry breaking. According to Eq. (271) this results in a negative pressure, which has been computed numerically by Jokinen [?]. The results are shown in Figs. (3) and (4).

The pressure-to-energy density ratio; $w = \gamma - 1$ for the gravity mediated case is plotted in Fig. (3a), which shows that there are time-dependent oscillations in pressure [?]. The average pressure is slightly on the negative side. The average value of $w$ is
Figure 3: Pressure-to-energy density ratio, $p/\rho \equiv w = \gamma - 1$, in the gravity mediated case vs. (a) time in logarithmic units for $d = 4$, (b) different initial conditions for $d = 4, 6$; (c) ellipticity $\varepsilon = B/A$ vs. initial conditions for $d = 4$ (thin lines), $d = 6$ (thick lines), D-term (solid), F-term (dashed) with $K = -0.01$ and $t = 100m_{3/2}^{-1}$. In (b) $w$ is shown at $t = 300m_{3/2}^{-1}$ with dotted lines for the $d = 4$ D-term case, from [?].

shown in Fig. (3b) at $t \sim 100m_{3/2}^{-1}$ for a few different initial conditions. In Fig. (3c), the ellipticity of the orbit, $\varepsilon = B/A$, is plotted to show that $w$ is more negative if $\varepsilon$ is small. It should be noted that $w$ achieves values which are more negative than the absolute lower bound coming from pure oscillation.

A similar analysis has been made for the gauge mediated supersymmetry breaking case. The results have striking similarities [?]. In Figs. (4a) and (4b), the time development of the pressure-to-energy density ratio, $w$, for $d = 4$ and $d = 6$ has been depicted. One can see that the pressure is always negative. The calculation of average pressure is even more involved than in the gravity mediated case, since the oscillation frequency becomes very large. In Fig. (4c), the ellipticity of the orbit is shown as a function of different initial conditions. Jokinen [?] has pointed it out that quite generically $\varepsilon \lesssim 0.1$.

5.9.2 Growth of perturbations in the AD condensate

As a result of internal negative condensate pressure the quantum fluctuations in the scalar condensate grow according to [?]

\[ \ddot{\delta}_k = -Kk^2\delta_k \, . \]  

\[ (276) \]
Figure 4: Pressure-to-energy density ratio, $w$, in the gauge mediated D-term case (without Hubble induced $A$-term) vs. time in logarithmic units for (a) $d = 4$ and (b) $d = 6$; (c) ellipticity of the orbit where $d = 4$ (thin lines) and $d = 6$ (thick lines). The scalar masses $m_\phi = 1, 10, 100$ TeV are denoted respectively with solid, dotted and dashed lines, from [?].

Figure 5: Energy-to-charge ratio, $x$, in the gravity mediated case vs. (a) time in logarithmic units for $d = 4, 6$; (b) the D-term case; (c) the F-term (with Hubble induced $A$-term) case with $d = 4, 5, 6, 7$ (solid, dash-dot, dashed and dotted lines), $K = -0.01$ and $t = 100 m_{3/2}^{-1}$, from [?].
Figure 6: Energy-to-charge ratio, $x$, in the gauge mediated case vs. (a) time in logarithmic units of time $d = 4$, (b) $d = 4$ and (c) $d = 6$ with D-term (thin lines, without Hubble induced A-term) and F-term (thick lines, with Hubble induced A-term) and $m_\phi = 1, 10, 100$ TeV (solid, dashed, dotted lines) at $t = 4 \cdot 10^5 m_\phi^{-1}$, $10^5 m_\phi^{-1}$, $4 \cdot 10^4 m_\phi^{-1}$ ($d = 4$) and $t = 4 \cdot 10^9$, $10^9$, $4 \cdot 10^8 m_\phi^{-1}$ ($d = 6$), from [?].

If $K < 0$, quantum fluctuations of the condensate field at the scale $\lambda = 2\pi/|k|$ will grow exponentially in time as

$$
\delta \phi_k(t) = \delta \phi(0) \exp \left( -Kk^2t \right).
$$

In reality the onset of non-linearity sets the scale at which the spatial coherence of the condensate can no longer be maintained and the condensate fragments. For the AD condensate the initial perturbation originates from inflation. Note that since the AD condensate carries a global charge, due to charge conservation the energy-to-charge ratio changes as the the condensate fragments.

The energy-to-charge ratio has been estimated numerically for both gravity and gauge mediated cases by Jokinen [?]. The time evolution of the energy-to-charge ratio $x$ is shown in Fig. 5, where $x$ is also plotted for various initial phases in F- and D-term inflation models. For the gauge mediation case the plots are quite different from the gravity mediated case, see Fig. (6).

5.9.3 The true ground state

Under the negative pressure the homogeneous AD condensate fragments and forms lumps. The question then is, what is the true ground state? The answer is, a non-topological soliton with a fixed charge, called the $Q$-ball [?, ?, ?], which in general
is made up of a complex scalar field with a global $U(1)$ symmetry, for which the Lagrangian is

$$\mathcal{L} = \partial_\mu \phi \partial^\mu \phi^* - U(\phi \phi^*) .$$  \hfill (278)

When supplemented by the CP violating terms, this is the Lagrangian for the MSSM flat directions.

The conserved current is $j^\mu = i(\phi \partial^\mu \phi^* - \phi^* \partial^\mu \phi)$, and the conserved charge, and energy are given by

$$Q = \int d^3 x j^0 ,$$  \hfill (279)

$$E = \int d^3 x [|\dot{\phi}|^2 + |\nabla \phi|^2 + U(\phi \phi^*)] .$$  \hfill (280)

If the charge is kept fixed, the state of lowest energy is found by minimizing $E_\omega$

$$E_{\omega} = E - \omega Q$$

with respect to variations in $\phi$ and the Lagrange multiplier $\omega$. From $\delta_\phi E_{\omega} = 0$, it follows that $\dot{\phi} - i\omega \phi = 0$, so that we may write

$$\phi(t, x) = e^{i\omega t} \varphi(x)$$  \hfill (282)

where $\varphi$ may be chosen real by virtue of $U(1)$ invariance. The charge and energy of such a configuration read

$$Q = 2\omega \int d^3 x \varphi^2 ,$$  \hfill (283)

$$E = \int d^3 x \left[ |\nabla \varphi|^2 + U(\varphi^2) + \omega^2 \varphi^2 \right] ,$$  \hfill (284)

and one has to minimize $E_{\omega} = \int d^3 x [(|\nabla \varphi|^2 + \hat{U}_\omega(\varphi^2)] + \omega Q$ where

$$\hat{U}_\omega(\varphi^2) = U(\varphi^2) - \omega^2 \varphi^2 .$$  \hfill (285)

To find a localized configuration that vanishes at spatial infinity one may make use of the spherical rearrangement theorem, which implies that $E_{\omega}$ is minimized by $\varphi(x)$ which is spherically symmetric and monotonically decreasing. This is equivalent to solving the equation of motion

$$\frac{d^2 \varphi}{dr^2} + \frac{2}{r} \frac{d \varphi}{dr} - \varphi \frac{d U_\omega}{d \varphi^2} = 0 .$$  \hfill (286)
If by convention we set the globally symmetric minimum to $\varphi = 0$ with $U(0) = 0$, one can then show that a non-trivial solution to Eq. (286) is obtained whenever $U(\varphi^2)/\varphi^2$ has a minimum at $\varphi \neq 0$, i.e. $U(\varphi^2)$ grows more slowly than $m_\phi^2 \varphi^2$ over some range. We will discuss $Q$-balls in detail in the next Sects. 6 and 7.

5.10 Numerical studies of fragmentation

Although the homogeneous AD condensate is not the ground state, it is not obvious that the ground state should always be reached within cosmic time scales. It is then essential to study the dynamical evolution of the AD condensate. Since the $Q$-ball formation is inherently a non-linear phenomenon, analyzing small perturbations is not sufficient to determine the full dynamical evolution of the AD condensate. One can nevertheless gather some information about the gross features of the condensate fragmentation by perturbative considerations alone.

5.10.1 Perturbation theory

Negative pressure is equivalent to an attractive force between the condensate quanta which induces a growing mode in spatial perturbations. A linearized description of the evolution of perturbations has been given by Kusenko and Shaposhnikov in [?], and by Enqvist and McDonald in [?]. For a maximally charged AD condensate ($B = 0$ in Eq. (272)), the linearized perturbation takes the form $\phi = \phi(t) + \delta\phi(x, t)$, and $\theta = \theta(t) + \delta\theta(x, t)$, where the homogeneous condensate is described by

$$\Phi = \frac{\phi(t)}{\sqrt{2}} e^{i\theta(t)},$$

(287)

with $\phi(t) = (a_{\phi}/a)^{3/2}\phi_o$ and $\dot{\theta}(t)^2 \approx m_\phi^2$. Initially $\delta\phi(x, t)$ and $\delta\theta(x, t)$ should satisfy the relationship [?, ?, ?]

$$\delta\theta_i \approx \left( \frac{\delta\phi}{\phi} \right)_i.$$

(288)

The solution of the linear perturbation equations then has the form [?]

$$\delta\phi \approx \left( \frac{a_{\phi}}{a} \right)^{3/2} \delta\phi_o \exp \left( \int dt \left( \frac{1}{2 a^2} \frac{|K|m_\phi^2}{\theta(t)^2} \right)^{1/2} \right) e^{ikx}$$

(289)
and
\[ \delta \theta \approx \delta \theta_i \exp \left( \int \frac{dt}{a^2} \left( \frac{1}{2} \frac{K|m_\phi^2}{\dot{\theta}(t)^2} \right)^{1/2} \right) e^{ik \cdot x}. \] (290)

For the gravity mediation case, the above condition applies if \(|k^2/a^2| \lesssim |2K| m_\phi^2|, and \(H^2\) is small compared with \(m_\phi^2\) and \(|K| < 1\). If the first condition is not satisfied, then the gradient energy of the perturbations produces a positive pressure larger than the negative pressure due to the attractive force from the logarithmic term, preventing the growth of the perturbations.

For the case of a matter dominated Universe, the exponential growth factor is then
\[ \int dt \left( \frac{1}{2} \frac{K|m_\phi^2}{|\dot{\theta}(t)|^2} \right)^{1/2} = \frac{2}{H} \left( \frac{|K| k^2}{2 a^2} \right)^{1/2}. \] (291)
The largest growth factor will correspond to the largest value of \(k^2\) for which growth can occur,
\[ \left| \frac{k^2}{a^2} \right|_{\text{max}} \approx 2|K|m_\phi^2. \] (292)
The value of \(H\) at which the first perturbation goes non-linear is \([?, ?]\)
\[ H_i \approx \frac{2|K|m_\phi}{\alpha(\lambda)}, \] (293)
with
\[ \alpha(\lambda) = -\log \left( \frac{\delta \phi_o(\lambda)}{\phi_o} \right), \] (294)
where \(\phi_o\) is the value of \(\phi\) when the condensate oscillations begin at \(H \approx m_\phi\). A typical value of \(\alpha(\lambda)\) (e.g. with \(d = 6\)) is \(\alpha(\lambda) \approx 30\). The initial non-linear region has a radius \(\lambda_i\) at \(H_i\), which is given by \([?, ?]\)
\[ \lambda_i \approx \frac{\pi}{|2K|^{1/2} m_\phi}. \] (295)

For the case of a non-maximally charged condensate the situation is slightly different. It is likely that the initial radius and the time at which the spatial perturbations initially go non-linear will roughly be the same \([?]\) as for the maximally charged condensate. In general, the charge density of the initial non-linear lumps will essentially be the same as that of the original homogeneous condensate.
The perturbative evolution of a single condensate lump was considered in [?]. In terms of \( \phi = (\phi_1 + i\phi_2)/\sqrt{2} \), the initial lumps are described by

\[
\phi_1(r, t) = A \cos(m_\phi t)(1 + \cos(\pi r/r_0)) \quad (296)
\]

\[
\phi_2(r, t) = B \sin(m_\phi t)(1 + \cos(\pi r/r_0)) \quad ,
\]

for \( r \leq r_0 \) and by \( \phi_{1,2} = 0 \) otherwise. The initial radius of the lump is \( 2r_0 \), where \( r_0 = \pi/(\sqrt{2}|K|^{1/2}m_\phi) \). The maximally charged condensate lump corresponds to \( A = B \), while the non-maximal lump has \( A > B \). The total energy and charge in a fixed volume are given by [?, ?]

\[
E = 4\pi \int_V drr^2 \rho \quad , \quad Q = 4\pi \int_V drr^2 q \sim AB ,
\]

with \( Q_{\text{max}} = A^2 \).

In [?], the behavior of the solutions was found in a perturbative analysis to depend on \( K \), and to a greater extent on \( Q/Q_{\text{max}} \). The condensate lump was found to pulsate while charge is flowing out until the lump reaches a (quasi-)equilibrium pseudo-breather configuration, also called \( Q \)-axiton, with the lump pulsating with only a small difference between the maximum and minimum field amplitudes. For the \( Q \)-axiton, in which the attractive force between the scalars is balanced by the gradient pressure of the scalar field, the energy per unit charge is much larger than \( m_\phi \); indeed, the \( Q \)-axiton exists even if \( Q = 0 \). Only for a maximally charged \( Q \)-axiton are the properties similar to that of the corresponding \( Q \)-ball. It is however unclear whether \( Q \)-axitons are just an artifact of perturbation expansion.

### 5.10.2 Lattice simulations

The features of the fragmentation of the AD condensate cannot be fully captured by studying various mean field theory approaches, such as in large \( N \)-approximation and Hartree-approximation [?, ?, ?]. The formation of a \( Q \)-ball is a non-linear process for which various mode-mode interactions become important. This can be seen by expanding the perturbed \( \phi \) and \( \theta \) as shown by Kasuya and Kawasaki in [?, ?, ?]

\[
\delta \ddot{\phi} + 3H\dot{\phi} - 2\dot{\theta}\dot{\phi} - \frac{\nabla^2}{a^2}\delta \phi + U''(\phi)\delta \phi = 0 ,
\]
\[ \phi \ddot{\theta} + 3H \phi \dot{\theta} + 2(\dot{\phi} \dot{\theta}) - 2 \frac{\phi}{\dot{\phi}} \dot{\theta} \delta \phi - \phi \frac{\nabla^2}{a^2} \delta \theta = 0. \] 

(299)

Although the potentials differ in the gauge and gravity mediated cases, it is nevertheless always possible to identify the fastest growing amplified mode. In the gravity mediated case we have already obtained that by inspecting Eq. (291). A similar analysis can be performed for the gauge mediated case by noting that \( U''(\phi) \approx -2m_\phi^4/\phi^2 \).

Taking into account the conservation of charge \( \dot{\theta} \phi^2 a^3 = \text{const.} \), along with the approximation of a circular orbit, one may simplify Eq. (299) by seeking a solution of the form \( \delta \phi = \delta \phi_0 \exp(\alpha t + ikx) \) and \( \delta \theta = \delta \theta_0 \exp(\alpha t + ikx) \). In order to further simplify the analysis, one can also assume \( a = \text{const.} \) and \( \phi = \phi_0 = \text{const.} \), so that the phase velocity \( \dot{\theta} = (U'/\phi)^{1/2} \approx \sqrt{2m_\phi^2}/\phi_0 \). If \( \alpha \) is real and positive, the fluctuations grow exponentially and become non-linear. Solving for \( \delta \phi_0, \delta \theta_0 \), Kasuya and Kawasaki finds for gauge mediated case [?]

\[ \alpha^4 + 2 \left( \frac{k^2}{a^2} + \frac{2m_\phi^4}{\phi_0^2} \right) \alpha + \left( \frac{k^2}{a^2} - \frac{4m_\phi^4}{\phi_0^2} \right) k^2 = 0. \] 

(300)

Note that in order for \( \alpha \) to be positive, one must require the expression in the second parenthesis to be negative. This means that the instability band for the fluctuations is given by

\[ 0 < \frac{k}{a} < \frac{2m_\phi^2}{\phi_0}. \] 

(301)

The most amplified mode appears at \( (k/a)_{\text{max}} \approx (3/2)^{1/2}m_\phi^2/\phi_0 \) in the gauge mediated case.

Various groups have studied the fragmentation of the AD condensate and the formation of \( Q \)-balls numerically. In [?], condensate fragmentation was simulated numerically on a \( 2 + 1 \) dimensional \( 100 \times 100 \) lattice, starting with a uniform AD-condensate with \( \phi_0 = 10^9 \text{ GeV} \) and with an arbitrary phase \( \omega, \phi = \phi_0 e^{i\omega t} + \delta \phi \) with uniformly distributed random noise \( \delta \phi \sim \mathcal{O}(10^{-13})|\phi_0| \) added to the amplitude and phase. The parameter values chosen for the simulations were \( m_\phi = 10^2 \text{ GeV}, \ K = -0.1, \) and \( \lambda = 1/2 \). The results indicate that first the charge density of the condensate decreases uniformly due to the expansion of the Universe. As time progresses a growing mode can be seen to develop. White noise is still present but the growing mode soon