Structure functions near the chiral limit

M. Gökcelera,b, R. Horsleyc, D. Pleitere,d, P.E.L Rakowc, and G. Schierholzd,f

aInstitut für Theoretische Physik, Universität Leipzig, D-04109 Leipzig, Germany
bInstitut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany
cSchool of Physics, University of Edinburgh, Edinburgh EH9 3JZ, UK
dDeutsches Elektronen-Synchrotron DESY, John von Neumann-Institut für Computing NIC, D-15735 Zeuthen, Germany
eTheoretical Physics Division, Department of Mathematical Sciences, University of Liverpool, Liverpool L69 3BX, UK
fDeutsches Elektronen-Synchrotron DESY, D-22603 Hamburg, Germany

We compute hadron masses and the lowest moments of unpolarized and polarized nucleon structure functions down to pion masses of 300 MeV, in an effort to make unambiguous predictions at the physical light quark mass.

1. INTRODUCTION

Understanding the structure of hadrons in terms of quark and gluon constituents (partons), in particular how quarks and gluons provide the binding and spin of the nucleon, is one of the outstanding problems in particle physics.

Moments of parton distribution functions, such as \( \langle x \rangle \) and \( g_A = \Delta u - \Delta d \), are benchmark calculations in lattice QCD. To compare the lattice results with experiment, one must extrapolate the data from the lowest calculated quark mass to the physical value. Up to now, most results are for quark masses of about twice the strange quark mass and larger. A naive, linear extrapolation of \( \langle x \rangle \) in the quark mass overestimates the experimental number by \( \approx 40 \% \) [1]. While most results are for quenched QCD, where one might expect that \( \langle x \rangle \) is larger than the experimental value, recent unquenched results [2–4] indicate that this problem remains.

It has been argued that a linear extrapolation must fail because it omits non-analytic structure associated with chiral symmetry breaking [5]. Indeed, chiral perturbation theory [5,6] suggests a large deviation of \( \langle x \rangle \) and \( g_A \) from linearity as the pion (quark) mass tends to zero:

\[
\langle x \rangle_{NS} = \langle x \rangle_{NS}^0 \left( 1 - \frac{3g_A^2 + 1}{(4\pi f_{\pi})^2} m_{\pi}^2 \ln \left( \frac{m_{\pi}^2}{m_{\pi}^2 + \Lambda^2} \right) \right) + O(m_{\pi}^2),
\]

\[
g_A = g_A^0 \left( 1 - \frac{2g_A^2 + 1}{(4\pi f_{\pi})^2} m_{\pi}^2 \ln \left( \frac{m_{\pi}^2}{m_{\pi}^2 + \Lambda^2} \right) \right) + O(m_{\pi}^2),
\]

where \( \Lambda \) is a phenomenological parameter. Equation (1) fits both the lattice data and the experimental value [5]. However, \( \Lambda \) is not yet determined by the lattice data. To constrain this parameter, and to perform an accurate extrapolation based solely on lattice results, data at smaller quark masses are crucial.

2. THE SIMULATION

To narrow the gap between the ‘chiral’ regime and the lowest calculated quark mass, we have started simulations at quark masses corresponding to about twice the physical pion mass. The calculations are done for Wilson fermions in the quenched approximation. Improved Wilson fermions are known to suffer from exceptional configurations, which would forbid such a calcu-
lation at currently accessible couplings.

We work at $\beta = 6.0$. Our present data sample consists of:

<table>
<thead>
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<th>$\beta$</th>
<th>$\kappa$</th>
<th>Volume</th>
<th>Configurations</th>
</tr>
</thead>
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<tr>
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<td>0.1515</td>
<td>$16^332$</td>
<td>O(5000)</td>
</tr>
<tr>
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<td>$16^332$</td>
<td>O(5000)</td>
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<td>O(5000)</td>
</tr>
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<td>0.1550</td>
<td>$24^332$</td>
<td>220</td>
</tr>
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</tr>
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<td>$32^348$</td>
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<td>$32^348$</td>
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</table>

The $24^332$ and $16^332$ lattices are from our previous runs [7]. On all our lattices $m_\pi L > 4$, so that finite volume effects may be expected to be small. We did not see any at $\kappa = 0.1563$. We are currently increasing our statistics at the smaller quark masses.

3. HADRON MASSES

We first looked at the chiral behavior of pion, $\rho$ and nucleon masses. We did not find compelling evidence for non-analytic behavior in any of the three cases. In Fig. 1 we plot the nucleon mass as a function of the pion mass. Quenched chiral perturbation theory (qCPT) predicts [8]

$$m_N = m_0^N + C_{1/2}m_\pi + C_1m_\pi^2 + C_{3/2}m_\pi^3$$  \(3\)

with

$$C_{1/2} = \frac{3\pi}{2}\left(D - 3F\right)^2 \delta \approx -0.5.$$  \(4\)

Empirically one finds, to a very high precision,

$$m_N = \sqrt{m_0^N + C_1m_\pi^2},$$  \(5\)

which is an analytic function in the quark mass. In Fig. 1 we fit (3) and (5) to the data. Both fits are hardly distinguishable in the range of the data. Fit (3) gives $C_{1/2} = 0.7(1)$ and $\chi^2 = 0.8$. Note that the coefficient $C_{1/2}$ comes out positive instead of negative as it should, according to qCPT. There is some preference for the single parameter analytic fit (5), which gives $\chi^2 = 0.4$. What comes as a surprise is that the resulting physical nucleon masses (denoted by open circles in Fig. 1) still differ by more than 10%. We conclude that pion masses of 300 MeV are not small enough to sufficiently constrain the fit function, so as to give unambiguous results.

4. STRUCTURE FUNCTIONS

Let us now turn to the moments of non-singlet, unpolarized and polarized structure functions. We consider $\langle x \rangle_{NS}$ first. On the lattice one does not compute $\langle x \rangle_{NS}$ directly but [1]

$$R = \langle x \rangle_{NS} m_N,$$  \(6\)

which can be interpreted as the fraction of the proton’s mass carried by the $u$ quark minus that carried by the $d$ quark. Because chiral perturbation theory does not provide much guidance at present quark masses, we seek a phenomenological extrapolation. In Fig. 2 we plot $R$ as a function of the pion mass. The data points have been properly renormalized and converted to renormalization group invariant (RGI) numbers following [2]. We find that the data lie precisely on a
Figure 2. The lowest moment of the non-singlet, unpolarized structure function against \( m_π \), together with a linear extrapolation to the physical pion mass. Also shown is the experimental result (*) taken from [9].

straight line, all the way from \( m_π \approx 1 \text{ GeV} \) down to our smallest pion mass at \( m_π = 300 \text{ MeV} \). The slope comes out to be \( \approx 0.4 \), which is close to the value \( 1/3 \) expected in the heavy quark limit. A linear extrapolation to the physical pion mass, \( R = R^0 + R_{1/2} m_π \), gives a value for \( \langle x \rangle_{RGI}^{NS} \) that is much closer to the experimental number than previous results. A fit of (1) to \( \langle x \rangle_{NS} \) still does not constrain \( \Lambda \). We find \( \Lambda = 360^{+120}_{-360} \text{ MeV} \). Our present data cannot distinguish between the fit function (1) and \( (R^0 + R_{1/2} m_π)/\sqrt{m_π^2 + C_1 m_π^2} \).

Let us now consider \( g_A \), the axial vector coupling of the nucleon. This quantity describes the fraction of the proton’s spin carried by the \( u \) quark minus that carried by the \( d \) quark. In Fig. 3 we plot \( g_A \) as a function of the pion mass. Again, the data lie on a straight line, and we do not see any sign of non-analytic behavior as suggested by (2).

5. CONCLUSION

It appears that at pion masses of 300 MeV we are not yet sensitive to the predictions of chiral perturbation theory. Whether the situation will improve on finer lattices has to be seen.

The numerical calculations were performed on the APE100 at NIC (Zeuthen) and on the Cray T3E at NIC (Jülich).

REFERENCES

3. D. Dolgov et al., hep-lat/021021.