Nano-constraints on the spatial anisotropy of the Gravitational Constant

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Abstract

We present constraints from various experimental data that limit any spatial anisotropy of the Gravitational constant to less than a part per billion or even smaller. This rules out with a wide margin the recently reported claim of a spatial anisotropy of $G$ with a diurnal temporal signature.

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The question whether the gravitational constant has some spatial anisotropy has been discussed in the context of nonstandard theories of gravity as well as in the context of experimental observations[1, 2]. The general setting for discussing this issue in the Parametrized Post-Newtonian framework is discussed extensively by Will[1]. There are severe constraints on the anisotropy parameters from gravimetric experiments[2] and Lunar Laser Ranging[3]. These anisotropies depend on the magnitude and direction of the velocity of the frame in which the experiments are done, with respect to a preferred frame. In the usual frame work of alternate theories of gravity, a spatial anisotropy of $G$ that depends merely on the relative angle between the line of interaction of two gravitating bodies and a reference direction with respect to the distant stars is not expected. However, recently there has been a claim for the observation of a significant anisotropy in $G$ from an experiment that monitored the gravitational interaction between a source mass and the mass element of a torsion balance over a period of 7 months [4]. The reported result was that $G$ changes with orientation of the line of interaction of the masses by at least 0.054%. This is a very large anisotropy, and of a different kind, when contrasted with what is usually discussed in the
context of theoretical models. In this paper we discuss possible constraints on such an anisotropy and show that results from several standard experiments that are sensitive to the gravitational interaction between the earth and another body constrain any spatial anisotropy of $G$ to less than a part in a billion. This rules out the reported claim of the spatial anisotropy of 0.054% by a factor of more than a million. While our discussion focuses on a recent experiment as an example[4], the constraints we present are very general and apply in a model independent way to the issue of spatial anisotropy of $G$ that depend on the direction of the line of interaction with respect to a preferred frame.

Theoretically, the issue of spatial anisotropy of $G$ is related to how nearby masses and their distribution can affect the gravitational interaction between two bodies, and also to preferred frame effects. If the modification of the gravitational interaction depends on the gravitational potential generated by other masses, then the most distant masses dominate and the spatial anisotropy is expected to be small. The galactic contribution to the anisotropy is then the dominant term in any reasonable expectation, and the order of magnitude is then of the order of $\delta \approx GM\delta/c^2R_g \approx 10^{-6}$. Anisotropy of even this level can be expected only in very special theoretical scenarios since $\delta$ is the value of the scalar potential and it does not single out a direction in space. Theoretically, existence of a preferred frame would give rise to the dependence of $G$ on the velocity with respect to this frame and such effects have been discussed extensively and completely in the context of the PPN formalism [1]. It is appropriate to summarize the relevant physical issues in the context of the PPN formalism. Local physics in General Relativity itself does not contain any dependence on the location, orientation and the velocity of a frame, except the standard couplings to gradients in the gravitational field (tidal forces). In particular, $G$ is truly a constant in space and time. This is the case in any theory that contains only the metric to represent long range fields. In a metric theory that contains a scalar field in addition (Scalar-Tensor theory), $G$ can depend on location and time through the variations in the scalar field, but not on velocity and orientation of the frame. In a more general type of theory, though, there could be local effects that depend on position, velocity and orientation and even perhaps angular velocity (spin). Note that any such effects point to a violation of the Strong Equivalence Principle, and it is in this context we analyze the possible variation of $G$ with orientation.

Experimentally the most common approach has been to look for such anisotropies as the laboratory reference frame assumes different velocities and positions as the earth moves with respect to a nonrotating reference frame fixed to the galactic centre. If one is searching for an effect that depends on the orientation of the line of interaction between two bodies, then the signal is typically of the diurnal period and its second harmonic. The line of interaction sweeps through a fixed plane containing the rotation axis of the earth twice during a diurnal period. Thus the primary signal should be at twice the diurnal frequency (semi-diurnal period), like a tidal force, and a signal at the diurnal period could also be present due to possible asymmetry in the second harmonic signal because of the inclination of the earth’s rotation axis, for example, with respect to the
This requires that the experimental conditions are controlled and monitored with great care since there are many sources – variations in ambient conditions like temperature, pressure and magnetic field, electromagnetic couplings, human interferences etc. – that can generate a large diurnal signal and its second harmonic in high sensitivity experiments. It is important to note that detecting variations in $G$ with a relative precision better than $10^{-5}$ is beyond the current experiments that measure the absolute value of $G$[5]. But, measuring variations in $G$ relative to fixed average value can be done with a precision better than $10^{-9}$, and this is what allows us to constrain the spatial anisotropy of $G$ to better than 1 part in a billion. A measurement of $\Delta g/g$ is equivalent to a measurement of $\Delta G/G$ in such experiments, after accounting for known effects that cause a nonzero $\Delta g/g$. This point has been noted long ago for constraining PPN parameters[1].

The Hughes-Drever type very high precision null experiments test the anisotropy of inertia[6]. They however do not test a spatial anisotropy of $G$ since the anisotropy of inertia affects the spectral line splitting through a nongravitational interaction involving the ratio of charge to inertial mass.

The experiment by Gershteyn et al consisted of repetitive measurements of $G$, separated by a time interval of about 1.2 hours. They measured the change in the period of a torsion pendulum in the presence of a heavy source mass[4]. The measurements were repeated round the clock, for a period of about 7 months. The periodogram showed a strong peak at the diurnal period and this was interpreted as evidence for a spatial anisotropy of the gravitational constant amounting to larger than $5 \times 10^{-4}$, revealed as the line of interaction of the masses swept around with the rotating earth with respect to the distant stars.

This is easily shown to be not due to a spatial anisotropy of the gravitational constant. There are several reasons that immediately reject the hypothesis that the observed signal was due to an anisotropy in $G$ detected as the line of interaction of the masses assumed different orientations with respect to a frame defined by distant stars. There was not a strong and significant signal at the semi-diurnal period (some of their data contains significant power at the semi-diurnal period, but the noise in the nearby bins are also large). Though this in itself could be considered as evidence that the signal in their experiment is spurious and not an indication of $G$-anisotropy, strong numerical constraints that we discuss below conclusively prove that any $G$-anisotropy signals above $\Delta G/G \simeq 10^{-10}$ should be considered spurious.

Before we discuss a stringent limit, we illustrate how to easily obtain a limit good enough to rule out anisotropy of $G$ at the level of $10^{-6}$, and thus conclusively rule out the result claimed by Gershteyn et al. A simple experiment that can be performed in almost any laboratory using a commercial electronic weighing balance is capable of constraining anisotropy of $G$ to below $10^{-6}$. Commercial electronic balances (like the Mettler AE series) have sensitivity of about 10 to 100 microgram for measuring masses in the range of 10 to 500 grams. Even after allowing for long term drifts (whose diurnal component will mimic a signal), these balances can measure weight changes with a precision better
than $10^{-6}$, especially if the signal is repetitive with a specific period, like the diurnal period. Since the weight measured is directly proportional to the local gravitational attraction of the earth on the sample being weighed, the fractional change in weight is equal to the fractional change in the gravitational constant. At the sensitivity of $10^{-6}$, tidal effects from the solid deformation of the earth or ocean loading are not observed, and there is no need for correction from tidal terms. Clearly, such an experiment looks directly for a spatial anisotropy of the gravitational constant with a sensitivity of $10^{-6}$ as the line joining the sample mass and earth’s centre sweeps out different directions with respect to the galactic centre. We monitored a weight of about 150 grams for a few days and did not observe any periodic component at the sensitivity limit of 100 µg. This means that this simple experiment gives the remarkable constraint $\Delta g/g \leq 6 \times 10^{-7}$, which translates to a similar constraint on $\Delta G/G$ with respect to orientation changes in the galactic plane, since the experiment is done at a relatively small latitude of 20 degrees. Absence of a signal at the diurnal period and its second harmonic rules out the possibility that what Gershteyn et al observed was a spatial anisotropy of $G$ by a wide margin – a factor of about 1000.

The simple yet significant constraint of $\Delta G/G < 10^{-6}$ we pointed out can be improved by a factor of 1000 or more using other standard experimental data, especially the kind obtained routinely from microgravimeters with a sensitivity exceeding 1 microgal ($10^{-8}$ m/s² $\simeq 10^{-9}$ g). All one needs is the residual diurnal and semi-diurnal component in the local gravitational field value after subtracting out the known tidal contributions. We had already used such data to constrain the Majorana shielding factor earlier [7, 8]. We had analyzed 2 weeks’ continuous gravimetric data from a Chinese experiment [9], obtained bracketing a solar eclipse, and found that the diurnal signal is smaller than 0.06 microgal. The signal at the second harmonic is even smaller. This translates to a fractional diurnal change in local gravity, after subtracting known tidal effects, smaller than $\frac{\Delta g}{g} \leq 6 \times 10^{-11}$. The experimental site was at a latitude of about 53 degrees, and the angle factors reduce the sensitivity of the experiment. (This would be the case if significant contributions are assumed from orientations in the galactic plane). Even allowing for a large sensitivity reduction of 0.1, we can conservatively obtain the constraint that $\Delta G/G < 6 \times 10^{-10}$. This constraint is almost a million times smaller than the spatial anisotropy claimed by Gershteyn et al, and therefore there is no doubt that they are observing a spurious systematic effect.

This constraint can be improved by another factor of 10 without much effort using data from a long term microgravimetry experiment at suitable locations. In fact, there is already data from a dedicated and pioneering experiment done long ago, using a superconducting gravimeter, that rules out such an anisotropy at the level of $\Delta G/G < 10^{-10}$. This experiment by Warburton and Goodkind[2] collected gravimetric data over an 18 month duration with sub-microGal sensitivity and put constraints on the PPN anisotropy parameters at the level of 0.1%. A detailed analysis of the data at various frequencies and phases of relat-
evance was done. The amplitude of the residuals after subtracting the known tidal terms at diurnal and semi-diurnal periods was at most 0.3 microGal and 0.1 microGal respectively, and phase analysis constrains this even better. Thus a model independent constraint at the level of \( \Delta G/G \leq 10^{-10} \) can be obtained from their data. In specific models with the galactic mass as the source of the anisotropy, phase analysis allows tighter constraints of \( \Delta G/G \leq 10^{-11} \) (the residual amplitude used for constraining the PPN preferred frame parameter corresponded to about 0.006 microGal in their data).

There is another source of data that constrains spatial anisotropy of \( G \) even better. The Lunar Laser Ranging (LLR) data has been used for constraining the fractional temporal variation of the gravitational constant to about \( 4 \times 10^{-12}/\text{year} \). The same analysis then also constrains the spatial anisotropy to similar levels, since a periodic signal with a period close to the lunar month (corrected for the earth yearly motion) will show up if there is spatial anisotropy in \( G \), as the line joining earth and moon sweeps out different directions with respect to the direction to the galactic centre. The sensitivity with which a periodic signal can be pulled out is of the same order (or sometimes better) than the sensitivity with which a long term drift can be analyzed. Absence of such a signal then constrains \( \Delta G/G < 4 \times 10^{-12} \) or so in the galactic neighbourhood. This remarkable constraint cannot be easily surpassed by laboratory experiments.

A large variety of other experiments could also be cited as evidence against the anisotropy claimed by Gershteyn et al. See the reviews of Gillies[5] and Unnikrishnan and Gillies[8] for some further discussion of anomalous gravitational effects and see the bibliographic listing compiled by Gillies[10] for citations to relevant experiments. While it is difficult to say what the origin of the effect might be in the results of Gershteyn et al., it is possible to consider some interesting alternatives. Their experimental apparatus has a long history of use for the measurement of the gravitational constant, see eg., [11, 12]. Their torsion pendulum system has been used for that purpose in both symmetric and asymmetric modes of operation, i.e., with the small mass system in the gravitational potential of either two attracting masses or just one, respectively. For the data taken in the experiments under consideration here, they used their system in the asymmetric mode, such that the dumbbell, which is a mass quadrupole, was coupled gravitationally to the point-source field of the large spherical attracting mass, which is a single mass monopole. Variations in the ambient environment as well as changing gravitational gradient couplings could be responsible for a signal at the diurnal period. We note that the most recent high-precision measurements of \( G \) incorporate mass distributions such that the gravitational interaction is as well-defined as possible in order to avoid such problems; see eg. [13]. However, absolute \( G \)-measuring experiments have not yet achieved the sensitivity to probe a variation in \( G \) less than 0.001%, though several experiments have recently reached enough sensitivity to probe the effect claimed by Gershteyn et al with a signal to noise ratio of about 10[13, 14].

We conclude by stressing that any claim for a spatial anisotropy of the gravitational constant that depends on the direction of line of interaction with
respect to a fixed frame should be considered as spurious if reported to be larger
than about $10^{-10}$, which is a conservative constraint obtained from laboratory
experiments.

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