Three-body monopole corrections to the realistic interactions

\begin{align}
\left( \frac{\tau_{\text{A}}}{\tau_{\text{B}}} \right) &= \frac{(\tau_{\text{A}} - \tau_{\text{B}})}{2} \left[ 1 + \left( \frac{\tau_{\text{A}}}{\tau_{\text{B}}} \right) \right] \\
\left( \frac{\tau_{\text{A}}}{\tau_{\text{B}}} \right) &= \frac{(\tau_{\text{A}} - \tau_{\text{B}})}{2} \left[ 1 + \left( \frac{\tau_{\text{A}}}{\tau_{\text{B}}} \right) \right] \\
\left( \frac{\tau_{\text{A}}}{\tau_{\text{B}}} \right) &= \frac{(\tau_{\text{A}} - \tau_{\text{B}})}{2} \left[ 1 + \left( \frac{\tau_{\text{A}}}{\tau_{\text{B}}} \right) \right]
\end{align}
they correct $\varepsilon_s$ which will be taken from experiment as traditionally done. The latter, together with the 3b part will transform the realistic (R) 2b centroid $V_{2b}^T(R)$ into

$$V_{2b}^T(R) = V_{2b}^T(R) + \varepsilon_s R_n \equiv V_{1b}^T(R) + \varepsilon_s R_n.$$  

$H_{1b}$ can be characterized by demanding correct single particle and single hole spectra around closed shell nuclei [18]. This set $(\varepsilon_s \pm 1)$ is taken to include the differences in binding energies (gaps) $2BE(\varepsilon_s) - BE(\varepsilon_s + 1) - BE(\varepsilon_s - 1)$. The major monopole correction involves the gaps around $^{12}$C, $^{28}$Si, $^{48}$Ca and $^{56}$Ni which are too small to produce the observed double magicity [19]. It will be taken care by a single linear form $\kappa \equiv \kappa(n)$. The generalization of Eq. (6) is then

$$V_{1b}^T(R) = V_{1b}^T(R) - (-)^T \kappa + \chi_{1b}^T,$$

$$V_{1b}^T(R) = (V_{1b}^T(R) - 1.5 \kappa) \delta_{T0} + \chi_{1b}^T,$$

$$V_{1b}^T(R) = V_{1b}^T(R) + \chi_{1b}^T.$$  

(8)

The single particle splittings above the $f$ closures are quite well given by some R interactions. Hence the corrective term $\chi_{1b}^T$, which will prove useful in the $sd$ shell—most likely to have a $2b$ origin, $\chi_{1b}^T$ is introduced only for completeness and will be altogether disregarded. $\chi_{1b}^r$, must play an important role because the single hole states (at $A = 15, 39$ and 79) [20] are severely missed. However, they have little influence on the nucleon we shall study (at the beginning of the shells).

For the $^{16}$O spectrum in Fig. 1 the black squares show the results of Nvratil and Ormand (NO) [14, Fig. 4, 6b] for the low lying $T = 0$ states in $^{16}$O. The black circles correspond to the bare KLS G-matrices [21, 22] used in [23]. The agreement with experiment (lines) [24] is poor but the agreement between the calculations is good. This is not a joke, but an important remark: NO provides the foundation for a conventional G-matrix study. As emphasized over the years [10, 23, 25], the realistic G-matrices are very close to one another and will provide good spectra once monopole corrected. Absolute energies and strength functions are another matter, and much remains to be learnt from exact and no-core results.

The open pentagons in Fig. 1 correspond to the classic Coulomb Kurath fit [20] (CK). The open squares and circles refer to the KLS interaction with a $\kappa = 1.1$ correction in Eq. (8). The open squares test the influence of the $\chi_{1b}^T$ term through a uniform attraction of 1.5 MeV in CK it is about 3 times as large. Conclusion: there is not much to choose between the two KLS corrected cases. Moreover, they are practically as good as CK except for the second $J = 3$ level.

There are two reasons not to dwell any longer in the $p$ shell. The first is that the aim of this letter is to show that the monopole corrections must be 3b, i.e., $\kappa$ must be linear in $n$, which demands examining cases of sufficiently different $n$. Unfortunately this is impossible without bringing in the other possible contributions: For example, $\chi_{1b}^r$, is not very significant in $^{16}$O ($n = 6$), but it is important in $^{12}$C ($n = 8$) and crucial in $^{14}$N ($n = 10$). Therefore, there is no way of exploring what a single term in Eq. (8) does: all must contribute. As it happens—and this is the second reason—the full exploration has been done [23], and the results were excellent. At this time, the problem was that the 3b contributions turned out to be large and important, and the authors did not know what to do with them.

For the $^{22}$Na spectrum in Fig. 2 the black squares show the results for the venerable KB [6]. The black circles correspond to the Bonn C (BC) G-matrices [27, 28]. The agreement with experiment (lines) [29] is poor but the agreement between the calculations is good. Again, this is not a joke, but an important remark: as mentioned, there are very little differences between the realistic G-matrices. The open pentagons correspond to Wildenthal’s USD [8]. The open squares and circles refer to the KB and BC interactions with $\kappa = 0.9$ and 0.85 corrections respectively. We shall come to the triangles soon. Though USD is closer to experiment, the corrected R interactions do definitely well.
FIG. 3: Excitation energies for \(^{23}\)Na and \(^{24}\)Mg referred to the \(J = 3/2\) and 0 lowest states respectively. See text.

The story repeats itself for \(^{23}\)Na and \(^{24}\)Mg in Fig. 3. The notations are the same as in Fig. 2. The agreement with experiment is now truly satisfactory, and the plotting technique adopted makes the physics quite evident: the trouble with a 2\(l\)-only description is that the excited band \(K = 1/2\) in \(^{23}\)Na, and the \(\frac{1}{2}^+ = 2 (\gamma)\) band in \(^{24}\)Mg are too low.

The open triangles in Figs. 2 and 3 show what happens with KB when—instead of keeping \(\kappa\) fixed—we increase it by steps of 0.5 per \(n\). Though there is an improvement, it is not sufficient to claim the irrebutability of a 3\(l\) mechanism. The proof comes when we move to Figs. 4: In \(^{27}\)Si, \(^{28}\)Si, and \(^{29}\)Si the local value of \(\kappa\) (open squares and circles) has decreased to 0.60 for KB and to 0.55 for BC. A constant \(\kappa\) is totally ruled out, while the linear law (triangles) does quite well. Clearly, the 3\(l\) terms are indispensable. The superb 2\(l\)-only USD fit was obtained mostly through the massacre of a strong \(JT = 20\) pairing term that is a constant feature of the R interactions, which makes USD R-incompatible [25, Section V]. This has been known for some time and it is only occasionally that trouble may arise. The problem has been the difficulty, so far, of obtaining an R-compatible fit of comparable quality. The mild exception comes from \(^9\)B where, as in the \(p\) shell, the 3\(l\) contributions turned out to be so large and important, that the authors did not know what to do with them.

FIG. 4: Excitation energies for \(^{27}\)Si, \(^{28}\)Si and \(^{29}\)Si referred to the \(J = 5/2\), 0 and 1/2 lowest states respectively. See text.

In the \(pf\) shell KB1 (or KB3) is very good for \(A = 47-52\) but it produces too large a gap at \(^{56}\)Ni (7.5 MeV against the observed 6.3 MeV). The most serious problem comes from the first \(BE2(2 \rightarrow 0)\) transition in \(^{58}\)Ni which falls short of the observed value \((140 e^2 fm^6)\) by a factor \(\approx 0.4\). Here it is expedient to replace the constants in Eq. (6) by linear terms that have the same value at \(A = 48\), and are reduced by a factor \(\approx 0.7\) at \(A = 56\) (Eq. (8) works as well). The situation in \(^{56}\)Ni becomes consistent with experiment but in \(^{58}\)Ni it remains unacceptable. The problem is solved by BonnC (BC) [27], with the same \(\kappa\) reduction of \(\approx 0.7\) in going from \(A = 48\) to \(A = 56\). The key difference between KB and BC is that the intensity of the quadrupole force (in MeV, extracted as in [25]) is 2.7 for KB and 3.2 for BC. This discrep-
ancy is somewhat disturbing, but it does alter the basic fact that 3b monopole terms are necessary. Fig. 5 shows that for $\kappa = 0.43$, BC produces a backbending pattern in $^{48}$Cr that is practically as good as the KB3 one. At $\kappa = 0.28$, the correct value around $A = 56$—the agreement with experiment is destroyed.

There are several other indications that a 3b interaction is essential. Perhaps the most significant is the following: The monopole centroids $\tilde{T}_{i}$ must be such that when $f_{j\ell}$ fills the $d$, $(\ell = 2)$ orbit is depressed with respect to the $s$, $(\ell = 0)$ one [18]. However, it is clear from the spectrum and the spectroscopic factors in $^{29}$Si that the filling of $d_{5/2}$ favours the $p$, $(\ell = 1)$ orbit(s) over the $f$, $(\ell = 3)$ ones [29]. A 2b-only assumption leads to a contradiction: if $f_{j\ell}$ acting on the sd shell favours the larger $l$ orbits, $d_{5/2}$ acting on the pf shell must do the same. Without unacceptable ad-hoc assumptions, a 2b mechanism cannot do otherwise but a 3b one can.

From what we have seen, 3b monopole forces make things simpler, and there are good reasons to believe that the formidable task of a full treatment—including multipole terms—need not be inevitable. A recent generation of 3b potentials [30] has made it possible for the exact solutions to eliminate the more offending aspects of the 2b $^{13}$B spectrum [31]. It will be of much interest to check whether the underlying mechanism corresponds to the one proposed in this letter. At any rate, a full characterization of the 3b potentials is not an easy matter, and it could be hoped that information coming from shell model studies may prove valuable. Especially at a time when GFMC and no-core calculations have rigorously established the basic reliability of such studies.

Several observations of Alfredo Poves and Frédérique Nowacki have been of great help.

[17] For “historical” reasons, these calculations use the KB3 interaction [12], which differs from KB1 in some innocuous non-monopole changes.
[19] The closed shell component in the ground state of these nuclei may be as small as 30% in $^{28}$Si but the large gaps and the spectroscopic factors for the $cs \pm 1$ states indicate strong magicity.
[20] $^{29}$Zr is not a closure, but we can always calculate the monopole properties around a closed pf shell.
[28] The $X_{ij}$ correction amounts to a repulsion of 150 keV (BC), and 50 keV (KB) for the $r = dx_{j\ell}$ orbit.