Solitonic $D$-branes and brane annihilation

Gia Dvali$^a$ and Alexander Vilenkin$^b$.

$^a$Department of Physics, Center for Cosmology and Particle Physics, New York University, New York, NY 10003

$^b$Institute of Cosmology, Department of Physics and Astronomy, Tufts University, Medford, MA 02155, USA

Abstract

We point out some intriguing analogies between field theoretic solitons (topological defects) and $D$-branes. Annihilating soliton-antisoliton pairs can produce stable solitons of lower dimensionality. Solitons that localize massless gauge fields in their world volume automatically imply the existence of open flux tubes ending on them and closed flux tubes propagating in the bulk. We discuss some aspects of this localization on explicit examples of unstable wall-anti-wall systems. The annihilation of these walls can be described in terms of tachyon condensation which renders the world-volume gauge field non-dynamical. During this condensation the world volume gauge fields (open string states) are resonantly excited. These can later decay into closed strings, or get squeezed into a network of cosmic strings formed at a cosmological phase transition. Although, as in the $D$-brane case, perturbatively one can find exact time-dependent solutions, when the energy of the system stays localized in the plane of the original soliton, such solutions are unstable with respect to decay into open and closed string states. Thus, when a pair of such walls annihilates, the energy is carried away (at least) by closed string excitations ("glueballs"), which are the lowest energy excitations about the bulk vacuum. Suggested analogies can be useful for the understanding of the complicated $D$-brane dynamics and of the production of topological defects and reheating during brane collision in the early universe.

1 Branes and Solitons

Unstable $D$-brane systems may play an important role in cosmology, since they can give an explicit string realization of the inflationary scenario [1]. Their post
-inflationary evolution can provide simple mechanisms for baryogenesis [2], and for
generation of dark matter in the form of open strings stretched between the $D$-
branes [3]. Significant progress has been recently made in the understanding of the
dynamics of unstable non-BPS $D$-brane configurations [4]. Some interesting features
have emerged [5], which very briefly can be summarized as follows.

Annihilation of $D$-brane anti-$D$-brane pair can be described in the language of a
(single) tachyon condensation. This tachyon is the tachyon of the bosonic open string
theory, living on the world-volume of an unstable non-BPS brane. Tachyonic vacuum
is the closed string vacuum, about which there are no open string excitations. Due
to this, the energy initially stored in $D$-anti-$D$ system cannot dissipate in the bulk,
and stays localized in the plane of the original $D$-brane. Energy-momentum of a
rolling tachyon asymptotically reach the form characteristic of a pressureless gas.

When the tachyon rolls away from the maximum of its potential, it can produce
topologically stable knots in the world volume. These knots are stable BPS $D$-
branes of lower dimensionality. Thus, annihilating $D$-branes can produce branes of
lower dimensions.

The above features can have important cosmological implications in the context
of $D$-brane inflation [1] (see [6, 7, 8] for recent progress in model building). In
this setup, a pair (or number) of originally separated $D$-branes or brane-anti-branes
slowly fall towards one another. During this process the Universe undergoes a stage
of exponential expansion. The scalar excitation corresponding to the inter-brane
distance plays the role of a slowly rolling inflaton field. Gradually $D$-branes accelerate,
and slow-roll inflation stops. Soon after this point, branes collide and reheat the
Universe. This marks the beginning of the standard big bang cosmology, in which
the subsequent evolution is determined by the dynamics of brane collision. While
in the case of $D$-branes this dynamics is well understood (at least at the level of an
effective low-energy gauge theory), for a $D$-$\bar{D}$ system it is rather non-trivial. For
instance, it has been suggested that the energy stored in the rolling tachyon can
play the role of dark matter[5]. However, it was shown recently in [9], that unless the
energy from the tachyon condensate dissipates very efficiently, the tachyon will in
fact overclose the Universe. Therefore, it is very important to understand the na-
ture of energy transfer from the tachyon to the ordinary particles. Moreover, stable
$D$-branes of lower dimensionality, produced by annihilating $D$-branes, can play the
role of topological defects, such as cosmic strings, monopoles, or domain walls, and
can be potentially detected by observations [10]. They may also help in liberating
part of the tachyonic energy[11].

The aim of the present paper is twofold. We point out some close analogies
between $D$ branes and field-theoretic topological defects. We also identify some new
decay mechanisms for defect-anti-defect pairs and argue that similar mechanisms
should operate in the decay of $D - \bar{D}$ pairs. The close analogy between $D$-branes
and field theory topological defects is interesting for two reasons. First, it becomes
possible to model at least some aspects of $D$-brane dynamics on simpler field the-
oretical models. Second, since analogies are much closer than one might naively
expect, they may enable us to better understand the solitonic nature of $D$-branes.

In the present paper, we shall explicitly demonstrate the following properties.

1) Annihilation of a pair of field theoretic topological defects (of opposite topological charge) can produce stable lower dimensional topological defects.

2) This annihilation can also be described in the language of tachyon condensation.

3) Topological defects that support massless gauge field excitations in their world-volume necessarily allow for the electric flux tubes (open strings) ending on their world volume. They also imply the existence of closed flux tubes (closed strings) in the bulk. Thus, such solitons are analogous to $D$-branes.

4) The energy of annihilating defects is dissipated in the form of strings and scalar waves. In some exceptional models the scalar wave production can be suppressed. Much as in the case of unstable $D$-branes, one can construct classically-exact, time-dependent localized lump solutions. These have the property that, although the lump evolves in time, the energy stored in the original soliton classically stays in the same plane and cannot dissipate into scalar waves.

5) Nevertheless, such solutions are unstable with respect to the resonant production of world-volume states, such as the localized gauge fields. Production of the world-volume gauge field (open string states) during tachyon condensation is a generic feature of unstable solitons that localize massless gauge field in their world volume. We expect similar effects to take place during the decay of unstable $D$-brane systems.

6) Since the soliton localizes a massless gauge field, it inevitably couples to the open and closed string sectors. Both of these states are produced during the tachyon condensation. Closed strings are expected to be the primary energy carriers in the bulk. Such models share closest similarities with unstable $D$-branes as far as energy dissipation is concerned.

7) When a pair of defects annihilates, the electric field excited along the world-volume is squeezed into electric flux tubes (strings), while the rest of the energy dissipates away. The strings form a random network consisting of infinite strings and closed loops, and their subsequent evolution is expected to be similar to that of the conventional cosmic strings.

2 Defects Creating Lower Dimensional Defects.

Topological defects are stable field configurations that appear in theories with topologically non-trivial vacuum manifolds. Most commonly (but not necessarily) this happens when, as a result of the Higgs effect, a certain symmetry group $G$ breaks spontaneously down to its subgroup $H$. Defects can be characterized by homotopy classes $\pi_n(G/H)$, which classify uncontractible $n$-dimensional surfaces in $G/H$. Stable field configurations can exist whenever the configuration of the order parameter (Higgs vacuum expectation value (VEV)) $\Phi$ at space infinity is isomorphic to an
uncontractible n-surface in G/H. In a D-dimensional spacetime a stable defect has
n + 1 transverse and p = D − 2 − n longitudinal (world-volume) space dimensions.
We shall call such a defect a topological (or solitonic) p-brane. In the simplest cases
the VEV of the Higgs field vanishes in the core of the defect, so that G is restored.
However, such a configuration may not necessarily be energetically favored, and the
actual structure of the core may be more complicated. At the moment we shall
discuss the simplest case.

Topological p-branes are characterized by a topological charge qn, corresponding
to a given homotopy class of n-surfaces. In such a case the two branes with opposite
topological charge can annihilate into the vacuum state. However, as noticed in [12],
n-homotopy topological charge may not be the only characteristic of a (D − 2 − n)-
brane. A given p-brane may be characterized by a pair of topological charges qn, qn′
with respect to different homotopy groups. The existence of such a system can be
understood as follows. Let the vacuum manifold G/H have two nontrivial homotopy
classes corresponding to πn and πn′, respectively. Let us assume that n < n′. Then
there are stable topological (D − 2 − n)-branes and (D − 2 − n′)-branes respectively,
created by the same order parameter Φ. Each of these objects is stable due to a
topological obstruction at infinity. So if the two p-branes are separated far enough
in the transverse dimension, they continue to be stable.

Let us ask what happens if we try to place a (D − 2 − n′)-brane with the charge qn
in the world volume of a (D − 2 − n)-brane with the charge qn′? When we do so, there
is no topological reason for the (D − 2 − n′)-brane to be stable, and it will unwind.
This effect was called “defect erasure” in [12]. A lower-dimensional defect is erased
by a higher-dimensional one. The existing topological charge cannot disappear,
however, and gets transferred to the higher dimensional brane. As a result, we shall
get a (D − 2 − n)-brane with a pair of homotopy charges qn, qn′. In other words, the
qn′ charge of a (D − 2 − n)-brane is determined by how many (D − 2 − n′)-branes
it has erased.

Now, if such a (D − 2 − n)-brane with charge qn, qn′ encounters a (D − 2 − n)-
brane of charge −qn, 0, the result of the annihilation will not be the vacuum state,
but rather a stable (D − 2 − n′)-brane of charge qn′. This is closely analogous to the
annihilation of string theory D(p)-branes.

We shall now study this effect in more detail on an explicit example of [12]. This
example considers the interaction between a magnetic monopole and a domain wall.
On the D-brane side it shares some analogies with D(p)-D(p + 2) brane system
studies by Gava, Narain and Sarmadi [13].

We consider the SU(5) Grand Unified model with an adjoint, Φ, (and a fundamental)
scalar field. We take the standard Higgs potential for Φ,

\[ V(\Phi) = -\frac{1}{2} m^2 \text{Tr} \Phi^2 + \frac{h}{4} (\text{Tr} \Phi^2)^2 + \frac{\lambda}{4} \text{Tr} \Phi^4. \]

This potential has a Z2 symmetry: Φ → −Φ.
The spontaneous symmetry breaking

\[ SU(5) \times Z_2 \rightarrow [SU(3)_c \times SU(2)_L \times U(1)_Y]/Z_6 \]  

occurs when \( \Phi \) acquires a vacuum expectation value

\[ \Phi_0 = \frac{v}{\sqrt{30}} \text{diag}(2, 2, 2, -3, -3), \]

where

\[ v = m\sqrt{\frac{30}{30h + 7\lambda}} \equiv \frac{m}{\sqrt{\lambda'}}. \]  

To pick out this direction, we need the following constraints on the parameters in
the Higgs potential

\[ \lambda > 0, \quad h > -\frac{7}{30} \lambda \quad (i.e. \lambda' > 0). \]

The VEV \( \Phi = -\Phi_0 \) also leads to the symmetry breaking (1). The two discrete vacua \( \Phi = \pm \Phi_0 \) are degenerate due to the exact \( Z_2 \) symmetry. It is well known that the symmetry breaking (1) leads to magnetic monopoles and \( Z_2 \) domain walls.

Following [12], let us ask what happens when a monopole hits a domain wall? Here, there are two possibilities. The first is when \( \Phi = 0 \) and the other is when \( \Phi \neq 0 \) inside the wall. Both cases are possible, but here we shall discuss only the first possibility.

The most likely result of the monopole-wall encounter is that the monopole will unwind on entering the wall where the full \( SU(5) \) symmetry is restored. The indications for that are as follows:

(i) Perturbatively, there is a short range attractive force between the monopoles and the walls coming from the Higgs exchange. This is because the Higgs field inside the monopole, as well as inside the domain wall, is not in the vacuum, which is costly in Higgs energy. So a monopole can save the Higgs energy of its core by entering the core of the wall where the zero Higgs VEV is supported by the wall. Thus, the monopole tends to form a bound state with the wall \(^1\).

(ii) Once such a bound state is formed, as there is no topological obstruction to the unwinding of monopoles on the wall, the monopoles can continuously relax into the vacuum state. Their magnetic charge will then spread out along the wall.

(iii) Energetically, the most favored state is where the monopole unwinds.

(iv) Finally, in [14] the interaction of domain walls and monopoles was studied numerically. It was found that monopoles indeed unwind when passing through the wall.

\(^1\)It was observed [12] that the domain wall and monopole bound state can lead to a classical realization of a D-brane if the \( SU(3)_c \) symmetry group further breaks to \( Z_3 \) since now the monopoles bound to the walls will be connected by strings. These strings will be the magnetic flux tubes of \( SU(3)_c \) field. We will show below that if \( SU(3)_c \) stays unbroken, there still are open strings ending on the wall. These are the \( SU(3)_c \)-electric flux tubes.
The fundamental magnetic monopole in the above model is essentially an SU(2) monopole embedded in the full theory. The monopole solution has the following form:

\[
\Phi_M \equiv \sum_{a=1}^{3} \Phi^a T^a + \Phi^4 T^4 + \Phi^5 T^5 ,
\]

where the subscript \( M \) denotes the monopole field configuration,

\[
T^a = \frac{1}{2} \text{diag}(\sigma^a, 0, 0, 0) , \quad T^4 = \frac{1}{2\sqrt{3}}(0, 0, 1, 1, -2) , \quad T^5 = \frac{1}{2\sqrt{15}}(-3, -3, 2, 2, 2) ,
\]

\( \sigma^a \) being the Pauli spin matrices,

\[
\Phi^a = P(r)x^a , \quad \Phi^4 = M(r) , \quad \Phi^5 = N(r) ,
\]

where \( r = \sqrt{x^2 + y^2 + z^2} \) is the spherical radial coordinate. The ansatz for the gauge fields of the monopole is:

\[
W^a_i = \varepsilon^a_{ij} x^j (1 - K(r)) , \quad (a = 1, 2, 3) , \quad W^b_i = 0 , \quad (b \neq 1, 2, 3).
\]

The exact solution is known only in the case of a vanishing potential (the BPS limit). In the non-BPS case, the profile functions \( P(r), K(r), M(r) \) and \( N(r) \) need to be found numerically. In [16] they were found by using a relaxation procedure with the BPS solution serving as the initial guess.

The solution for a domain wall located in the xy-plane is

\[
\Phi_{DW} = \eta 2\sqrt{15} \tanh(\sigma z)(2, -3, 2, 2, -3) ,
\]

where \( \sigma = \eta \sqrt{\lambda/2} \).

When the monopole and the domain wall are very far from each other, the joint field configuration is given by the product ansatz:

\[
\Phi = \tanh(\sigma(\bar{z} - z_0))\Phi_M ,
\]

where \( z_0 \) is the position of the wall and \( \Phi_M \) is the monopole solution in eq. (3).

As we already mentioned, monopoles unwind when they are placed on top of a domain wall, with their magnetic energy spreading along the wall. To visualize this process, let us imagine the interaction of a spherical domain wall and the monopole. For a large radius \( r >> \sigma^{-1} \) such a domain wall can be approximated by

\[
\Phi = \eta 2\sqrt{15} \tanh(\sigma(\rho - r)) ,
\]
where $\rho$ is a radial coordinate. When such a domain wall erases a monopole, the magnetic charge of the monopole gets uniformly distributed on the sphere. The field configuration outside the sphere is then similar to the field of a magnetic monopole placed at $\rho = 0$. The spherical domain wall carries zero $q_0$ topological charge, but nonzero $q_2$ magnetic charge. Such a system is unstable and will collapse, producing a magnetic monopole.

If a monopole is erased by an infinite planar domain wall, its magnetic charge spreads along the wall in the form of Higgs and gauge field excitations, but at any finite time it will be within a finite radius of the point of the original encounter. If the wall then annihilates with an anti-wall, the magnetic charge will recollapse and form a stable magnetic monopole.

Magnetic monopoles will be produced in wall annihilations even if the colliding walls have no net magnetic charge. This is due to quantum (or thermal) fluctuations of the Higgs field and of the magnetic charge density along the walls. This mechanism of defect formation, which is similar to the usual Kibble mechanism, will be analyzed in a separate paper. In the context of $D$-brane annihilation, it was recently discussed by Sarangi and Tye [10].

3 Open Strings Ending on Domain walls

3.1 General Mechanism for Gauge Field Localization

It has been known for some time [17] that domain walls, like $D$-branes [18], can support massless gauge fields in their world volume. It is interesting that such a situation automatically implies that there must also be open strings ending on such domain walls. Strings in question are the electric flux tubes. In addition, there must be closed strings in the bulk.

This result is very general and is independent of the detailed mechanism of gauge field localization. Consider a domain wall (or any solitonic brane) and assume that it localizes a massless Abelian gauge field in its world-volume. We shall assume that there is a mass gap and no massless photon in the bulk theory. This implies that test electric charges located on the wall must be in the Abelian Coulomb phase at low energies. This can only happen if the test charges outside the brane are in the confining phase. Indeed, if the bulk test charges were in the Higgs phase, then the photon on the brane would be massive, due to charge screening. If instead they were in the Coulomb phase, then the gauge field could not be localized, simply by charge conservation and universality. Charges cannot go where there is no massless photon. So the only regime that is compatible with the charge-universality and flux conservation is the confining phase outside the wall. This automatically implies that the electric field outside the brane can only exist either in the form of flux tubes attached to the wall (open strings), or the closed flux tubes (closed strings) that can move freely in the bulk. This striking analogy with the $D$-brane picture
may indicate an intrinsic connection between the existence of massless world-volume gauge field and of open strings in the same theory.

In this section, we shall discuss this effect in more detail and generalize it to unstable wall systems. We shall first review the general mechanism of Ref. [17] and later discuss the analogies with D-branes. We shall also show that the SU(5) domain walls discussed above localize massless gauge fields by this mechanism. Due to this, there are automatically open strings (QCD flux tubes) ending on such domain walls.

Following [17], let us consider a toy model with SU(2) \( \otimes Z_2 \) symmetry and two Higgs fields \( \Phi \) and \( \chi \). \( \chi \) is a real SU(2)-singlet field, which changes sign under the \( Z_2 \) symmetry. \( \Phi \) is a \( Z_2 \)-even SU(2)-triplet field. The Lagrangian of the system is

\[
\mathcal{L} = -\frac{1}{4g^2} G_{\mu \nu}^a G^{a \mu \nu} + \frac{1}{2} \left( D_\mu \Phi^a \right)^2 - \frac{1}{2} \lambda' \left( \chi^2 + \kappa^2 - v^2 + \Phi^2 \right)^2 + \frac{1}{2} \left( \partial_\mu \chi \right)^2 - \lambda \left( \chi^2 - v^2 \right)^2 ,
\]

where \( G_{\mu \nu}^a \) is the gluon field strength tensor, \( v \) and \( \kappa \) are positive parameters having the dimension of mass and assumed to be much larger than the scale parameter \( \Lambda \) of the SU(2) gauge theory at hand \( ^2 \), \( \lambda \) and \( \lambda' \) are (small) dimensionless coupling constants, and \( g \) is the gauge coupling constant.

In the true vacuum of the theory, \( Z_2 \) symmetry is spontaneously broken, and the field \( \chi \) develops a vacuum expectation value,

\[
\chi = v \text{ or } -v .
\]

Correspondingly, the self-interaction potential for \( \Phi \) is stable, and the gauge SU(2) is not spontaneously broken. The theory is in the confining phase. All observable degrees of freedom are bound states of gluons and/or matter fermions (if such are included in the model), with masses of the order \( \Lambda \). The mass of the \( \chi \) quantum is

\[
m = \sqrt{2\lambda v} .
\]

The theory has a stable domain wall interpolating between the two different vacua in Eq. (10),

\[
\chi_0 = v \tanh (mz) .
\]

For definiteness the wall is placed in the \( \{x, y\} \) plane; the width of the wall in the \( z \) direction is of order \( m^{-1} \).

Although \( \Phi \) is zero in the vacuum, it can condense on the wall, thereby breaking gauge symmetry. To see that this is indeed the case, consider the behavior of \( \Phi \) in the classical wall background. Consider a linearized equation for small perturbations in \( \Phi^a = \delta_{34} \Phi_0 e^{-i\omega t} \) in the kink background (12),

\[
\left\{ -\partial_z^2 + \lambda' \left[ \kappa^2 + v^2 (\tanh^2 (mz) - 1) \right] \right\} \Phi_0 = \omega^2 \Phi_0 .
\]

\(^2\)Thus, we ignore the shift of the vacuum energy due to the gluon condensate outside the wall.
A close examination of this equation shows [17] that in a wide range of the parameter space, Φ becomes tachyonic and condenses in the core of the defect. Thus, inside the wall the $SU(2)$ gauge symmetry is spontaneously broken down to $U(1)$. Two out of three gluons acquire very large masses of order of $v$ in the vicinity of the wall. The third gluon becomes a photon.

The phase portrait of the theory emerging in this way is the following. Outside the wall the theory has a wider gauge invariance, $SU(2)$, and is in the non-Abelian confining phase. The $U(1)$ gauge invariance is maintained everywhere – inside and outside the wall. The test charges inside the wall interact through the photon exchange. The theory inside the wall is in the Abelian Coulomb phase. The photon and the test charges cannot escape in the outside space because there they become a part of the confining $SU(2)$ theory with no states lighter than $\Lambda$. The three-dimensional low-energy observer confined inside the wall sees a massless photon.

### 3.2 Flux Spreading in the Wall

Let us now study in more detail how the Coulomb phase appears in the world volume theory of the wall. As discussed above, the key point is that the non-Abelian theory is in a partially-Higgs phase inside the wall, and confinement cannot penetrate there. That is, the wall plays the role of a vacuum layer inside a dual superconductor. The non-abelian fields are repelled away from the wall by a dual Meissner effect. In order to study this dynamics more explicitly, it is useful to have a model in which the thickness of the non-confining layer could be arbitrarily changed. Let us introduce a model of this sort. We shall see that at the same time the example gives an explicit realization of an unstable brane, which supports a massless gauge field in the world-volume.

Our task is rather simple. The general mechanism of [17] tells us that whenever we create a layer of a non-confining phase in the confining vacuum, a massless $U(1)$ gauge field will be trapped in the layer. Thus all we need is to find a theory that could supports both confining and Coulomb vacua. In order to do this, let us modify the model (9) in such a way that one of the degenerate vacua is at $\Phi = 0$, while the other is at $\Phi \neq 0$. This will enable us to construct a stable wall interpolating between the confining and Coulomb phases. Next, by taking a wall-anti-wall pair we can realize an unstable lump supporting a $(2+1)$-dimensional Coulomb phase embedded in a $(3+1)$-dimensional confining space.

In order to achieve this goal, we have to modify the Higgs potential of the $SU(2)$ model of [17]. The needed potential is provided by an $N = 1$ supersymmetric theory with the superpotential

$$W = \frac{\text{Tr} \Phi^3}{3} + \text{Tr} \Phi^2 \chi - \chi \mu^2 + \frac{\chi^3}{3}.$$  \hspace{1cm} (14)

The scalar fields of the previous model are promoted to chiral superfields. Note that the $Z_2$ symmetry of the previous example now becomes an $R$-symmetry, under
which
\[ W \rightarrow -W, \ \Phi \rightarrow -\Phi, \ \chi \rightarrow -\chi. \quad (15) \]
Transformation properties under the gauge $SU(2)$ are unchanged. The vacua of the theory can be found by solving the standard $F$-flatness and $D$-flatness conditions. The latter one,
\[ [\Phi, \Phi^*] = 0, \quad (16) \]
suggests that $\Phi$ can be brought to a diagonal form $\Phi = \frac{\phi}{\sqrt{2}} \text{diag}(1, -1)$ by a gauge transformation. With this form, the only remaining vacuum conditions are
\[ W_\chi = \phi^2 - \mu^2 + \chi^2 = 0 \quad (17) \]
and
\[ W_\phi = 2\phi \chi = 0. \quad (18) \]

There are four degenerate vacuum states ($\phi = \pm \mu, \chi = 0$) and ($\chi = \pm \mu, \phi = 0$). There will be corresponding domain walls. We are interested in domain walls that interpolate between $\phi = \mu$ and $\phi = 0$ vacua. Notice that the value of the superpotential is different on different sides of such walls,
\[ W(+\infty) - W(-\infty) = \frac{2}{3} \mu^3. \quad (19) \]

On such a background, $N = 1$ supersymmetric algebra admits a central extension with the central charge given by (19)[17, 19]. This implies the existence of BPS saturated walls (see Appendix). Indeed, the BPS conditions
\[ \partial_z \chi \pm (\phi^2 - \mu^2 + \chi^2) = 0 \quad (20) \]
and
\[ \partial_z \phi \pm 2\phi \chi = 0 \quad (21) \]
can be easily solved, yielding exact solutions for the domain wall profile ($z$ is the coordinate perpendicular to the wall). Choosing the upper signs in Eqs.(20),(21), we have
\[ \phi = \frac{\mu}{2} (\tanh(z\mu) + 1) \quad (22) \]
and
\[ \chi = \frac{\mu}{2} (\tanh(z\mu) - 1) \quad (23) \]

The domain wall divides space into two regions. $z < 0$ is the region in which the full $SU(2)$ is unbroken and perturbatively all three gauge bosons ($W_\mu^\pm, A_\mu$) are massless. But below certain scale $\Lambda$, the $SU(2)$-gauge theory is strongly coupled and is in the confinement phase. The lowest-energy degrees of freedom are glueballs of $SU(2)$ of mass $\sim \Lambda$, and there are no massless excitations in the spectrum. We shall assume that $\Lambda << \mu$. In the $z > 0$ domain, $SU(2)$ is broken to $U(1)$ at the scale $\mu$, and the theory is in a partially Higgs phase. Two ($W_\mu^\pm$) out of three gauge fields gain
masses $\sim \mu$. As a result, the only degree of freedom below this scale is a massless photon $A_\mu$, and the theory is in the Abelian Coulomb phase. Ignoring confinement, we can find the low energy perturbative spectrum by solving the linearized equation for the gauge fields about such a background. For the charged gauge fields this equation has the following form:

$$\left( \partial^2 - g^2 \frac{\mu^2}{4} \left( \tanh(z\mu) + 1 \right)^2 \right) W_\mu^{\pm} = 0 \quad (24)$$

This is just a Schrodinger equation with an infinitely wide barrier. Modes with $z$-momentum lower than $\mu$ cannot propagate in the $z > 0$ domain and have exponentially suppressed wavefunctions,

$$\Psi(z) \sim \exp(-g\mu z). \quad (25)$$

This is nothing but the non-Abelian Meissner effect. On the other hand, for the photon the barrier is transparent. Thus, the test charges localized in $z > 0$ domain can effectively interact only via exchange of photons. Now let us take into account the effect of the confinement in $z < 0$ domain. Since there is a mass gap, the low energy photon cannot penetrate in this domain. However, since the effective low energy theory in $z > 0$ domain is an unbroken $U(1)$, an observer in this domain must see a massless photon. Thus, there is a massless photon, but its wave function gets exponentially suppressed in the left domain, due to dual Meissner effect. Charges in the right domain continue to be in the Coulomb phase, but the photon flux gets repelled from the left domain.

If we try to move a test charge from the right domain into the left one, the tube of the photon electric flux will be stretched between the charge and the wall. The throat of the tube will open up in the right domain where flux can freely spread out. The throat of the tube will be seen by inner observers as the charge of the same magnitude that was taken out. Thus, an observer in this domain will see charge conservation.

Now it is not hard to see what happens if an anti-wall is placed parallel to the wall at some $z = \Delta > 0$. This anti-wall takes $\phi$ back to the trivial vacuum $\phi = 0$ with the confinement and the mass gap. For $\Delta \gg \mu^{-1}$, the wall-anti-wall configuration can be approximated by the product ansatz:

$$\phi = \frac{\mu}{4} \left( \tanh((z - \Delta)\mu) + 1 \right) \left( -\tanh(z\mu) + 1 \right). \quad (26)$$

Now the photon wave function is repelled from both sides, and the photon is trapped in the layer where $SU(2)$ is in the partially-Higgs phase. This layer is nothing but an unstable non-BPS brane, which supports a massless photon in its world volume. We can integrate out all the heavy degrees of freedom and write down an effective low energy theory. This theory consists of only $U(1)$ gauge field, with the gauge-kinetic function of the following form

$$\psi(z) F_{\alpha\beta} F^{\alpha\beta} + \tilde{\psi}(z) F_{\alpha z} F^{\alpha z} + \text{high - derivative operators scaled by} \Lambda, \quad (27)$$

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where $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha$, the indices $\alpha, \beta = 0, 1, 2$ correspond to the world-volume coordinates, and $\psi(z)$ and $\tilde{\psi}(z)$ are localized functions of width $\Delta$. Note that they are in general different due to the spontaneous breaking of four-dimensional translational invariance by the wall. Since the localization is due to the bulk confinement effects, the exact form of the functions $\psi(z), \tilde{\psi}(z)$ is unknown, but they must fall off exponentially fast,

$$\psi(|z| \gg \delta) \sim \tilde{\psi}(|z| \gg \delta) \sim e^{-k\Lambda |z|}$$

with $k \sim 1$. This fall-off is simply a consequence of the fact that the light gauge degrees of freedom cannot penetrate in the bulk, where there are no states lighter than $\Lambda$. Hence, all the correlation functions must fall off exponentially fast. The only behaviour compatible with this fact and with the unbroken $U(1)$-gauge invariance, is that shape functions $\psi, \tilde{\psi}$ must die-off, as given above. Then it is obvious that $A_\alpha$ has a $z$-independent zero-mode component that satisfies the three-dimensional massless equation

$$\nabla_{2+1}^2 a_\alpha(x) = 0.$$  

This component is normalizable due to the finiteness of the following integral

$$\int dz \psi(z).$$

Consequently, there are open strings ending on the wall; these are the electric flux tubes of the confining bulk theory. When wall-anti-wall annihilate, the remaining vacuum will have no open string excitations, but only closed strings (glueballs of the bulk $SU(2)$).

Again, this effect is in full analogy with the $D(p)$-brane picture. If this analogy is indeed deep, it may sheds some light on the $D$-brane dynamics. For instance, in the $D$-brane picture, all the open string modes must decouple in the tachyonic vacuum. Here, we see this effect explicitly. In this language, it is obvious why there are no tachyonic modes in the gauge field, but only a zero mode. When branes annihilate, the wave function of the photon simply vanishes, which signals that the description in terms of a massless photon is no longer possible. Photon becomes non-dynamical and the only relevant degrees of freedom are glueballs (closed strings) of the strongly coupled confining theory.

Moreover, once the brane becomes thinner than the penetration length, $\Delta \ll \mu^{-1}$, it can no longer support the spread-out of the flux. At this point, there are only closed (or infinitely long) strings in the theory.

### 3.3 Spread-out of Flux in the Dual Example

The spread-out of flux in the world-volume of the layer is the key ingredient for establishing the Coulomb phase there. Dynamics of this spread-out can be best
understood on a toy "dual" example, in which the roles of magnetic and electric charges are interchanged. That is, now the magnetic charges are in the Coulomb phase on the brane and in the confining phase in the bulk. Let us construct such a dual example. In fact all we have to do is to take the previous example and substitute $SU(2)$ by $U(1)$, and adjoint $\Phi$ by a pair of singlet fields $\Phi, \bar{\Phi}$, with opposite $U(1)$-charges. The superpotential now becomes:

$$W = \bar{\Phi}\Phi - \chi\mu^2 + \frac{\chi^3}{3}. \quad (31)$$

Much in the same way, this model also has a $Z_2$ $R$-symmetry

$$W \rightarrow -W, \quad \Phi, \bar{\Phi} \rightarrow -\Phi, -\bar{\Phi}, \quad \chi \rightarrow -\chi. \quad (32)$$

The four degenerate vacuum states are $(\Phi = \bar{\Phi} = \pm \mu, \chi = 0)$ and $(\chi = \pm \mu, \Phi = 0)$. The domain walls of our interest are the ones that interpolate between $\Phi = \mu$ and $\Phi = 0$ vacua. These walls are again BPS states, due to the existence of the central charge,

$$W(+\infty) - W(-\infty) = \frac{2}{3}\mu^3. \quad (33)$$

The wall solutions can be explicitly found and are similar to the ones in the previous model,

$$\Phi = \bar{\Phi} = \frac{\mu}{2}(-\tanh(z\mu) + 1) \quad (34)$$

and

$$\chi = \frac{\mu}{2}(-\tanh(z\mu) - 1). \quad (35)$$

Now, in the $z < 0$ domain $U(1)$ is in the Higgs phase and there are topologically stable magnetic flux tubes (cosmic strings). These strings can end on the wall, since the flux can freely spread out in the right domain, where $U(1)$ is unbroken. Magnetic charges are in the Coulomb phase everywhere in this domain, but electric charges are partially screened close to the boundary, due to presence of the infinite superconductor. Let us again consider an anti-wall placed parallel to the wall at some $z = \Delta$ plane. This anti-wall takes $\bar{\Phi}, \Phi$, back to the Higgs phase. For $\Delta >> \mu^{-1}$ the wall-anti-wall configuration can be approximated by the product ansatz:

$$\bar{\Phi} = \Phi = \frac{\mu}{4}(-\tanh((z - \Delta)\mu) + 1)(\tanh(z\mu) + 1) \quad (36)$$

Thus, we have created a layer of vacuum in an infinite superconductor. This layer is our non-BPS brane, and is unstable. However, we want to know whether magnetic charges in the layer are in the Coulomb phase. To answer this question, consider the following experiment. Take an infinite straight magnetic flux tube parallel to the layer. What happens if the tube enters the layer? Will the magnetic flux spread uniformly or stay in the tube? This question is simplest to answer for the cylindrical layer. That is, consider a cylindrical layer created by a cylindric wall-
anti-wall system concentrically embedded inside one another. Inside and outside
of the cylindrical layer, the theory is in the Higgs phase. Let us assume that the
cylinder is extended in the \( z \) direction. If the radius of the inner wall \( R \), as well as
the wall-anti-wall separation \( \Delta \), is large, the configuration can be approximated by
the following function

\[
\bar{\Phi} = \Phi = \frac{\mu}{4} (-\tanh(\mu(r - R)) + 1)(\tanh(\mu(r - (R + \Delta))) + 1).
\] (37)

This configuration is unstable for two reasons. First, wall and anti-wall tend to
attract and annihilate. Second, each of them individually wants to collapse under the
tension force. However, if \( R >> \Delta >> \mu^{-1} \), we can ignore the time evolution. Now,
let us see what will happen with the straight infinite string after entering the layer.
First off all, notice that there is a short range attractive force between the string
and the layer, due to the Higgs energy: inside the layer the Higgs VEV vanishes,
and thus the string will cost no additional zeros of the Higgs field, whereas outside
the very existence of strings requires extra zeros, which are costly in energy. This is
due to the same reason as the attractive force between monopole and domain wall in
the \( SU(5) \) example. Thus, the string will form a bound state with the layer. Once
inside the layer, there is no topological obstruction for the flux to spread uniformly
around the cylinder. Whether this will happen is now a dynamical question. To
answer it, it is useful to first consider the case of a global string.

**Global Flux Spread-out**

Assume for a moment that the \( U(1) \) symmetry is not gauged. Then strings in
question are global strings. In such a case, there is no gauge field to compensate
gradient energy at infinity, and the string energy per unit length diverges logarhythmically in the \( r \) direction. If the flux stays localized in the tube, then it’s energy will
effectively be equal to the one of the global string with an effective short distance
cutoff at \( r \sim \Delta \). That is [20],

\[
E_{\text{localized}} \sim \mu^2 \ln(r_{\text{max}}/\Delta),
\] (38)

where \( r_{\text{max}} \) is the large distance cutoff. On the other hand, for the uniformly spread
flux the energy is

\[
E_{\text{localized}} \sim \mu^2 \ln(r_{\text{max}}/R)
\] (39)

and is by \( \sim \mu^2 \ln(R/\Delta) \) smaller. So the flux will spread.

**Magnetic Flux Spread-out**

Let us now come back to the gauged \( U(1) \). Now the gradient energy of the phase
is compensated by the gauge field, which assumes a pure gauge form

\[
A_g = n,
\] (40)

where \( n \) is the winding number. This compensation occurs at the expense of the
magnetic energy. The reason is that the gauge field must vanish at the origin
and cannot be pure gauge everywhere. This results in a finite magnetic energy
Now let us estimate the magnetic energy of the configuration when the flux is uniformly spread around the cylinder. Since $\Phi$ vanishes at the layer, $A_\theta$ can be zero everywhere in the inner domain and be $A_\theta = n$ outside. Thus $A_\theta(r)$ changes from 0 to $n$ in a small interval $\sim \Delta$. The resulting magnetic energy of such a configuration is

$$E_{\text{magnetic}} \sim \mu^2 \int_0^\infty dr \frac{dr}{r} (\partial_r A_\theta)^2 \sim n^2 \mu^2 \ln(1 + \Delta/R),$$

which vanishes for $R \to \infty$. This indicates that the magnetic flux would prefer to spread. Thus, magnetic charges inside the layer will be in the $(2+1)$-dimensional Coulomb phase.

4 Brane Annihilation and Energy Dissipation in the Bulk

When $D(p) - \bar{D}(p)$ system annihilates, its energy can dissipate in the bulk in the form of closed strings (or lower dimensional D-branes). The same is true for solitons that support massless gauge fields in their world volume. Indeed, according to the mechanism of [17], in order for a massless gauge field to appear in the world volume of a domain wall, the electric charges must be in the confining phase in the bulk. The bulk theory then has a mass gap, and the lowest mass excitations are closed strings. These are the “glueballs” of the confining bulk theory. There is an intrinsic connection between the existence of a massless photon on the brane and closed strings in the bulk. These strings are the lightest agents that can carry away the energy of the unstable wall configurations.

It has been suggested that in $D$-brane annihilation, at least in the limit of decoupled closed strings, the energy classically stays localized, although the tachyon evolves in time. This behavior is very different from that of annihilating solitons. The difference could be attributed to the fact that solitons are made of a scalar field, which is a bulk mode. Thus, there are excitations of the scalar field (e.g., plane waves) about the bulk vacuum which can carry away the energy of the annihilating solitons, while there are no obvious analogous constituents for $D$-branes.

Below we shall discuss solitonic solutions with similar properties. Much as in the $D$-brane case, in these solutions the tachyon evolves in time, but classically the energy density stays localized. Although the solitons in question are made of a scalar field, the quanta of this field are infinitely heavy, and dissipation into scalar waves is suppressed. Nevertheless, we show that there is a universal source of instability,

\footnote{Strictly speaking in the configuration described by (37) $\Phi$ is nowhere zero, but is exponentially small. For $A_\theta$ to vanish in the inner domain, $\Phi$ must become strictly zero on some circle. This will only cost exponentially small energy, as we shall see, negligible with respect to the relatively enormous gain due to resulting spread-out of the magnetic flux.}

\footnote{We assume there are no massless “pions” in the bulk.}
which is due to the resonant production of the localized world-volume gauge field. This mechanism is largely insensitive to the detailed structure of the soliton, and we expect that a similar effect should take place for $D$-branes.

We shall now discuss this issue in more detail. In some models, a wall-anti-wall pair decays predominantly into Higgs particles, even though they are much more massive than glueballs. Annihilation of kinks and anti-kinks in the $(1 + 1)$-dimensional $\phi^4$ theory has been extensively studied (see, e.g., [21] and references therein) and reveals surprisingly rich physics. Depending on the impact velocity, the kink and anti-kink are either reflected from one another, or they form a relatively long-lived bound state. In the latter case, the bound pair oscillates and gradually dissolves by radiating $\phi$-waves. In the models of interest to us, $\phi$ is coupled to gauge fields, and this oscillating $\phi$ background will produce pairs of gauge quanta. The energy of the quanta will be determined by the frequency of the oscillation, which is typically comparable to the scalar field mass $m$. If $m$ is large, the initial energy of the gauge quanta can be much greater than the confinement scale $\Lambda$, but the quanta will gradually degenerate into quubeballs. Thus, the energy of the annihilating walls is carried away by $\phi$-particles and by closed string excitations. The fraction of energy dissipated into each channel is model-dependent.

Here, we shall be interested in models where the classical soliton decay into scalar waves is suppressed and can be ignored in the first approximation. To this end, we can consider the extreme case, in which the Higgs field excitations in the bulk are infinitely massive, although the domain walls and strings have a finite tension. An explicit model of this type can be easily constructed. We take $N = 1$ supersymmetric $SU(2)$ gauge theory with one chiral superfield $\Phi$ in the adjoint representation. We choose the superpotential in the form

$$ W = \frac{1}{4} \text{Tr} \Phi^2 (\ln(\text{Tr} \Phi^2) - 1) \quad (42) $$

The above equation is written in units of some fundamental mass scale. This theory has two degenerate vacuum states: (1) an $SU(2)$-invariant vacuum $\Phi = 0$, and (2) a vacuum with $\Phi = \frac{1}{\sqrt{2}} (1, -1)$, where $SU(2)$ is broken to $U(1)$. (As usual, all non-diagonal components can be set to zero due to $D$-flatness conditions). Let us investigate both of these vacuum states. In the $SU(2)$-invariant vacuum, the mass of $\Phi$-quanta is infinite. So the low energy theory is a pure supersymmetric $SU(2)$-gluodynamics. It is well known [24] that this theory exhibits confinement and a mass gap at a scale $\Lambda << 1$. There is an anomalous $U(1)_R$ chiral global symmetry, which is broken down to $Z_2$ by instantons. Perturbatively, the value of the superpotential is zero in this vacuum. Non-perturbatively, there is a low-energy superpotential generated though the gaugino condensation [22],

$$ W_{\text{dynamical}} = \langle \tilde{\lambda} \lambda \rangle \sim \Lambda^3. \quad (43) $$

This condensate spontaneously breaks the anomaly-free $Z_2$. The low energy degrees of freedom in this vacuum are glueballs (or closed strings) of mass $\sim \Lambda$.  

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In the $\Phi = \frac{1}{\sqrt{2}}(1, -1)$ vacuum, $\Phi$-quanta and the two out of three $SU(2)$ gauge superfields have masses $\sim 1$. The only massless degree of freedom is the photon and the theory is in the Abelian Coulomb phase.

Due to the vacuum degeneracy, there are domain walls that interpolate between $SU(2)$-confining and $U(1)$-Coulomb vacua\(^5\). These walls satisfy the BPS equation

$$\partial_z \phi + \frac{\phi}{2} \ln \phi^2 = 0,$$

where

$$\Phi = \frac{\phi}{\sqrt{2}}(1, -1).$$

This can be explicitly integrated and gives

$$\phi_w(z) = \exp(-e^{-z}).$$

The tension of the wall is given by the difference of the superpotential VEVs on the two sides of the wall and is perturbatively equal to

$$T = W(-\infty) - W(+\infty) = \frac{1}{4}.$$ 

Non-perturbative effects due to gaugino condensation in the confining vacuum introduce negligible corrections $\sim \Lambda^3 << 1/4$. Since the wall is the boundary between confining and Coulomb phases, there are flux tubes that end on it. We are interested in an unstable layer of Coulomb vacuum bounded by two confining regions. As discussed above, there is a massless photon localized within the world volume theory of the layer. Such a layer is formed by a wall-anti-wall system and is unstable, since wall and anti-wall tend to annihilate. The energy stored in the walls will be released in the form of closed string excitations.

The model (42) was constructed so that in the $\Phi = 0$ vacuum the Higgs field has an infinite mass, and thus has no harmonic plane wave excitations in the bulk. Nevertheless, there are other types of waves. For $\Phi$ of the form (45), the field equation is

$$\partial^2 \phi + V'(\phi) = 0,$$

where

$$V(\phi) = |W'(\phi)|^2 = \frac{1}{4} \phi^2 \ln^2(\phi^2).$$

For homogeneous, small-amplitude oscillations, this has an approximate solution

$$\phi(t) \approx A \sin[m(A)t]$$

\(^5\)Note that in addition there are BPS domain walls interpolating between the two degenerate confining vacua, due to spontaneous breaking of $Z_2$ [17, 26]. However, we shall only be interested in the walls that interpolate between confining and Coulomb phases.
with

\[ m(A) = \sqrt{2}|\ln A|. \quad (51) \]

This is accurate as long as \((\ln A)^2 \gg 1\).

Travelling wave solutions can be obtained from (50) by applying a Lorentz boost. They have the form of the usual plane waves for a massive particle, the only unusual feature being that the particle’s mass depends on the amplitude \(A\) of the wave. The effect of this feature on wave propagation is not difficult to figure out. For annihilating defects of codimension \((n + 1)\), the amplitude of massless radiation decreases with the distance as \(r^{-n/2}\), and for usual massive particles there is an additional decrease due to the spreading of the wave packets (this is the only effect in the case of codimension-1 defects). In our model, as the amplitude of the waves decreases, the effective Higgs mass (51) will grow, and the speed of the waves will decrease. Asymptotically, the propagation speed approaches zero, but in our model this slow-down process is very slow, since the mass dependence on \(A\) is only logarithmic. As in the \(\phi^4\) model, closed string states will also be produced, but again, the relative efficiency of the two channels is hard to assess.

If one wants to suppress the scalar radiation altogether, one has to construct a model admitting non-dissipative “breather” solutions. One example is the sine-Gordon model, where the breather describes an oscillating kink-anti-kink pair. Another interesting example is the (non-supersymmetric) model with the potential

\[ V(\Phi) = -\frac{1}{4}\ln(\text{Tr} \Phi^2). \quad (52) \]

Such a potential was first considered by Minahan and Zwiebach in [23] as a model for an unstable brane solution. (Note, however, that our treatment of the gauge fields is very different.) This model has an unstable (but static) lump solution [23]

\[ \phi_{\text{lump}}(z) = e^{-\frac{z^2}{4}}. \quad (53) \]

\(\phi\) vanishes outside the lump, and the mass of \(\Phi\) is infinite. Since in our case \(\Phi\) is an adjoint field, the theory is in the confining phase outside, and in the Coulomb phase inside the lump, and the lump supports a massless photon in its world-volume. The lump is unstable, however, and will decay.

Remarkably, there is a (perturbatively) exact time-dependent solution, with a localized energy density:

\[ \phi(z, t) = A(t)\phi_{\text{lump}}(z), \quad (54) \]

where \(A(t)\) satisfies the oscillatory equation

\[ \frac{d^2 A(t)}{dt^2} = \frac{1}{2}A(t)\ln(A^2(t)). \quad (55) \]

Although the lump vibrates locally, the energy density does not dissipate and stays localized in the plane of the lump.
To investigate the stability of this vibrating lump solution, we consider linearized scalar perturbation on this background. These perturbations satisfy the following equation:

\[
\left[ \nabla^2 + \left( \frac{z^2}{4} - \frac{3}{2} - \frac{\ln A(t)^2}{2} \right) \right] \delta \phi = 0 \tag{56}
\]

For homogeneous perturbations, after the separation of variables

\[
\delta \phi(t, z) = f(t) \rho(z), \tag{57}
\]

we find that \( \rho(z) \) and \( f(t) \) satisfy

\[
\left[ -d_z^2 + \left( \frac{z^2}{4} - \frac{3}{2} \right) \right] \rho = m^2 \rho \tag{58}
\]

and

\[
\left[ d_t^2 - \frac{\ln A(t)^2}{2} \right] f(t) = -m^2 f(t). \tag{59}
\]

Equation (58) has a single negative eigenvalue solution \( m^2 = -1 \), which corresponds to the time-dependence of the vibrating background itself. There is also a zero mode \( m^2 = 0 \),

\[
\delta \phi = -e^{-\frac{z^2}{4}} \frac{z}{2} f_0(t), \tag{60}
\]

where \( f_0(t) \) is given either by

\[
f_0(t) = A(t) \tag{61}
\]

or by

\[
f_0(t) = A(t) \int \frac{dt'}{A^2(t')} \tag{62}
\]

The rest of the eigenvalues are positive. This indicates that there are no other tachyonic small perturbations in \( \phi \). That is, all the modes are oscillatory about the background. Thus, the classical decay of this system into scalar waves is suppressed.

These arguments, however, do not take into account the possible parametric resonance effects. Although there are no homogeneous imaginary-frequency modes, the amplitude of some of the positive-frequency modes can grow due to parametric resonance amplification. To see that such an effects may indeed take place, we consider perturbations of finite wavelength and write

\[
\delta \phi(t, z, \vec{x}) = f(t, \vec{x}) \rho(z) \tag{63}
\]

where \( \vec{x} \) are world-volume space coordinates. Taking the Fourier transform with respect to \( \vec{x} \), we obtain the following equation for \( f(t, k) \)

\[
\left[ d_t^2 + (k^2 + m^2 - \frac{\ln A(t)^2}{2}) \right] f(t, k) = 0, \tag{64}
\]
while \( \rho(z) \) still satisfies Eq. (58). This is an equation of an oscillator with a time-dependent mass term. The time dependence is periodic through the function \( A(t) \). It is well known that such systems exhibit parametric resonance effects. That is, for some values of the parameter the amplitude grows exponentially. For a small amplitude, this happens when the average value of the mass is approximately equal to half integer times the frequency of mass-oscillation. For large amplitudes, the resonance bands become wider. Based on this analogy, the above equation is expected to exhibit parametric resonance for certain resonant values of \( k^2 + m^2 \). The situation is somewhat complicated by the fact that \( \frac{\ln A(t)}{2} \) blows up every time \( A \) passes through zero. However, \( A \) spends near zero a very small portion of the full period, and for an approximate analysis one can smooth out the singularity.

Another source of instability is due to the bulk gauge fields. Coupling to these fields is crucial for localizing the gauge field on the lump, and thus cannot be neglected. In other words, there is no limit in which we can ignore coupling to the bulk confining theory without undoing the localization of the world-volume photon. Therefore, even though production of scalar waves is suppressed in this particular example, the energy density of the vibrating lump will still dissipate in the form of glueballs (closed strings) of the confining theory. This dissipation can go in two channels. One is the direct production of \( W^\pm \) bosons that get localized masses from the lump field. When the lump vibrates, so does the effective mass of these particles, and this leads to particle creation. However, there is also a second channel, which is more efficient. As we show below, the vibrating lump causes a resonant amplification of the localized \( U(1) \) gauge field, that is, of open string states. These states later annihilate into the bulk glueballs, or survive in the form of long flux tubes, similar to cosmic strings.

Before proceeding, we remark that non-dissipative breather solutions are not known for defects with codimension greater than one, and we expect that radiation of scalar waves cannot be avoided when such defects annihilate.

5 Production of World-Volume Gauge Fields (Open Strings)

We now want to show that the tachyon condensation in the above model leads to the production of localized gauge field quanta, that is, of the open string states. This effect is generic for any unstable solitons that localize a massless gauge field via the mechanism of bulk confinement, and we expect it should also persist for \( D \)-branes. Thus, we expect that open string states are produced during the tachyon condensation in an unstable \( D \)-brane decay. We shall first review the generic mechanism of the production and later discuss an explicit model.

The effective low-energy action for the zero mode world-volume photon \( a_\alpha(x^\beta) \)
is the following:

$$\int d^3 x d z \psi(z, t) F_{\alpha\beta} F^{\alpha\beta} + \ldots \quad (65)$$

The function $\psi(z, t)$ is a localized function which is determined by the lump profile and the bulk confinement; its precise form is not important for us. What is important is that $\psi$ evolves together with the lump. In the limit when the lump vanishes, so does $\psi$. This is obvious, since for the vanishing lump, there is no dual Meissner effect that protects the localized photon from the bulk confinement. The linearized equation for the world-volume zero mode photon on such a time-dependent background is

$$\nabla_{2+1}^2 a_\alpha + \frac{\dot{\kappa}}{\kappa} \dot{a}_\alpha = 0, \quad (66)$$

where $\nabla_{2+1}^2$ act only on the $2+1$-dimensional world-volume coordinates $x^\alpha$ and

$$\kappa(t) = \int dz \psi(z, t). \quad (67)$$

Taking a Fourier transform with respect to world-volume spatial coordinates, and keeping the explicit time dependence, we get

$$\ddot{a}_\alpha(k) + k^2 a_\alpha(k) + \frac{\dot{\kappa}}{\kappa} \dot{a}_\alpha(k) = 0. \quad (68)$$

This is an equation for an oscillator with a time-dependent friction term. Note also that it coincides with the equation for a scalar field in a $(2+1)$-dimensional Robertson-Walker universe, where the role of the scale factor is played by $\kappa(t)$ and $t$ plays the role of the conformal time coordinate.

Depending on the dynamics, we can distinguish three cases: (1) $\kappa(t)$ decreases asymptotically approaching zero; (2) $\kappa(t)$ oscillates around zero; (3) $\kappa(t)$ oscillates without crossing zero. We shall consider these cases separately.

**1) Asymptotically decreasing $\kappa$**

In this case, generically there are growing perturbations due to the negative friction term. This growth indicates instability with respect to particle production. In many cases the above equation can be integrated explicitly. For instance, for an exponentially decreasing $\kappa(t)$,

$$\kappa(t) = e^{-\beta t}, \quad (69)$$

all the modes are exponentially growing as

$$a(k, t) \sim \exp \left( \frac{\beta \pm \sqrt{\beta^2 - 4k^2}}{2} t \right). \quad (70)$$

For a Gaussian time-dependence, $\kappa(t) = \exp(\beta^2 t^2)$, Eq. (68) is solved by Hermite polynomials, with $k^2 = 2n\beta^2$:

$$a(k, t) = H_n(t\beta) = (-1)^n e^{(t\beta)^2} \frac{d^n e^{-(t\beta)^2}}{d(t\beta)^n}, \quad (71)$$
which grow as \( H_n(t \beta) \sim t^n \). However, other modes grow much faster. For instance, the mode \( k = 0 \) exhibits an "explosive" growth:

\[
a_\alpha(0, t) = \int_0^t e^{(\nu \beta)^2} \, dt'.
\] (72)

In fact, for the zero mode there is an explicit generic growing solution for an arbitrary decreasing \( \kappa(t) \):

\[
a_\alpha(0, t) = \int_0^t \frac{dt'}{\kappa(t')}.
\] (73)

This solution can be viewed as generation of a uniform electric field on the brane,

\[|E| = 1/\kappa(t).\] (74)

After annihilation, this electric field can only survive in the form of straight, infinitely long strings. At sufficiently late time, all modes in the Gaussian model grow like the zero mode (72). This can be checked by verifying that (73) gives an approximate solution to Eq. (68) for \( t \gg k/\beta^2 \). The effect of these inhomogeneous modes is that the electric field will vary from place to place, and the resulting strings will be rather irregular, resembling a cosmic string network formed in a phase transition.

As our final example we consider a power-law dependence, \( \kappa(t) \sim \propto t^{-2\nu} \), with \( \nu > 0 \). In this case, Eq. (68) has a solution in terms of Bessel functions,

\[
a(k, t) = t^{\nu + \frac{1}{2}} Z_{\nu + \frac{1}{2}}(kt),
\] (75)

which grows like \( t^\nu \) at late times.

(2) Oscillating \( \kappa(t) \) and parametric resonance effects

Such behavior would be expected in situations where the tachyonic vacuum is at a finite distance from the origin. If \( \kappa \) goes through zero during this oscillation, the friction term becomes infinite and our approximation breaks down. Again, as in the case of monotonically decreasing \( \kappa \), there are growing mode solutions at the beginning of the evolution. For instance, the growing solution for the zero mode (73) is still valid, until \( \kappa \) gets very small. In the vicinity of \( \kappa = 0 \), the time dependence should be well approximated by a linear function. Choosing the origin of \( t \) at the moment when \( \kappa \) crosses zero, we then have

\[
\kappa(t) \approx Ct,
\] (76)

and the solution of (68) is

\[
a(k, t) = Z_0(kt).
\] (77)
Like the zero mode, these functions generally diverge at $t = 0$. At some point the energy in the gauge field becomes comparable to that of the soliton itself, and backreaction has to be taken into account. Even before this happens, the time evolution of $\kappa(t)$ near $t = 0$ may become too fast for the effective low-energy theory to apply.

In some models, like the one discussed in the next section, $\kappa$ reaches zero, but instead of crossing over to negative values, it starts growing again. In such models, Eqs. (76), (77) are replaced by

$$\kappa(t) \approx Ct^2,$$

$$a(k,t) = t^{-1/2}Z_{-1/2}(kt),$$

while the qualitative picture remains the same.

In the case when $\kappa$ oscillates without ever crossing zero, the electric field (74) oscillates as well. However, for inhomogeneous modes with $k \neq 0$ particle production can occur due to the parametric resonance effect. This can be seen in the following way. By making a substitution

$$a(k,t) = \frac{u(k,t)}{\sqrt{\kappa(t)}}$$

we can bring equation (68) to the following form:

$$\ddot{u}(k,t) + \left[ k^2 - \frac{1}{2} \left( \frac{\dot{\kappa}}{\kappa} - \frac{1}{2} \frac{k^2}{\kappa^2} \right) \right] u(k,t) = 0$$

Since $\kappa(t)$ oscillates, so does the function in the square brackets. Thus, the equation for $u(k,t)$ is an equation for a scalar field with a periodically changing mass. It is well known that such systems exhibit parametric resonance behavior, the net result of which is that occupation numbers for certain resonant values of $k^2$ grow exponentially[28]. Thus, an explosive production of particles takes place.

6 Explicit Model of Gauge Field Production

We shall now make our analysis more concrete by considering a model in which the gauge field localization on an unstable lump and its subsequent intensive production can be demonstrated explicitly, without need to consider the bulk confinement. In this model, the role of the bulk confinement is mimiced by a tree-level coupling of a $U(1)$ gauge field to the lump profile. This coupling renders the gauge field outside the brane non-dynamical and produces a localized photon. We choose the action of the model in the form

$$\int d^3x dz \left[ \frac{1}{2} (\partial_\mu \phi)^2 + \frac{\phi^2}{4} \ln(\phi^2) - \phi^2 F_{\mu\nu} F^{\mu\nu} + ... \right],$$

where $F_{\mu\nu}$ is the field strength of the $U(1)$ gauge field $a_\mu(x, z)$. The crucial point is the tree-level coupling between $\phi$ and the gauge field. It is this coupling that
forces the kinetic term of the gauge field to vanish outside the lump. Thus, from the low-energy perspective, such a coupling to the lump has the same effect as the bulk confinement. It makes the photon a non-dynamical degree of freedom in the bulk. This similarity is not surprising, since the above model can be viewed as the effective low-energy theory obtained after integrating out the confining bulk dynamics. Such an integration would in general generate other possible couplings between $\phi$ and $a$, including high-derivative interactions. We shall keep here the simplest coupling sufficient to produce a localized photon (note that a linear coupling in $\phi$ is forbidden by the discrete symmetry $\phi \to -\phi$).

We first consider the spectrum on a static lump background. The linearized equation for the gauge field takes the form

$$\phi_{\text{lump}} \partial^\nu F_{\nu \mu} - 2 \phi_{\text{lump}} F_{z \mu} = 0,$$

(83)

where primes denote $z$-derivatives. We perform the standard separation of variables by demanding

$$a_\alpha(z, x) = b^{(m)}(z) \tilde{a}^{(m)}_\alpha(x), \quad a_z(z, x) = b^{(m)}(z) a^{(m)}_z(x)$$

(84)

and introduce the new fields

$$a^{(m)}_\alpha(x) = \tilde{a}^{(m)}_\alpha(x) - \partial_\alpha a^{(m)}_z(x),$$

(85)

which satisfy the $(2 + 1)$-dimensional massive vector field equation

$$\partial^\beta F^{(m)}_{\beta \alpha} = -m^2 a^{(m)}_\alpha.$$

(86)

Here, $F^{(m)}_{\beta \alpha} = \partial_\beta a^{(m)}_\alpha - \partial_\alpha a^{(m)}_\beta$, indices $\mu, \nu$ take values from 0 to 3, corresponding to the bulk spacetime, and indices $\alpha, \beta$ take values 0, 1, 2 on the worldsheet. Due to eq. (86), all modes with $m \neq 0$ are transverse, $\partial^\alpha a^{(m)}_\alpha = 0$.

The functions $b^{(m)}(z)$ satisfy the following equation:

$$d_z^2 b^{(m)} - zd_z b^{(m)} + m^2 b^{(m)} = 0.$$  

(87)

The solutions of this equation are Hermite polynomials

$$b^{(\sqrt{n})}(z) = H_n(z/\sqrt{2})$$

(88)

with $m = \sqrt{n}$, where $n$ is an integer. These functions are othonormalized,

$$\int dz e^{-z^2/4} H_n(z/\sqrt{2}) H_m(z/\sqrt{2}) = \sqrt{2\pi} 2^n n! \delta_{nm}$$

(89)

Thus, all the modes are localized on the lump. Notice that the mass levels grow as $m \sim \sqrt{n}$, as in the string spectrum, as opposed to $m \sim n$ that one would expect if the gauge fields were localized by compactification.
Now, to study the gauge field production during the tachyon condensation, let us consider the linearized equation for the gauge modes on the time-dependent background of Eq.(54),

$$\partial^\alpha F_{\alpha\mu} - \partial^z F_{z\mu} + \frac{2}{A} \dot{A} F_{0\mu} + z F_{z\mu} = 0. \tag{90}$$

Since the $z$-dependence of the lump profile does not change in time in the first approximation, we shall look for solutions in the factorized form given by (84), with $b^{(m)}(z)$ satisfying eq.(87). For simplicity, we shall look for solutions of (90) satisfying the condition $a^{(m)}_0 = \vec{\nabla} a^{(m)} = 0$. (Note that for $m = 0$, this condition can be imposed due to the gauge freedom, without loss of generality.) Performing a Fourier-transform with respect to the world-volume space momenta, we get the following equation for $a^{(m)}(\vec{k},t)$

$$\ddot{a}^{(m)}(\vec{k},t) + (k^2 + m^2) a^{(m)}(\vec{k},t) + 2 \frac{\dot{A}}{A} \dot{a}^{(m)}(\vec{k},t) = 0. \tag{91}$$

This is nothing but equation (68), discussed in the previous section, with the substitutions $\kappa(t) \rightarrow 2 A(t)$ and $k^2 \rightarrow k^2 + m^2$. Hence, all the results of the previous section are applicable here. In particular, there is a growing solution for the $\vec{k} = m = 0$ mode,

$$\vec{a}^{(0)}(0,t) = \vec{c} \int_0^t \frac{dt'}{A(t')}^2, \tag{92}$$

where $\vec{c}$ is an arbitrary constant vector along the world volume. This amounts to a growing uniform electric field. This solution breaks down when $A(t)$ approaches zero and the electric field blows up. At that point the back reaction must be taken into account.

The behavior described above is obtained for the simplest possible coupling in the action (82). With this choice, we have neglected all possible high-derivative interactions, such as

$$- \partial_\gamma \phi \partial^\mu \phi F_{\mu
u} F^{\mu\nu}. \tag{93}$$

Inclusion of such terms would replace the function $A^2(t)$ by some combination of $A$ and its derivatives, which need not go through zero. In such a case, the equation for gauge quanta may exhibit a parametric resonance behavior, as discussed at the end of the last section. For example, consider the effect of adding the term (93) to the original interaction in (82). On the lump background, the gauge-kinetic term now takes the form

$$- \exp(z^2/2) \left[ A^2 \left( 1 - (z/2)^2 \right) + \dot{A}^2 \right] F_{\mu\nu} F^{\mu\nu}. \tag{94}$$

Now the localizing function does not have a simple factorizable form, and the analysis of $z$-dependent excitations of the gauge field (the ones with $m^2 \neq 0$) becomes more
complicated. For the zero mode, the linearized equation takes the form (81), where now \( \kappa(t) \) is given by the following equation:

\[
\kappa(t) = \sqrt{\frac{\pi}{2}} \left[ \frac{3}{4} A^2 + \dot{A}^2 \right].
\]  

(95)

This is an oscillatory function which never becomes zero. Thus, as discussed in the previous section, we expect the model to exhibit parametric resonance production of states with non-zero \( k^2 \).

7 Fate of the World-Volume States

The question is what is the fate of the world-volume particles after the lump completely annihilates? The world-volume particles cannot exist in the vacuum, since they are open string states. In the vacuum we can have either closed or infinitely long strings (electric flux tubes). Therefore, some part of the created open strings can be expected to survive in this form. As we have seen, states of different possible values of \( \vec{k} \) are produced. Hence, we expect the strings will be brownian, due to the amplification of short-wavelength modes. Not all of these will decay into closed strings. The evolution will be more like the evolution of cosmic strings [29], with the characteristic length scale growing at the speed of light.

We would like to make a rough qualitative connection with the production and evolution of cosmic string networks in ordinary cosmology. Consider a phase transition with a spontaneous breaking of certain gauge \( U(1) \) symmetry. After the transition, the Universe becomes superconducting, and the space gets populated by cosmic strings carrying the \( U(1) \) magnetic flux. We distinguish two independent sources of string production. First, any pre-existing magnetic field (that was created before the Universe became superconducting) gets trapped into the flux tubes. In addition, magnetic flux tubes are created by the Kibble mechanism, due to the winding of the Higgs phase. We shall ignore the second mechanism. Then, all the flux tubes come from the pre-existing magnetic field, and the properties of the string network will be determined by the mechanism that produced that field. The typical scale of the string network will be given by the wavelength of the primordial magnetic field that gives the highest contribution to the energy density. The above situation is analogous to our story.

The role of the phase transition with \( U(1) \) breaking is played by the brane annihilation. The difference is that the roles of magnetic and electric fields are interchanged in our picture. Instead of a superconducting phase (Higgs phase), our Universe ends up in a confining phase. As a result, we end up with electric flux tubes (or fundamental strings in the \( D \)-brane picture) instead of magnetic ones. An important feature of brane annihilation is that it provides a mechanism for efficient creation of the electric field.

In Appendix B we analyze the world-volume gauge field amplification in the model with an exponential time dependence (69). We find that the energy density
of the field grows as
\[ \rho \sim (\beta/t)^{3/2} e^{\beta t}, \] (96)
and the main contribution to the energy is given by the modes of momenta
\[ k_* \sim (\beta/t)^{1/2}. \] (97)

Because of this exponential growth, the gauge field energy becomes comparable to that of the soliton itself on a timescale \( \Delta t \sim (\text{few}) \times \beta \). At this point the back-reaction becomes important, and the field amplification terminates. The characteristic length scale of the field variation at this time is \( l_* \sim 1/k_* \sim 1/\beta \); we expect this to be the initial scale of the string network.

In this paper we have mostly discussed \((2+1)\)-dimensional branes, but the effect of gauge field amplification is quite generic, and in Appendix B we considered the case of \((d+1)\) dimensions with an arbitrary \( d \). We found that in this general case Eq. (96) is replaced by
\[ \rho \sim (\beta/t)^{d+1/2} e^{\beta t}, \] (98)
while the characteristic momentum is still given by Eq. (97).

At the beginning, the strings are located in the plane of the original branes, but later they may depart from that plane as a result of inter-string interactions. We expect the evolution of strings to be similar to that of a cosmic string network. Since the extra dimensions are presumably compact, the strings will have gravitational effects similar to the ordinary cosmic strings.

The string evolution may be modified in the presence of some additional branes, which survive brane annihilation. (Such branes must be present in realistic scenarios: there should be at least one surviving brane where the standard model particles are localized.) What happens when a string hits one of the surviving branes? This is likely to result in some particle production, but the most dramatic effect occurs if the gauge field trapped in the strings is massless on the brane. In this case, the strings can end on the brane, and a string segment passing through the brane will generally break into two, with their ends attached to the brane. Thus, the interaction of strings with surviving branes can result in chopping up of the strings into pieces ending on the branes.

Apart from producing a string network, the remaining part of the open string energy will be annihilated in closed string states, and in particular in gravitational radiation. We have not studied the back reaction of the created states on the tachyon condensation. It is certainly possible that this back reaction may resist the phase transition and delay it, as in the case of parametric resonance preheating [28].

8 \textit{SU}(5) Domain Walls as D-branes.

Let us now show that the \textit{SU}(5)-domain walls discussed in Section 2 localize massless gauge fields in their world-volume by the mechanism of ref. [17] discussed above.
The key point is that part of the non-Abelian symmetry $SU(3)_c \times SU(2)_L \times U(1)_Y$, that is unbroken away from the wall, gets spontaneously broken in the wall. As we have seen, in the simplest wall configuration, the Higgs field VEV vanishes in the core of the defect, and the $SU(5)$ symmetry is fully restored. In this solution, the only relevant component of the adjoint Higgs field $\Phi$ is the one corresponding to hypercharge direction (2), and all the other components are zero. This, however, is not necessarily the energetically most favorable configuration. In fact, other components of the adjoint Higgs field $\Phi$, that vanish in the vacuum, may get destabilized in the wall, triggering the symmetry breaking. The fact that such destabilization can indeed happen was already shown in [12], and a more detailed analysis was performed in [14]. According to these studies, walls with broken symmetries in the core have lower energy for all values of the parameters. For our purposes, it is enough to show that such a situation occurs at least for some values of the parameters.

To see that symmetry breaking inside the wall is possible, consider the Higgs potential with $\Phi$ restricted to lie along the diagonal

$$\Phi = a \lambda_3 + b \lambda_8 + c \tau_3 + v Y,$$

where $\lambda_3$ and $\lambda_8$ are matrices from the $SU(3)_c$ Cartan subalgebra, $\tau_3$ is the weak isospin and $Y$ is the hypercharge generator:

$$\lambda_3 = \frac{1}{2} \text{diag}(1, -1, 0, 0, 0),$$

$$\lambda_8 = \frac{1}{2 \sqrt{3}} \text{diag}(1, 1, -2, 0, 0),$$

$$\tau_3 = \frac{1}{2} \text{diag}(0, 0, 0, 1, -1),$$

$$Y = \frac{1}{2 \sqrt{15}} \text{diag}(2, 2, 2, -3, -3),$$

all matrices being normalized to unity. For the simplest wall, only the $Y$-component is non-zero. Let us now show that, at least for a region of the parameter space, some other component(s) among $\lambda_3, \lambda_8$ and $\tau_3$ (which vanish in the vacuum) can pick up a VEV and break the gauge symmetry inside the wall in a different fashion. This can be simply understood by examining the linearized Schrodinger equation for small excitations, $\epsilon = \epsilon_0(x) e^{-i \omega t}$ ($\epsilon$ is either the $a, b$ or $c$ component), in the wall background

$$\left[-\partial_x^2 + (-m^2 + (v(x))^2(h + \lambda r))\right] \epsilon_0 = \omega^2 \epsilon_0,$$

where the wall is taken to lie in the $x = 0$ plane, and for $\epsilon$ being in the $a, b, c$ directions we have $r = 2/5, 2/5$ and $9/10$, respectively. The function $v(x)$ is the profile of the wall, which for a planar infinite wall can be approximated by the kink solution,

$$v(x) = \frac{m}{\sqrt{\lambda}} \tanh \left( \frac{mx}{\sqrt{2}} \right).$$

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It is obvious that there is a wide region of the parameter space for which the above Schrodinger equation has bound state (negative $\omega^2$) solutions. For instance, it is enough to take
\[
(h + \lambda r) = \lambda'
\] (105)
In such a situation, the symmetry inside the wall will be less than the symmetry outside. To be concrete, we shall take an example of a stable wall found in [14], in which the Higgs VEV inside is \text{diag}(1, -1, 0, 1, -1) and thus the symmetry is $SU(2) \otimes SU(2) \otimes U(1)_{\text{color}} \otimes U(1)_{\text{weak}}$. Here, the subscripts indicate that the corresponding $U(1)$-s belong to the $SU(3)$ and $SU(2)$ subgroups of the symmetry group that is unbroken in the vacuum outside the brane. Let us concentrate on one of them, say $U(1)_{\text{color}}$. The corresponding photon gauge field is a $\lambda_8$-gluon $g^8_{\mu}$ of QCD, and becomes a part of the confining $SU(3)$-theory outside the brane. But on the brane the $SU(3)$ is in the Higgs phase and the $\lambda_8$-gluon is a part of an Abelian symmetry in the coulomb phase. Thus, the $SU(5)$ domain wall supports a massless gluon excitation in its world volume.

We can consider a wall in which there is a uniform $g^8_{\mu}$-electric field. Such a wall can anihilate with an anti-wall of opposite topological charge that carries no electric field. The result of the anihilation will be a set of infinitely long electric flux tubes localized in the plane of the original wall. In the absence of fundamental charges, the flux tubes are stable due to charge conservation. This is closely analogous to what happens in anihilation of a $D(p)$-brane system with a Born-Infeld electric field.

9 Appendix A

In this Appendix we shall briefly review the central extension of $N = 1$ globally supersymmetric algebra and its implications for the domain wall solution. We shall mostly follow [17]. In $N = 1$ SUSY, localized objects (e.g. particles or monopoles) cannot be BPS saturated, but domain walls can. The reason is that in the wall background the 4D $N = 1$ SUSY algebra admits the central extension,
\[
\{ \tilde{Q}_\alpha Q_\beta \} = 2(\gamma^\mu P_\mu)_{\alpha\beta} + 2 \left( \gamma^5 \sigma_{\mu\nu} J^{\mu\nu} \right)_{\alpha\beta},
\] (106)
where $\sigma_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu]$ and
\[
J^{\mu\nu} = \int d^3 x \epsilon^{0\mu\nu\omega} \partial_\omega W
\] (107)
is the trivially conserved topological charge. Here, $W$ is the superpotential. Due to this central extension, backgrounds with non-zero $J^{\mu\nu}$ may preserve half of the original supersymmetry (and thus be BPS-saturates states), even though the energy is non-zero. Such configurations are domain walls across which superpotential changes. Let us consider theories with a single chiral superfield $\Phi$. We shall denote
the scalar component of the superfield by the same symbol, and the fermionic member by $\psi$. We choose the basis in which all the $\gamma_\mu$ matrices are real. For static field configurations that depend on a single space coordinate $z$, the energy functional has the form

$$
\int dz \left( K_{\Phi^*\Phi} \partial_z \Phi^* \partial_z \Phi + K_{\Phi^*\Phi}^{-1} W_{\Phi^*\Phi} \right),
$$

(108)

where $K(\Phi^*\Phi)$ is the Kähler function, and subscripts denote derivatives with respect to the fields. Rewriting the above as a perfect square and integrating the rest, we get

$$
\int dz K_{\Phi^*\Phi} \left| \partial_z \Phi \pm K_{\Phi^*\Phi}^{-1} W_{\Phi}^* \right|^2 \equiv (W(+\infty) - W(-\infty)).
$$

(109)

For the BPS-saturated wall the first term vanishes,

$$
\partial_z \Phi \pm K_{\Phi^*\Phi}^{-1} W_{\Phi}^* = 0,
$$

(110)

and the tension is given by

$$
T = W(+\infty) - W(-\infty).
$$

(111)

The important result is that, due to non-renormalization of the superpotential, this tension is exact to all orders in perturbation theory. The profile of the wall can receive radiative corrections due to renormalization of Kähler, but the tension is fixed once and for all. The BPS condition (110) guarantees that half of the supersymmetric transformations annihilate the wall. As a result, the low-energy world volume theory is a $(2 + 1)$-dimensional $N = 1$ supersymmetric theory. In the case of a single chiral superfield, it consist of a real zero mode scalar and a fermion. $z$-dependence of their profiles must be identical, due to unbroken supersymmetry. This can be seen explicitly for an arbitrary $W$ without actually solving the equations. Let us for simplicity restrict the analysis to real solutions. Then the wall is given by a profile $\Phi_w(z)$, which satisfies equation (110). The profile of the scalar zero mode is simply given by $\Phi_w(z)'$, and thus by $W_{\Phi}(\Phi_w)$. The profile of the fermionic zero mode satisfies the equation

$$
(\gamma_z \partial_z + W_{\Phi}(\Phi_w))\psi(z) = 0,
$$

(112)

which is solved by

$$
\psi(z) = \epsilon \exp(\pm \int_0^z W_{\Phi}(z')dz'),
$$

(113)

where $\epsilon$ is an eigenspinor of $\gamma_z \epsilon = \mp \epsilon$. Differentiating the BPS condition (110) and integrating with respect to $\Phi$, we get

$$
\int_0^z W_{\Phi}(z')dz' = \ln \left( \frac{\Phi_w(z)'}{\Phi_w(0)'} \right).
$$

(114)

Plugging this back into (113) we obtain

$$
\psi(z) = \frac{\Phi_w(z)'}{\Phi_w(0)'}.
$$

(115)
Thus, the wave functions of (canonically normalized) fermionic and bosonic zero modes are identical, as they should be by supersymmetry.

10 Appendix B

In this Appendix we shall study the world-sheet field amplification during the unstable brane decay. For simplicity, we shall consider a massless scalar field. As we saw in Section 5, it is described by the same field equation as the gauge field; thus, the two cases are equivalent, apart from the number of components. The corresponding effective action is

\[ S = \int dt \, d^d x \, \kappa(t) \partial_\mu \varphi \partial^\mu \varphi, \]

(116)

where \( d \) is the number of spatial dimensions of the worldsheet.

We shall adopt a simple model where \( \kappa(t) = 1 \) at \( t < 0 \) and

\[ \kappa(t) = e^{-\beta t} \]

(117)
at \( t > 0 \). For \( t < 0 \), \( \varphi \) satisfies the usual free scalar field equation, and we can represent the field operator as

\[ \varphi(x) = \sum_k \left( a_k \varphi_k(t) e^{ikx} + a_k^* \varphi_k^*(t) e^{-ikx} \right), \]

(118)

with

\[ \varphi_k(t) = \frac{1}{\sqrt{2kV}} e^{-ikt}, \]

(119)

where \( V \) is the \( d \)-dimensional normalization volume, to be taken to infinity at the end.

At \( t > 0 \), the mode functions are linear combinations of the solutions (70),

\[ \varphi_k(t) = A_k \exp(\nu_k^+(t)) + B_k \exp(\nu_k^-(t)), \]

(120)

where

\[ \nu_k^{(\pm)} = \frac{1}{2} (\beta \pm \sqrt{\beta^2 - 4k^2}). \]

(121)

The coefficients \( A_k \) and \( B_k \) can be found by matching the mode functions and their derivatives at \( t = 0 \). This gives

\[ A_k = -\frac{\nu_k^- + ik}{\sqrt{2kV} (\nu_k^+ - \nu_k^-)} \]

(122)

and

\[ B_k = \frac{\nu_k^+ + ik}{\sqrt{2kV} (\nu_k^+ - \nu_k^-)}. \]

(123)
The expectation value of the energy density in the in-vacuum state is given by

$$\rho = \frac{\kappa(t)}{2} \sum_k (\dot{\varphi}_k \dot{\varphi}_k^* + k^2 \varphi_k \varphi_k^*). \quad (124)$$

The effective field theory (116) is valid only up to some scale $k \sim M$, so the summation in (124) has to be cut off at that scale. We should also subtract the zero-point energy of the field prior to the decay,

$$\rho_0 = \frac{1}{2} \sum_k k, \quad (125)$$

with the same cutoff. This is not important, however, since we are interested only in the time-dependent part of $\rho$.

It is clear from the mode function solutions (120) that the fastest growing modes are those with the lowest $k$. Hence, we expect the low-$k$ modes to give the dominant contribution at late times. For $k < \beta/2$, both $\nu_k^{(+)}$ and $\nu_k^{(-)}$ are real, and we obtain the following expression after substituting (120)-(123) into (124):

$$\rho = \frac{1}{2} e^{-\beta t} \sum_k \frac{1}{2kV(\beta^2 - 4k^2)} [(\nu_k^{(+)})^2 + k^2][(\nu_k^{(-)})^2 + k^2] \left( e^{2\nu_k^{(+)} t} + e^{2\nu_k^{(-)} t} \right). \quad (126)$$

As expected, al large $t$ this sum is dominated by small $k$. Expanding $\nu_k^{(\pm)}$ in powers of $k$, keeping only the leading terms, and replacing summation by integration, we have

$$\rho \approx e^{\beta t} \frac{d^d \! k}{(2\pi)^d} \int \! k e^{-4k^2/\beta t} \sim \left( \frac{\beta}{t} \right)^{d/2 \! + 1} e^{\beta t}. \quad (127)$$

The dominant contribution to the integral is given by the modes with

$$k \sim (\beta/t)^{1/2}. \quad (128)$$

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