A note on CFT dual of RS model with gauge fields in bulk

K. Agashe∗ and A. Delgado†

Department of Physics and Astronomy
Johns Hopkins University
3400 North Charles St.
Baltimore, MD 21218-2686

Abstract

It has been conjectured that the (weakly coupled) Randall-Sundrum (RS) model with gauge fields in bulk is dual to a (strongly coupled) 4D conformal field theory (CFT) with an UV cut-off and in which global symmetries of the CFT are gauged. We elucidate features of this dual CFT which are crucial for a complete understanding of the proposed duality. We argue that the limit of no (or small) brane-localized kinetic term for (bulk) gauge field on RS side (often studied in literature) is dual to no bare kinetic term for the gauge field which is coupled to the CFT global current. In this limit, in the CFT dual, the gauge field kinetic term is “induced” by CFT loops. Then, this CFT loop contribution to the gauge field 1PI two-point function is dual (on the RS side) to the full gauge propagator (i.e., including the contribution of Kaluza-Klein and zero-modes) with both external points on the Planck brane. We also emphasize that loop corrections to the gauge coupling on RS side are dual to sub-leading effects in a large-\(N\) expansion on CFT side; these sub-leading corrections to the gauge coupling in the dual CFT are (in general) sensitive to the strong dynamics of the CFT.

∗email: kagashe@pha.jhu.edu
†email: adelgado@pha.jhu.edu
1 Introduction

The Randall-Sundrum (RS) proposal of a warped extra dimension [1] is interesting from both theoretical and phenomenological points of view, for example, the RS1 model can solve the Planck-weak hierarchy problem. For the case of RS1, the extra dimension is an orbifolded circle of radius $r_c$ with the Planck (or UV) brane at $\theta = 0$ and the TeV (or IR) brane at $\theta = \pi$. The geometry is a compact slice of AdS (with curvature scale, $k$ of order $M_4$, the 4D Planck mass):

$$ds^2 = e^{-2kr_c|\theta|}\eta_{\mu\nu}dx^\mu dx^\nu + r_c^2d\theta^2, \quad -\pi \leq \theta \leq \pi$$

$$= \frac{1}{(kz)^2} \left( \eta_{\mu\nu}dx^\mu dx^\nu + dz^2 \right), \quad z_{UV} \leq z \leq z_{IR}.$$ (1.1)

In terms of the coordinate $z \equiv 1/k e^{-kr_c|\theta|}$, the Planck brane is at $z_{UV} = 1/k$, the TeV brane is at $z_{IR} = 1/k e^{kr_c}$ and, in order to solve the Planck-weak hierarchy problem, $r_c$ is chosen so that $1/z_{IR} \sim \text{TeV}$. The RS2 model corresponds to $r_c \to \infty$.

The AdS/CFT correspondence [2–5] suggests that RS models are dual to (deformed) conformal field theories (CFT’s) [6–9]. In this paper, we are interested in the CFT dual of RS model with gauge fields in bulk [10–12]. In this case, the dual is a CFT with a UV cut-off and with global symmetries of the CFT gauged by a gauge field external to the CFT [4, 7]. In this paper, we study aspects of this dual CFT which are required for a better understanding of the AdS/CFT correspondence as applied to this RS model. Although these CFT duals have been discussed before, the properties we focus on have not been studied in detail before.

The central point of our study is that the bare kinetic term (or kinetic term at the UV cut-off) for gauge field external to the CFT is to be identified with Planck brane-localized kinetic term for (bulk) gauge field on the RS side. As a check, we show that (the form of) the propagator for gauge field coupled to the global current of the dual CFT (including bare kinetic term and CFT loop correction) agrees with the gauge propagator in RS model with both external points on the Planck brane (including the effects of bulk and brane-localized kinetic term).

This implies that the limit of no (or small) (Planck) brane-localized kinetic term on RS side is dual to no bare kinetic term for gauge field coupled to global current of the CFT, so that the kinetic term for this gauge field is “induced” by CFT loops. This “induced gauge theory” is similar to CFT loops inducing gravity (or Newton constant) in the limit of very large (or infinite) bare gravitational constant [7, 8, 13] – we extend this analysis to the case of the gauge field in order to have a complete understanding of the AdS/CFT duality in this case. We also study in more detail the implications of induced gauge theory and induced gravity. For example, in the induced gauge theory (gravity) limit, the CFT contributions to the (external)
gauge field (graviton) 1PI two-point function are dual (on RS1 side) to the contribution of zero and Kaluza-Klein (KK) modes to the Planck brane-to-Planck brane propagator.

Another consequence of induced kinetic term for gauge field external to the dual CFT is large kinetic mixing between this gauge field and CFT bound states [7]. This leads to a dual interpretation of flat profile of gauge field zero-mode and sizable coupling of gauge KK modes to Planck brane fields. We compare the case of gauge field to that of gravity, where we argue that this mixing is small (even in the induced gravity limit) which matches with localization of graviton zero-mode near Planck brane and weak coupling of graviton KK modes to Planck brane fields. We also give a dual interpretation of coupling of gauge KK modes to TeV brane fields.

The AdS/CFT correspondence says that tree-level effects in AdS are dual to leading effects in a large-N expansion on the CFT side (for example, if the CFT is a SU(N) gauge theory), whereas loop effects on AdS side are dual to sub-leading large-N effects on the CFT side [2–5]. In this paper, we focus on the CFT dual of the simple RS model with scalar QED in bulk. In this case, the sub-leading large-N corrections to the (external) gauge coupling in the dual CFT are due to the loop of a fundamental scalar (external to the CFT) and sub-leading part of the vacuum polarization (running) due to CFT charged matter. We point out that these two effects are comparable in size and that the latter (i.e., sub-leading part of CFT loop correction) is sensitive to the strong CFT dynamics. Thus, the loop corrections to the gauge coupling in this RS model are difficult to compute using the dual theory.

The paper is organized as follows. In section 2, we define the RS model (the dual of which is to be studied), namely scalar QED in bulk, and give expressions for the classical and one-loop corrected low-energy gauge coupling in this model. In section 3, we discuss dual of the RS2 model and present the central point mentioned above, in particular, the limit of induced kinetic term for the gauge field external to the CFT. Section 4 deals with CFT dual of the RS1 model. Specifically, in section 4.1, we discuss kinetic mixing between fields external to CFT and CFT bound states and also give a dual interpretation of coupling of KK modes. In section 4.2, we discuss low-energy gauge coupling in the dual CFT including the sub-leading large-N effects (which are dual to loop corrections on RS side).
2 Review of RS model with bulk gauge fields

For simplicity, we consider (massless) scalar QED in bulk – extension to non-abelian gauge fields in bulk should be straightforward. The bulk 5D action is:

\[ S_{\text{bulk}} = \int d^4x \, r_c d\theta \sqrt{-G} \left( -\frac{1}{4g_5^2} F_{MN} F^{MN} + G^{MN} D_M \phi (D_N \phi)\right). \]  \hspace{1cm} (2.1)

\( A_\mu \) and \( \phi \) are taken to be orbifold-even while \( A_5 \) is taken to be orbifold-odd.

In general, there are brane-localized kinetic terms for gauge and scalar field: even if absent at tree-level, these are generated by bulk loop effects (due to breaking of translation invariance by orbifold) [14]. Thus, the brane action is (in addition to action for brane-localized fields):

\[ S_{\text{UV(IR)}} = \int d^4x \sqrt{-g_{\text{UV(IR)}}} \left( -\frac{1}{4} \phi^{\mu \nu} F_{\mu \nu} + \sigma_{\text{UV(IR)}} (D_\mu \phi)^\dagger D^\mu \phi \right). \]  \hspace{1cm} (2.2)

We assume that these couplings are perturbations to the bulk couplings, for example, they are of same order as one-loop processes involving bulk couplings.

The zero-mode of the gauge field has a flat profile and hence the 4D low energy effective gauge coupling at classical level is

\[ \frac{1}{g_4^2} = \tau_{\text{UV}} + \tau_{\text{IR}} + \frac{\pi r_c}{g_5^2}, \]

\[ = \tau_{\text{UV}} + \tau_{\text{IR}} + \log \left[ \frac{O(M_4) / \text{TeV}}{k g_5^2} \right] \]  \hspace{1cm} (2.3)

(\( g_4^2 = \frac{1}{2 (24\pi^2)} \) is the \( 4D \) \( \beta \)-function coefficient of a charged scalar and \( c \) is of order a 5D loop factor.

\[ \frac{1}{g_4^2(q)} = \tau_{\text{UV}}(k) + \tau_{\text{IR}}(k) + \frac{\pi r_c}{g_5^2 R(k)} \]

\[ + b_4 \left( \log \frac{k}{q} + \xi k \pi r_c + O(1) \right), \]  \hspace{1cm} (2.4)

where \( \xi \sim O(1) \) and \( \tau_{\text{UV(IR)}}(k) \equiv b_4/4 \log (\Lambda/k) + \tau_{\text{UV(IR)}} \), \( 1/g_5^2 R(k) \equiv c(\Lambda - k) + 1/g_5^2 \) are renormalized couplings. \( b_4 = 1/(24\pi^2) \) is the 4D \( \beta \)-function coefficient of a charged scalar and \( c \) is of order a 5D loop factor.

3 Duality for RS2 with bulk gauge fields

We now discuss the 4D CFT dual of this model. The (usual) AdS/CFT correspondence [2–5] suggests that any 5D gravity theory on (infinite) AdS_5 is dual to some 4D CFT. In particular,
for every 5D bulk field, \( \phi \), there corresponds an operator, \( O \) in the CFT and the value of the (bulk) field at the 4D boundary of \( \text{AdS}_5 \) (at \( z = 0 \)), \( \phi_0 \) acts as a source for the operator. The AdS/CFT correspondence tells us that

\[
\left\langle \exp \left( i \int d^4x \phi_0 O \right) \right\rangle_{\text{CFT}} = \exp (i \Gamma [\phi_0]),
\]

where \( \Gamma [\phi_0] \) is the effective action for the (unique) solution for \( \phi \) in bulk with \( \phi \) at boundary \( = \phi_0 \) [3, 4]. In general, RHS of Eq. (3.1) is \( \int_\phi \text{at boundary}=\phi_0 D\phi \exp (i S [\phi]) \).

The 5D theory with bulk gauge fields is dual to a 4D CFT with (unbroken) global symmetries [4]. Then, the source (gauge field) couples to the (conserved) current corresponding to a subgroup of this global symmetry, i.e., \( O \sim J^\text{global}_\mu \) and \( \phi_0 \text{(source)} \sim A_\mu (x, z = \text{boundary}) \). In the case of a massless bulk scalar, the corresponding \( O \) has conformal dimension 4 (marginal operator).

The modification of the AdS/CFT correspondence in the case of RS2, which is \( \text{AdS}_5 \) cut-off by a Planck brane (i.e., with the boundary) at \( z_{\text{UV}} > 0 \), is that the presence of the Planck brane corresponds to putting a UV cut-off on the CFT: \( \Lambda_{\text{CFT}} \sim 1/z_{\text{UV}} \) (but in such a way that the theory remains conformal in the IR) [6–9].

Since bulk fields evaluated on \( \text{AdS}_5 \) boundary are dual to sources in the CFT, the (Planck) brane-localized kinetic terms for gauge and scalar field in Eq. (2.2) should correspond to bare kinetic terms for sources in the dual CFT – we will check this ansatz in what follows. We will also show in what follows that even if these bare/brane-localized kinetic terms are small (or absent), kinetic terms for the sources will be “induced” by CFT loop effects due to the presence of a UV cut-off. This is dual to the fact that the boundary fields become dynamical in RS2, unlike in the case of the boundary at \( z = 0 \). Since sources/boundary values of bulk fields are dynamical, we have to include both sides of Eq. (3.1) in a path integral over \( \phi_0 \) (for scalar fields, these issues were studied in reference [9]). However, on the CFT side, we will still refer to these (dynamical) fields as “sources” to avoid confusion with CFT fields (or with the bulk fields on RS side).

Thus, the action for dual CFT at cut-off \( \sim O (M_4) \) (i.e., the bare action) is

\[
S \sim S_{\text{CFT}} + \int d^4x \left( A_\mu J^\mu - \frac{1}{4} \tau_{\text{UV}} F_{\mu\nu} F^{\mu\nu} + \sigma_{\text{UV}} |D\phi|^2 + \frac{1}{k} \phi O \right).
\]

Here, \( S_{\text{CFT}} \) denotes the pure CFT action, \( A_\mu \) is the (source) gauge field (“photon”), \( \phi \) is the (source) charged scalar\(^1\), \( O \) is a dimension four CFT operator with zero anomalous dimension and \( J^\mu \) is a \( U(1) \) global symmetry current of the CFT.

\(^1\)Again, these source fields are dual to boundary values of bulk AdS fields, but for notational ease, we drop the subscript 0.
Next, we include effects of CFT loops and the $\phi$ loop on the $A_\mu$ propagator assuming that, at the cut-off, the external gauge field is weakly coupled to the CFT, i.e., $1/\tau_{UV} \ll 16\pi^2$. We obtain, at energy scale $q \ll k$, the following expression for the 1PI two-point function (inverse propagator) for $A_\mu$ – an explanation is given below (also see discussion in references [7, 19]):

$$
(\eta_{\mu\nu}q^2 - q_\mu q_\nu) \left( \tilde{\tau}_{UV} + [b_{CFT} + b_{\text{scalar}}] \log \frac{\Lambda_{CFT}}{q} \right).
$$

(3.3)

Here, $\Lambda_{CFT} \sim k \sim O(M_4)$ is UV cut-off of CFT. The tensor structure is fixed by current conservation. We assume that the CFT is a (strongly coupled) large-$N$ $SU(N)$ gauge theory, but the global symmetry group (the $U(1)$ subgroup of which is gauged) acts on fundamentals (and not, for example, adjoints) of the $SU(N)$ gauge theory. We also assume that the number of these fundamentals is fixed in the large-$N$ limit. Since “color” of CFT acts as “flavor” for photon, running due to charged CFT fields is given by $b_{CFT} \log (\Lambda_{CFT}/q)$ with $b_{CFT} \sim N/(16\pi^2)$ (this is the effect of the $\langle J_\mu J_\nu \rangle$ correlator). Although $b_{CFT}$ looks like a one-loop $\beta$-function coefficient, it is not (entirely) fixed by (gauge group) representations (i.e., quantum numbers) of the CFT charged matter (unlike in one-loop perturbation theory), but it depends on the strong CFT dynamics. However, the above $q$ dependence of this running (i.e., the fact that $b_{CFT}$ is a constant) is fixed by conformal invariance.

Next, we argue that the running due to scalar loop has the form of the $b_{\text{scalar}}$ term in Eq. (3.3) even though this includes dressing of scalar propagator by the $\langle \mathcal{O}\mathcal{O} \rangle$ correlator and scalar-photon vertex by the $\langle J_\mu \mathcal{O}\mathcal{O} \rangle$ correlator (since $\phi$ couples to CFT fields through $\phi\mathcal{O}$).\(^2\) Note that the $\langle \mathcal{O}\mathcal{O} \rangle$ correlator has a quadratic divergence which cancels the $1/k$ suppression in the $\phi\mathcal{O}$ coupling: $\langle \mathcal{O}(p)\mathcal{O}(0) \rangle \sim N/(16\pi^2) p^2 [\Lambda_{\text{CFT}}^2 + p^2 \log (\Lambda_{\text{CFT}}/p) + \text{finite}]$ (see reference [9]). The quadratically divergent part of this correlator results in wavefunction renormalization for the scalar and, in particular, “induces” a kinetic term for $\phi$ in the limit $\sigma_{UV} \to 0$. A similar argument applies to the dressing of the scalar-photon vertex (i.e., the $\langle J_\mu \mathcal{O}\mathcal{O} \rangle$ correlator). The leading order (i.e., quadratic divergence of) dressing of scalar propagator is related to that of the scalar-photon vertex by the Ward identity (i.e., gauge invariance). The logarithmic divergence and finite part (suppressed by $\sim p^2/k^2$) of CFT dressing of scalar propagator and scalar-photon vertex can be neglected for momenta smaller than $k$. Using these arguments, one can show that (for $q \ll k$) running due to scalar has the form shown above with $b_{\text{scalar}}$ given by $b_4$, the (one-loop) 4D beta-function [19].

Any remaining CFT loop corrections (at sub-leading order in a large-$N$ expansion) are incorporated in $\tilde{\tau}_{UV}$ (as a correction to $\tau_{UV}$). The precise coefficients of the CFT loop corrections in $b_{CFT}$ and $\tilde{\tau}_{UV}$ are sensitive to the strong CFT dynamics.

\(^2\)We thank Walter Goldberger for discussions on this point.
Since the source is dual to the gauge field evaluated on the Planck brane, the above 1PI two-point function (henceforth referred to as “kinetic term”) for source should correspond (on RS side) to (inverse of) gauge propagator with both external points on Planck brane (henceforth referred to as the “Planck brane propagator”).

Let us see if the two sides agree. In reference [16], the tree-level gauge propagator in RS2 (for arbitrary external points) was computed using Neumann boundary condition at the Planck brane, i.e., with no brane-localized gauge kinetic term \( \tau_{UV} = 0 \). To compute any gauge propagator with \( \tau_{UV} \neq 0 \), we have to modify the (Neumann) boundary condition at the Planck brane. Solving the classical wave equation of motion (including the effect of \( \tau_{UV} \)), we get (inverse of) wavefunction of a continuum mode of mass \( m \) at Planck brane (for \( m \ll k \)) is

\[
(\psi_m)^{-1} \sim \sqrt{\frac{m}{k}} \left( -\log \left( \frac{m}{2k} \right) + \gamma \right) + \left( kg_5^2 \right) \tau_{UV},
\]  

(3.4)

where \( \gamma \approx 0.58 \) is the Euler constant. The Planck brane propagator (including the gauge coupling) is given by an integral over continuum modes \( \sim \int \frac{q}{m} \left[ \log \left( \frac{2k}{q} \right) - \gamma \right] \).

The AdS/CFT correspondence tells us that the classical AdS action is captured by leading large-\( N \) effects of the dual CFT [2–5], where we can neglect \( b_{scalar} \) and set \( \tilde{\tau}_{UV} = \tau_{UV} \) in Eq. (3.3). Then, we see that the two sides (Eqs. (3.3) and (3.5)) agree if \( b_{CFT|_{large-N}} = 1/(kg_5^2) \): this is consistent with the fact that \( b_{CFT|_{large-N}} \) has an IR-free sign (even if \( A_\mu \) is non-abelian as long as rank of the gauge group is fixed in the large-\( N \) limit) since it is dominated by the running due to charged CFT matter which comes in complete large-\( N \) representations.

**“Induced” gauge theory:** Let us study the limit \( \tau_{UV} \to 0 \) on both RS2 and dual CFT sides. On RS2 side, this is the (simple) limit of only bulk gauge kinetic term studied in reference [16]. In this case, for \( q \ll k \), the tree-level Planck brane propagator is \( g_5^2 k / \left( q^2 \left[ \log \left( \frac{2k}{q} \right) - \gamma \right] \right) \) [16]. We see that the classical gauge coupling measured on Planck brane (defined as \( q^2 \times \text{propagator} \), i.e., \( \sim g_5^2 k / \left[ \log \left( \frac{2k}{q} \right) - \gamma \right] \), see also reference [23]) becomes very large (i.e., the theory becomes strongly coupled) as \( q \to 2e^{-\gamma} k \sim k \). Whereas, with \( \tau_{UV} \neq 0 \), gauge coupling measured on Planck brane as \( q \to k \) can be small (see Eq. (3.5)).

3We get the same result by solving for the propagator (Green function) directly, i.e., repeating the calculation of reference [16], but now modifying the Neumann boundary condition to include the effect of \( \tau_{UV} \).
Also, even if $\tau_{UV} = 0$ at tree-level, bulk loops generate a (logarithmically divergent) brane-localized kinetic term [14], thus requiring a brane-localized counterterm so that it is unrealistic to have no tree-level $\tau_{UV}$.

In the dual CFT, $\tau_{UV} \to 0$ corresponds to no (or very small) bare kinetic term for source – in other words, the limit of infinite bare gauge coupling – so that perturbation theory is not really valid for $q \sim k$ (this is dual to the classical strong coupling behavior for $q \to k$ on RS2 side mentioned above). However, the limit $\tau_{UV} \to 0$ is a smooth one (at the classical level) on the RS2 side (for $q \ll k$) and so we might expect that the expression for the source propagator in the dual CFT (for $q \ll k$) also has a smooth $\tau_{UV} \to 0$ limit. Therefore, we assume that Eq. (3.3) continues to hold even for $\tau_{UV} \to 0$. Then, we see that even though the gauge coupling (in the dual CFT) is strong at the cut-off, it does become weak for $q \ll k$ (so that perturbation theory is valid). Also, with $\tau_{UV} \to 0$ and for $q \ll k$, it is obvious that the gauge coupling in the dual CFT (in the large-$N$ limit and for $b_{\text{CFT}}|_{\text{large-}N} = 1/(kg_5^2)$) agrees with the classical gauge coupling measured on Planck brane in RS2.

It is clear that in the limit $\tau_{UV} \to 0$, the kinetic term for source (or gauge coupling) in the dual CFT (for $q \ll k$) is “induced” by CFT loop effects. Of course, in general, $\tau_{UV} \neq 0$ so that bare gauge coupling (in the dual CFT) can be weak (i.e., perturbation theory is valid even at the cut-off) and source kinetic term (for $q \ll k$) is not fully induced by CFT loop correction. In fact, since the source kinetic term generated by the CFT loop is divergent, we have to add such a counterterm (i.e., add $\tau_{UV}$) to cancel this divergence (i.e., cut-off dependence).

We now briefly discuss other aspects of this correspondence which illustrate the fact that classical effects on AdS side are dual to quantum (running) effects in CFT.

For a fixed $q$ (and $\tau_{UV}$), if the UV cut-off of CFT, $\Lambda_{\text{CFT}} \to \infty$, then the (infinite amount of) running due to matter fields results in vanishing of the propagator (or gauge coupling) for the source (see Eq. (3.3)). Since $\Lambda_{\text{CFT}} \sim 1/z_{UV}$, on the AdS side, this corresponds to the case where Planck brane is at the AdS boundary, i.e., $z_{UV} \to 0$ (with $k$ fixed). Then, gauge field evaluated at boundary (which is dual to source in CFT) is not dynamical (in agreement with the CFT side). The reason is that there is no normalizable zero-mode, whereas the continuum modes cannot reach the Planck brane (due to infinite potential barrier near the Planck brane in the analog quantum mechanics problem). In fact, we can repeat the calculation of reference [16] for a general position of Planck brane (i.e., for any $z_{UV}$, not necessarily $z_{UV} = 1/k$) and show that the boundary propagator (with $\tau_{UV} = 0$ and for $qz_{UV} \ll 1$) is $\sim -g_5^2k/[q^2 \log (qz_{UV})]$ (which vanishes as $z_{UV} \to 0$). This agrees with the source propagator in the dual CFT (Eq. (3.3)) since $\Lambda_{\text{CFT}} \sim 1/z_{UV}$. Since source/boundary value of bulk field is not dynamical in the limit $\Lambda_{\text{CFT}} \to \infty/z_{UV} \to 0$, path integral over these fields is not performed in Eq. (3.1) – this

---

4 Induced gauge theory has been discussed in other contexts, see, for example, [24].
is the limit of infinite AdS studied in the usual AdS/CFT duality [2–5].

With a Planck brane at \( z_{\text{UV}} \neq 0 \), but no TeV brane, the zero-mode is still not normalizable since it has a flat profile constant in \( z \) – both branes are required to get a normalizable zero-mode for the gauge field, unlike for graviton (or massless scalar field), where only the Planck brane is required. Nevertheless, for \( q \neq 0 \), some of the continuum modes do reach the Planck brane (by tunneling through the finite barrier in the analog quantum mechanics problem) so that the Planck brane propagator does not vanish. However, as \( q \to 0 \), the gauge coupling measured on Planck brane (\( \sim g_5^2 k / \log (k/q) \) for \( \tau_{\text{UV}} = 0 \)) vanishes since there is no zero-mode (and continuum modes do not contribute as \( q \to 0 \)). In the CFT dual, this corresponds to \( \Lambda_{\text{CFT}} \sim 1/z_{\text{UV}} \neq 0 \) and hence for \( q \neq 0 \), the source propagator is non-vanishing (see Eq. (3.3)), whereas propagator for source vanishes as \( q \to 0 \) since (as before) matter fields cause a gauge theory to be IR free [7,23]. As mentioned earlier, since the source/boundary value of bulk gauge field is dynamical in the case of \( \Lambda_{\text{CFT}} \neq 0 \) (i.e., in the case of RS model), we have to include both sides of Eq. (3.1) in a path integral over these fields.

4 Duality for RS1 with bulk gauge fields

When we have a TeV brane (RS1), on AdS side, there is a (normalizable) zero-mode (which is flat). The AdS/CFT correspondence as applied to RS1 [7–9] says that the IR brane at \( z_{\text{IR}} \) corresponds to spontaneous breaking of conformal invariance at \( \mu_{\text{CFT}} \sim 1/z_{\text{IR}} \sim \text{TeV} \). Also, KK modes of graviton (gauge boson) (with masses quantized in units of \( 1/z_{\text{IR}} \sim \text{TeV} \)) are dual to massive spin-2 (spin-1) bound states of CFT while massless bound states of CFT map on to fields on the TeV brane [7].

For \( q \gg \text{TeV} \), the kinetic term for the source in the dual CFT is still given by Eq. (3.3) (since for \( q \gg \text{TeV} \), the effect of breaking of conformal invariance is not important). Let us compare it to RS1 side. In reference [17], the tree-level gauge propagator in RS1 (for arbitrary external points) was computed using Neumann boundary condition at both Planck and TeV branes, i.e., again with \( \tau_{\text{UV,IR}} = 0 \) – for \( q \gg \text{TeV} \), the Planck brane propagator in RS1 is same as in that in RS2 (this propagator obviously includes effects of KK and zero modes). Thus, it agrees with source propagator in the dual CFT (in the large-\( N \) limit and with \( \tau_{UV} = 0 \)) for \( b_{\text{CFT}}|_{\text{large-N}} = 1/(g_5^2 k) \). Again, in the limit \( \tau_{UV} \to 0 \) studied (on RS1 side) in reference [17], the kinetic term for source in the CFT dual is induced by CFT loops. Thus, (in this limit) the CFT loop contribution to the source 1PI two-point function corresponds (on the RS1 side) to the contribution of KK and zero modes to the Planck brane propagator.

As in the case of RS2, we can compute the tree-level Planck brane propagator in RS1 for \( \tau_{UV} \neq 0 \) – for \( m_n \gg k \), wavefunction of KK modes at Planck brane is as in Eq. (3.4) (up to
a factor of $1/\sqrt{z_{IR}}$ to go from continuum to discrete normalization).\(^5\) Approximating the sum over (propagators of) KK modes lighter than $q$ by an integral (which is justified if $q \gg \text{TeV}$, i.e., splitting between KK masses) and adding the zero-mode contribution, we get the same propagator as in RS2 (Eq. (3.5)) which again agrees with source propagator in the dual CFT (Eq. (3.3)).

A brief comment as an aside: $\tau_{UV}$ is necessary for “holographic” RG \([25]\) which says that changing UV cut-off of CFT from $k$ to $k'$ (i.e., Wilsonian RG) corresponds to moving the position of Planck brane ($z_{UV}$) from $1/k$ to $1/k'$. On AdS side, to keep physical 4D gauge coupling (see Eq. (2.3)) invariant as we move the Planck brane (with the TeV brane, i.e., $z_{IR}$ fixed), we must include a boundary gauge kinetic term and change it: $\tau_{UV} \rightarrow \tau_{UV} + 1/(g^2 k) \log (k/k')$ (note that, in general, $\pi r_c \sim 1/k \log (z_{IR}/z_{UV})$).\(^6\) On CFT side, the cut-off dependence of CFT loop correction (to the external gauge coupling) is absorbed by $\tau_{UV}$ (as mentioned earlier).

### 4.1 Dual interpretation of coupling of KK modes

Actually, there is a subtlety in the CFT loop “calculation”: we expect there to be kinetic mixing between source (again, this is a dynamical field, but external to CFT) and spin-1 CFT bound states \([7]\) (similar to $\gamma - \rho$ mixing in QED coupled to QCD). For example, if we cut the Feynman diagrams which give the leading contributions to $\langle J_{\mu} J_{\nu} \rangle_{CFT}$, we get (only) one-“meson” intermediate states (as in the case of large-$N$ QCD – see, for example, reference \([26]\)). Thus, in the large-$N$ limit, $\langle J_{\mu} J_{\nu} \rangle_{CFT}$ is a sum of tree-level diagrams in which $J_{\mu}$ creates a $\rho$ meson with an amplitude $\langle 0| J_{\mu} |\rho \rangle$ which then propagates and is absorbed by $J_{\nu}$. Since $\langle J_{\mu} J_{\nu} \rangle_{CFT} \propto N/(16\pi^2)$, we get $\langle 0| J_{\mu} |\rho \rangle \propto \sqrt{N}/(4\pi)$ for the lightest $\rho$ meson with a mass of $\sim \mu_{CFT}$ (there might be a suppression by powers of $m_{\rho}/\mu_{CFT}$ for heavier (“$\rho$”) mesons). Then, “$\gamma - \rho$” mixing can be represented by $\sim \sqrt{N}/(4\pi) F_{\mu\nu} \rho^{\mu\nu}$.

In the case of small $\tau_{UV}$ (i.e., large bare gauge coupling), the above kinetic mixing is important since the CFT loop correction, which generates this mixing, also induces the kinetic term for source.\(^7\) As a result, the massless spin-1 state (which is dual to zero-mode of gauge field on RS1 side) is mixture of source and CFT fields – this is expected since zero-mode of RS1 has a flat profile and hence cannot correspond to just the source (which is dual to the gauge field evaluated on the boundary). Also, massive spin-1 states (which were pure CFT

\(^5\)For $\tau_{UV} = 0$, these wavefunctions were computed in \([11]\).

\(^6\)We can analyze the gauge coupling as measured on the Planck brane (instead of 4D gauge coupling) with the same result.

\(^7\)Conversely, this mixing is small in the limit $\tau_{UV} \rightarrow \infty$, i.e., small bare gauge coupling as in the case of QED (with a UV cut-off at, say, $M_4$) coupled to QCD.
bound states in the absence of mixing) now contain a part of the source. Hence, fields external to CFT which couple to the CFT only via the source (for example, the electron in the case of QED coupled to QCD) have a significant coupling to a single massive state [7]. In fact, using the above $\gamma - \rho$ kinetic mixing, we can estimate this ("$\rho - e$") coupling to be $\sim \sqrt{N} (4\pi) \times 1/[N/(16\pi^2) \times \log (k/\text{TeV})]$, where the first factor is from $\gamma - \rho$ mixing (which converts $\rho$ to $\gamma$) and the second factor accounts for $\gamma$ propagating to couple to the electron.

We have used Eq. (3.3) (evaluated at $q \sim m_\rho$) for the $\gamma$ propagator with $b_{CFT} \sim N/(16\pi^2)$ and $\tau_{UV} = 0$. Thus, the source propagator has poles corresponding to masses of these states – Eq. (3.3) is strictly speaking valid only for Euclidean momenta.

This is dual to the fact that the coupling of a gauge KK mode (which is dual to a massive spin-1 state in the CFT) to Planck brane fields (which are dual to fields external to CFT), given by $g_5 \psi_m$ (see Eq. (3.4)), is sizable. For the lightest KK mode, this coupling is $\sim g_5/\sqrt{k\pi r_c} \sim 0.2 \times g_4$ (using $k\pi r_c \sim \log (k/\text{TeV})$) in the case of $\tau_{UV} = 0$ [11]. In fact, this coupling agrees with our dual CFT estimate since $1/g_4^2 \sim k\pi r_c/ (g_5^2 k)$ (see Eq. (2.3)) and $1/(kg_5^2) = b_{CFT}|_{\text{large-}N} \sim N/(16\pi^2)$. Also, since the Planck brane propagator is a sum over (propagators of) these KK modes, it has poles at the KK masses – the expression for RS1 Planck brane propagator in Eq. (3.5) is also strictly speaking valid for Euclidean momenta.

The case of gauge field is to be compared to that of gravity, where mixing between source (which is dynamical) and spin-2 CFT bound states is very small for $q \ll k$. The reason is that, in the limit of very large (or infinite) bare gravitational constant, gravity (or Newton constant) is induced by the quadratically divergent part of CFT loop correction [7,8,13], i.e., the $\langle T_{\mu\nu}T_{\rho\sigma} \rangle$ correlator (which has a form similar to the $\langle \mathcal{O}\mathcal{O} \rangle$ correlator given earlier). Whereas, the mixing should be due to logarithmically divergent contribution and hence is suppressed by $\sim q^2/k^2$. Since the mixing is small, the source (which is dual to graviton field evaluated at the boundary) is mostly the massless spin-2 eigenstate and massive spin-2 states are mostly CFT bound states (with very small admixture of source). Thus, fields external to CFT (which couple to the CFT only via the source) couple weakly to a single massive spin-2 state. It is also clear that since the contribution of massive states to the source propagator (and hence to a scattering process involving fields external to the CFT) is due to mixing, it is suppressed by $\sim q^2/k^2$ compared to the contribution of the massless state.

On the RS side, this matches with localization of graviton zero-mode (which corresponds to massless spin-2 state in dual CFT) near the Planck brane and weak coupling of a KK graviton mode (which is dual to massive spin-2 state in CFT) with mass $\ll k$ to Planck brane fields (which are dual to fields external to the CFT) [1]. Also, the contribution of KK graviton modes

\textsuperscript{8} As in the case of the gauge field, we can estimate the coupling of (lightest) massive spin-2 state in the dual CFT to fields external to the CFT using the form of the $\langle T_{\mu\nu}T_{\rho\sigma} \rangle$ correlator. This coupling “agrees” with the

...
to the static potential between two masses on the Planck brane is suppressed by $\sim 1/(kr)^2$ compared to that of the zero-mode [1]. Obviously, a similar analysis holds for the case of a massless bulk scalar.

Of course, it is difficult to compute (analytically) the mixing between source and spin-1 CFT bound states since the CFT loop involves strong dynamics (just as one has to resort to, for example, lattice techniques to derive $\gamma - \rho$ mixing starting from QCD). Nevertheless, one can do the following check (which is an extension of the analysis of reference [7] to the case of $\tau_{UV} \neq 0$ on RS1 side).

We can obtain the (exact form of) CFT loop correction (self-energy of photon) from the AdS side using the correspondence [7]: Eq. (3.1) implies that the (large-$N$ limit of) correlator $\langle J_\mu J_\nu \rangle_{\text{CFT}} \equiv \int \mathcal{D}\psi J_\mu J_\nu \exp[iS_{\text{CFT}}(\psi)] \ (\psi$ are CFT fields) is given by dependence of (tree-level) 5$D$ action on boundary value of gauge field (with brane localized coupling, $\tau_{UV} = 0$).$^9$

Then, we can compare the result of resumming the source propagator (i.e., bare coupling $\tau_{UV} + \text{exact CFT loop}$)$^{10}$ (see Eq. (39) in (preprint version of) reference [7]) to the RS1 Planck brane propagator as modified by the addition of the brane-localized coupling, $\tau_{UV}$. This resummed source propagator has poles corresponding to masses $m_n$ of RS1 KK modes [7].$^{11}$ Away from poles (for example, with Euclidean momenta), for $q \gg \text{TeV}$, this source propagator (with the exact CFT loop correction) agrees with the RS1 Planck brane propagator which we computed above in Eq. (3.5) with a modification of the (Neumann) boundary condition at the Planck brane to include the effect of $\tau_{UV}$. We already mentioned this agreement based on the general form of CFT loop correction (i.e., based on Eq. (3.3) which is reasonably correct away from poles). The coupling of a massive spin-2 state to fields external to the CFT can be computed from the residue at the pole $m_n$ (see Eq. (44) in (preprint version of) reference [7]). One can show that it agrees with wavefunction of the RS1 KK mode of mass $m_n$ at the Planck brane, i.e., $g_5 \psi_{m_n}$ which we computed above in Eq. (3.4) (again, with a modified boundary condition at the Planck brane to include the effect of $\tau_{UV}$).

What is the dual interpretation of coupling of KK modes of gauge field to TeV brane fields? On RS1 side, KK modes of gauge field couple strongly to TeV brane fields [10,11]: the coupling is $\sim g_4 \sqrt{k\pi r_c} \sim O(10) \times g_4$ (since $k\pi r_c \sim \log \left[O(M_4)/\text{TeV}\right]$) for $\tau_{UV,IR} = 0$. This is dual to coupling of (lightest) graviton KK mode to Planck brane fields.$^9$

$^9$This 5$D$ quantity, i.e., $\partial^2 \Gamma[\phi_0]/\partial \phi_0^2$ is equal to (inverse of) tree-level Planck brane propagator (with $\tau_{UV} = 0$).

$^{10}$We can show that, in the large-$N$ limit (which corresponds to classical limit on RS1 side), the $\langle J_\mu J_\nu \rangle$ correlator is the same as in the pure CFT case (i.e., $\langle J_\mu J_\nu \rangle_{\text{CFT}}$ with no propagating sources) since the virtual effects of the sources ($A_\mu$ and $\phi$) propagating (on this correlator) are sub-leading in a large-$N$ expansion. Also, in the large-$N$ limit, the scalar loop can be neglected.

$^{11}$The masses of RS1 KK modes which are much lighter than $k$ are the same as in the case $\tau_{UV} = 0$. 

11
the coupling of massive CFT bound states to massless bound states ("$\rho - \pi - \pi$" couplings) which is $\sim 4\pi/\sqrt{N}$ in the large-$N$ limit (so that $\pi - \pi$ loop contribution to $\rho$ propagator is $\sim O(1/N)$, i.e., sub-leading order in a large-$N$ expansion: see, for example, reference [26]). For $1(kg_3^2) = b_{CFT}|_{large-N} \sim N/(16\pi^2)$, this agrees with the coupling on RS1 side (using the fact that $g_4 \sim \sqrt{g_5^2}k/\sqrt{k\pi r_c}$ – see Eq. (2.3)).

4.2 Dual interpretation of (one-loop corrected) low energy gauge coupling

Finally, we discuss the low energy gauge coupling. For $q \ll \mu_{CFT} \sim \text{TeV}$, on CFT side, we get the following (inverse) source propagator:

$$\left(\eta_{\mu\nu}q^2 - q_\mu q_\nu\right) \left(\tilde{\tau}_{UV} + \left[b_{\text{scalar}} + b_{CFT}\right] \log \frac{Q (M_4)}{\text{TeV}} + \tilde{\tau}_{IR} + b_4 \log \frac{\text{TeV}}{q}\right)$$

(4.1)

since running (and dressing) due to CFT fields stops at $\mu_{CFT} \sim \text{TeV}$, where the conformal invariance is broken – at this scale, there can be threshold effects $\sim \tilde{\tau}_{IR}$. These threshold effects corresponds to TeV brane-localized coupling on AdS side ($\tau_{IR}$ of Eq. (2.2)) and hence this notation in CFT dual: $\tilde{\tau}_{IR} = \tau_{IR}$ up to CFT corrections at sub-leading order in a large-$N$ expansion. Below $\sim \text{TeV}$, we have only zero-modes of photon and scalar (from the above discussion these are mixtures of $\phi, A_\mu$ and CFT fields) so that we get the usual 4D running, $b_4 \log (\text{TeV}/q)$.

On AdS side, since mass of lightest KK state is $\sim \text{TeV}$, only zero-mode contributes to any gauge propagator for $q \ll \text{TeV}$, in particular, to the propagator on the Planck brane. So, kinetic term for source in the CFT (which is dual to the propagator on Planck brane) should match (for $q \ll \text{TeV}$) with zero-mode gauge coupling of RS1 (including loop corrections)\(^{12}\) and it does as follows.\(^{13}\) In large-$N$ limit, $\tilde{\tau}_{UV(IR)} = \tau_{UV(IR)}$ and we can neglect $b_{\text{scalar}}, b_4$. Then, the CFT gauge coupling in Eq. (4.1) agrees with tree-level zero-mode coupling on RS1 side (Eq. (2.3)) again for $b_{CFT}|_{large-N} = 1/(kg_3^2)$ [7]. We see that the classical 4D coupling $g_4^2$ becoming zero as $r_c \to \infty$ (see Eq. (2.3)) is dual to $\mu_{CFT} \to 0$ and hence (infinite) running due to CFT matter fields causing gauge coupling to become zero in the IR (see Eq. (4.1) with $\mu_{CFT}$ replacing TeV) [7,23] – this is a quantum effect.

As per the AdS/CFT correspondence, loop effects on AdS side are dual to sub-leading effects in a large-$N$ expansion in the CFT [2–5]. In this case, the sub-leading corrections to the gauge coupling in the dual CFT are the terms with $b_{\text{scalar}}, b_4$ and sub-leading (i.e., $O(1)/(16\pi^2)$) part

---

\(^{12}\)This point was also made in [19].

\(^{13}\)This matching goes through for general (i.e., not just TeV) values of $1/z_{IR}$ and $\mu_{CFT}$ as long as $1/z_{IR} \sim \mu_{CFT}$. 

of $b_{CFT}$ in Eq. (4.1) and also the corrections to $\tau$’s. The $b_{\text{scalar}}$ term is calculable ($b_{\text{scalar}}$ is $b_4$ as mentioned earlier) and so is $b_4 \log(\text{TeV}/q)$ which is the running due to zero-mode below TeV and which matches with the same term in Eq. (2.4). But, the precise coefficients in the other sub-leading CFT effects are sensitive to the strong CFT dynamics (and hence difficult to compute), in particular, the sub-leading part of $b_{CFT}$ which is clearly comparable to the $b_{\text{scalar}}$ term (see Eq. (4.1)). However, using $k\pi r_c \sim \log [O(M_4)/\text{TeV}]$, we see that the general form (i.e., up to uncalculable $O(1)$ coefficients) of the sub-leading CFT effects agrees with RS1 loop correction to the gauge coupling (see Eq. (2.4)) [15]. So, the dual CFT is not useful to compute loop corrections, but serves as a consistency check – for example, if loop correction on RS1 side is some other power of $k\pi r_c$, then there is no dual CFT interpretation of RS1 loop effects.

As this paper was in preparation, reference [20] appeared which has some overlap with our discussion of loop corrections in RS being dual to sub-leading CFT effects.

## Acknowledgments

K. A. is supported by the Leon Madansky Postdoctoral fellowship and by NSF Grant P420D3620414350. A. D. is supported by NSF Grants P420D3620414350 and P420D3620434350. We thank Raman Sundrum for many useful discussions and for reading the manuscript. We also thank Enrique Alvarez, Csaba Csaki, Walter Goldberger, Gregory Gabadadze and Alex Pomarol for discussions, Csaba Csaki for encouraging us to write up this work and the Aspen Center for Physics for hospitality during part of this work.

## References


