Quantum information processing by incoherent photon substitution

1 INTRODUCTION

Quantum information processing (QIP) is a paradigm of computation where quantum states and quantum operations are used to process information. In QIP, quantum bits (qubits) are manipulated using quantum gates, and quantum information is stored in quantum states. QIP is considered to be a more powerful model of computation than classical computation, and it is expected to be used in future quantum computers. However, QIP is not yet feasible due to the limitations of current quantum technology. In this paper, we propose a method of QIP using incoherent photon substitution (IPS). IPS is a method of replacing a quantum state with a classical state. By using IPS, we can simulate QIP using classical computing resources. This paper presents the theory and implementation of IPS, and we also demonstrate its feasibility through numerical simulations.
teleportation, provided that the initial TWB energy is below a certain value.

The paper is structured as follows: in Section II we describe in details the IPS scheme, whereas in Section III we analyze the use of the IPS output state in coherent state teleportation. Section IV closes the paper with some concluding remarks.

II. THE INCONCLUSIVE PHOTON SUBTRACTION SCHEME

In IPS the two channels of TWB impinge onto two beam splitters each with transmissivity $\tau$, which we consider equal, where they are mixed with the vacuum state $|0\rangle_c |0\rangle_d$ of modes $c$ and $d$ (see Figure I). The effect of the beam splitter on two modes, say $a$ and $c$, is described by the unitary operator

$$U_{ac}(\tau) = \exp \left[ \lambda \tau (a^\dagger c - ac^\dagger) \right],$$

where

$$\lambda \tau = \arctan \left( \frac{\sqrt{1 - \tau}}{\tau} \right).$$

and $a$, $c$, $a^\dagger$ and $c^\dagger$ are the annihilation and creation operators for modes $a$ and $c$, respectively. After the beam splitters the wave function of the system is

$$|\psi_{BS}\rangle = U_{ac}(\tau) U_{bd}(\tau) |\text{twb}\rangle |0\rangle_a |0\rangle_d = \sqrt{1 - x^2} \sum_{n=-\infty}^{n=\infty} \frac{(1 - \tau)^n}{\tau} \left( \sum_{p,q=0}^{\infty} \binom{n}{p} \binom{n}{q} |n-p\rangle_a |n-q\rangle_b |p\rangle_c |q\rangle_d. \right.$$  

Now, we perform a conditional inconclusive photon subtraction revealing the mode $c$ and $d$ by ON/OFF photodetection. The POVM $\{\Pi_0(\eta), \Pi_1(\eta)\}$ (positive operator-valued measure) of each ON/OFF detector is given by

$$\Pi_0(\eta) = \sum_{j=0}^{\infty} (1 - \eta)^j |j\rangle \langle j|, \quad \Pi_1(\eta) = 1 - \Pi_0(\eta),$$

$\eta$ being the quantum efficiency. Overall, the conditional measurement on the modes $c$ and $d$, is described by the POVM

$$\Pi_{00}(\eta) = \Pi_{0,c}(\eta) \otimes \Pi_{0,d}(\eta), \quad \Pi_{11}(\eta) = \Pi_{1,c}(\eta) \otimes \Pi_{1,d}(\eta).$$

We are interested in the situation when both the detectors click. The corresponding conditional state for the modes $a$ and $b$ will be referred to as the IPS state. Notice that this kind of measurement is inconclusive, i.e., it does not discriminate the number of photons present in the beams: one can only say that a certain unknown number of photons has been revealed and, then, subtracted from each mode and that this number, in general, is not the same for the two modes. The probability of observing a click in both the detectors is given by

$$p_{11}(x, \tau, \eta) = \text{Tr}_{abcd} \{ \rho_{BS} \otimes 1_b \otimes \Pi_{11}(\eta) \} = \frac{x^2 \eta^2 (1 - \tau)^2 \{ 1 + x^2 [1 - \eta (1 - \tau)] \} \{ 1 - x^2 [1 - \eta (1 - \tau)] \}^2}{\{ 1 - x^2 [1 - \eta (1 - \tau)] \} \{ 1 - x^2 [1 - \eta (1 - \tau)] \}^2},$$

where $\rho_{BS} = |\psi_{BS}\rangle \langle \psi_{BS}|$ and the corresponding conditional state reads as follows

$$\rho_{IPS}(x, \tau, \eta) = \frac{p_{11}(x, \tau, \eta)}{p_{11}(x, \tau, \eta)} \sum_{n,m=-\infty}^{n,m} \binom{n}{h} \binom{m}{k} \binom{m}{k} \sum_{h,k=0}^{\infty} f_{h,k}(\tau, \eta) \sqrt{\binom{n}{h} \binom{n}{k} \binom{m}{k}} \times |n-k\rangle_a |n-h\rangle_b |m-h\rangle_d |m-k|.$$
where

$$f_{b, k}(\tau, \eta) = \left[ 1 - (1 - \eta)^k \right] \left[ 1 - (1 - \eta)^k \right] \left( \frac{1 - \tau}{\tau} \right)^{b+k}.$$  \hfill (12)

The mixing with the vacuum in a beam splitter with transmissivity \(\tau\) followed by ON/OFF detection with quantum efficiency \(\eta\) is equivalent to mixing with an effective transmissivity

$$\tau_{\text{eff}}(\tau, \eta) = 1 - \eta(1 - \tau)$$  \hfill (13)

followed by an ideal \((i.e., \text{efficiency equal to one})\) ON/OFF detection. Therefore, the IPS state \(\text{II}\) can be studied for \(\eta = 1\) and replacing \(\tau\) with \(\tau_{\text{eff}}\). In this way, the conditional probability \(\text{II}\) of obtaining the IPS state rewrites as

$$p_{11}(x, \tau_{\text{eff}}) = \frac{x^2(1 - \tau_{\text{eff}})^2(1 + x^2 \tau_{\text{eff}})}{(1 - \tau_{\text{eff}})(1 - x^2 \tau_{\text{eff}})},$$  \hfill (14)

which, in general, is larger than the corresponding probability for conclusive photo-subtraction methods, where

$$\Delta_{s,b}(x, \tau_{\text{eff}}) = \frac{\langle d^2 \rangle - \langle d \rangle^2}{\langle n_a + n_b \rangle} = \frac{(1 - \tau_{\text{eff}})(1 - x^2 \tau_{\text{eff}})^2(2 - x^2 - x^4 \tau_{\text{eff}})}{(1 + \tau_{\text{eff}})(1 - x^2 \tau_{\text{eff}})[2 - x^2(1 + \tau_{\text{eff}} + \tau_{\text{eff}}^2) + x^6 \tau_{\text{eff}}^3]}.$$  \hfill (15)

In Figure 5, we plot the average photon number of TWB and of the IPS state: below a certain threshold value for \(x\) the energy of the IPS state is increased. As a matter of fact, the IPS state is no longer a pure state and, therefore, the excess Von-Neumann entropy cannot be used to quantify the degree of entanglement. In order to characterize the IPS state we analyze the quantity

$$\Pi_{s}(\beta) = \frac{1}{\pi} D(\beta) a^T D(\beta)$$  \hfill (16)

where \(\beta\) is a complex number, \(D(\beta) = \exp{\{\beta a^T - \beta^* a\}}\) is the displacement operator, \((\cdot)^T\) stands for the transposition operation. The probability for the outcome \(\beta\) is

$$p(x, \tau, \eta, \beta) = Tr_{ab} \{ \rho_{\text{IPS}} \Pi_{s}(\beta) \otimes I_b \},$$  \hfill (17)

and the conditional state is

$$\rho_{\text{out}}(x, \tau, \eta, \beta) = Tr_{b} \{ \rho_{\text{IPS}} \Pi_{s}(\beta) \otimes I_b \} / p(x, \tau, \eta, \beta),$$  \hfill (18)

which, after the displacement by Bob, becomes \(\rho_{\text{out}}(x, \tau, \eta, \beta) = D(\beta) \rho_{\text{out}} D(\beta)^T\). For coherent state teleportation \(\sigma = |\alpha\rangle \langle \alpha|\) we have
\[ \psi_{\text{out}}(x, \tau, \eta, \beta) = \frac{1 - x^2}{p_{11}(x, \tau, \eta) p(x, \tau, \eta, \beta)} \sum_{n, m=0}^{\infty} \left( x \tau \right)^n \sum_{b, k=0}^{\min[n, m]} f_{b, k}(\tau, \eta) \times \sqrt{\binom{n}{h} \binom{m}{k}} \frac{e^{-\alpha + \beta x} \left( \alpha \beta \right)^{n-k} e^{-\frac{\alpha + \beta x}{2} (m-m-k)}}{\sqrt{(n-k)! (m-k)! (n-h)! (m-h)!}} \times \left( m-h \right)_{n-h} \]  

where \( p_{11}(x, \tau, \eta) \), \( f_{b, k}(\tau, \eta) \) and \( p(x, \tau, \eta, \beta) \) are given by Eqs. (18), (19) and (20) respectively. Therefore the teleportation fidelity, \( F(x, \tau, \eta, \beta) \), and the average fidelity, \( \overline{F}(x, \tau, \eta) \), are given by

\[ F(x, \tau, \eta, \beta) \equiv \langle \alpha | \psi_{\text{out}} | \alpha \rangle \]
\[ = \frac{1 - x^2}{p_{11}(x, \tau, \eta) p(x, \tau, \eta, \beta)} \sum_{n, m=0}^{\infty} \left( x \tau \right)^n \sum_{b, k=0}^{\min[n, m]} f_{b, k}(\tau, \eta) \times \sqrt{\binom{n}{h} \binom{m}{k}} \frac{e^{-\alpha + \beta x} \left( \alpha \beta \right)^{n-k} e^{-\frac{\alpha + \beta x}{2} (m-m-k)}}{\sqrt{(n-k)! (m-k)! (n-h)! (m-h)!}} \]  

and

\[ \overline{F}(x, \tau, \eta) \equiv \int d^2 \beta p(x, \tau, \eta, \beta) F(x, \tau, \eta, \beta) \]
\[ = \frac{1 - x^2}{2 p_{11}(x, \tau, \eta)} \sum_{n, m=0}^{\infty} \left( \frac{x \tau}{2} \right)^n \sum_{b, k=0}^{\min[n, m]} 2^{b+k} f_{b, k}(\tau, \eta) \times \sqrt{\binom{n}{h} \binom{m}{k}} \frac{\left( n+m-h-k \right)!}{\sqrt{(n-h)! (n-k)! (m-k)! (m-k)!}} \]  

By the substitution \( \eta \rightarrow 1 \) and \( \tau \rightarrow \tau_{\text{eff}} = 1 - \eta (1 - \tau) \), Eq. (18) can be summed, leading to the following expression

\[ \overline{F}(x, \tau_{\text{eff}}) = \frac{1}{2} \left[ 1 + x \right] \left[ 1 + x \tau_{\text{eff}} \right] \frac{\left[ 2 - 2 x \tau_{\text{eff}} + x^2 \tau_{\text{eff}} \right]}{\left[ 2 - 2 + \left( 1 - \tau_{\text{eff}} \right) x \tau_{\text{eff}} \right]} \]  

In Figure 1 we plot the average fidelity for different values of \( \tau_{\text{eff}} \); the IPS state improves the average fidelity of quantum teleportation when the energy of the incoming TWB is below a certain threshold, which depends on \( \tau_{\text{eff}} \) and, in turn, on \( \tau \) and \( \eta \) (see Eq. (20)). When \( \tau_{\text{eff}} \) approaches unit (when \( \eta \rightarrow 1 \) and \( \tau \rightarrow 1 \)), Eq. (20) reduces to the result obtained by Milburn et al. in Ref. (11) and the IPS average fidelity (line labelled with “a” in Figure 1) is always greater than the one obtained with the TWB state (11), i.e.

\[ \overline{F}_{\text{TWB}}(x) = \frac{1 + x}{2} \]  

However, a threshold value, \( x_{\text{th}}(\tau_{\text{eff}}) \), for the TWB parameter \( x \) appears when \( \tau_{\text{eff}} < 1 \); only if \( x \) is below this threshold the teleportation is actually improved \( \overline{F}(x, \tau_{\text{eff}}) > \overline{F}_{\text{TWB}}(x) \), as shown in Figure 1. Notice that, for \( \tau_{\text{eff}} < 0.5 \), \( \overline{F}(x, \tau_{\text{eff}}) \) is always below \( \overline{F}_{\text{TWB}}(x) \). A fidelity larger than \( 1/2 \) is needed to show that a truly
nonlocal information transfer occurred. Notice that using both the TWB and the IPS state, this limit is always reached (Figure 6). Nevertheless, we remember that in teleportation protocol the state to be teleported is destroyed during the measurement process performed by Alice, so that the only remaining copy is that obtained by Bob. When the initial state carries reserved information, it is important that the only existing copy will be the Bob’s one. On the other hand, using the usual teleportation scheme, Bob cannot avoid the presence of an eavesdropper, which can clone the state, obviously introducing some error, but he is able to verify if his state was duplicated. This is possible by the analysis of the average teleportation fidelity: when fidelity is greater than 2/3, Bob is sure that his state was not cloned.

The dashed line in Figure 6 shows the values \( x_{\text{th}}(\tau_{\text{F}}) \) which give an average fidelity equal to 2/3; notice that when \( x_{\text{th}} < x < x_{\text{th}} \) both the teleportation is improved and the fidelity is greater than 2/3. Moreover, while the condition \( F_{\text{TWB}}(x) > 2/3 \) is satisfied only if \( x > 1/3 \), for the IPS state there exists a \( \tau_{\text{F}} \)-dependent interval of \( x \) values \( x_{\text{th}} < x < 1/3 \) for which teleportation can be considered secure \( F(x, \tau_{\text{F}}) > 2/3 \).

IV. CONCLUSIONS

We have analyzed a photon subtraction scheme similar to that of Refs. to modify twin-beam and improve coherent state teleportation. The difference in our analysis is that the conditional photodetection after the beam splitters is considered inconclusive, i.e. performed by ON/OFF detectors which do not discriminate among the number of photons. This is closer to the current experimental situations and provides a higher conditional probability. We found that fidelity is improved compared to that of TWB-based teleportation if the initial TWB parameter is smaller than a threshold value, which in turn depends on the beam splitter transmissivity and on the quantum efficiency of the photodetectors. For realistic values of these parameters (\( \eta \) larger than 90% and \( \tau \) larger than 99%) the threshold is close to unit. In addition, there exists an interval of \( x \) for which teleportation can be considered secure, i.e. the receiver is able to check whether or not the state has been duplicated before teleportation. We conclude that IPS on TWB is a robust and realistic scheme to improve coherent state teleportation using current technology.

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FIG. 1: Schematic diagram of continuous variable optical quantum teleportation assisted by inconclusive photon subtraction.

FIG. 2: Conditional probability $p_{12}(x, \tau_{\text{eff}})$ of obtaining the IPS state as a function of the TWB parameter $x$ for different values of the effective transmissivity $\tau_{\text{eff}} = 0.5$ (a), 0.8 (b), 0.9 (c) and 1 (d).

FIG. 3: Log-Linear plot of TWB (dashed line) and IPS state average photon number as a function of the TWB parameter for different values of $\tau_{\text{eff}} = 1 - \eta(1 - \tau)$ (solid lines from top to bottom: $\tau_{\text{eff}} = 1, 0.9, 0.8$ and 0.5).
FIG. 4: IPS average fidelity $\overline{F}(x, \tau_a)$ as a function of the TWB parameter for different values of $\tau_a = 1 - \eta(1 - \tau)$ ($\tau_a = 1$ (a), 0.9 (b), 0.8 (c) and 0.5 (d)); the dashed line is the average fidelity $\overline{F}_{TWB}(x)$ for teleportation with TWB.

FIG. 5: Threshold value $x_{th}(\tau_a)$ on the TWB parameter $x$ (solid line): when $x < x_{th}$ we have $\overline{F}(x, \tau_a) > \overline{F}_{TWB}(x)$ and teleportation is improved. The dot-dashed line is $x = 1/3$, which corresponds to $\overline{F}_{TWB} = 2/3$: when fidelity is greater than $2/3$ Bob is sure that his teleported state is the best existing copy of the initial state $\rho$. The dashed line represents the values $x_{2/3}(\tau_a)$ giving an average fidelity $\overline{F}(x, \tau_a) = 2/3$. When $x_{2/3} < x < x_{th}$ both the teleportation is improved and the fidelity is greater than $2/3$. 