Neutrinoless Double Beta Decay in Supersymmetric Seesaw model

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Abstract
Inspired by the recent HEIDELBERG-MOSCOW double beta decay experiment, we discuss the neutrinoless double beta decay in the supersymmetric seesaw model. Our numerical analysis indicates that we can naturally explain the data of the observed neutrinoless double beta decay, as well as that of the solar and atmospheric neutrino experiments with at least one Majorana-like sneutrino of middle energy scale in the model.

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1 Introduction

It is commonly considered[1] that neutrinoless double beta decay ($\beta\beta_{0\nu}$) is a very sensitive probe for new physics beyond the standard model (SM). The models, such as the Majorana mass of light neutrinos[2], right-handed weak couplings involving heavy Majorana neutrinos[3, 4], the Higgs-boson contribution to $\beta\beta_{0\nu}$ in an SU(2) $\times$ U(1) gauge model with left-handed Majorana neutrinos[5], as well as the R-parity violation supersymmetric model that was first proposed by Mohapatra[6] and later studied in detail by other groups[7, 8, 9, 10], can result in the observed neutrinoless double beta decay. Definitely, the steadily improved experimental bounds[11] on the $\beta\beta_{0\nu}$ lifetime can then be translated into more stringent limits[12] for the parameter space of these models. This would be extremely valuable information to our search for new physics beyond SM.

Recently, a positive indication of the neutrinoless double beta decay has been reported[13]. According to the announcement of the Heidelberg group, the half-life time of $\beta\beta_{0\nu}$ for nuclei $^{76}Ge$ is $(0.8 - 18.3) \times 10^{25}$ years (with the best value of $1.5 \times 10^{25}$). Many Physicists focus their attention on the work due to its important consequences to the particle physics and astrophysics. Barger et al. [14] pointed out that accurate measurements of neutrinoless double beta decay may constrain the neutrino component of the dark matter.
Combining the bound set by the CHOOZ reactor, authors of Ref.[15] derived constraints on the neutrino mixing angles; a model-independent constraint on the neutrino mass spectrum imposed by the HEIDELBERG-MOSCOW double beta decay experiment is also given in Ref.[16]. Assuming a reasonable hierarchy pattern of neutrino masses, the magnitude of neutrinoless double beta decay is estimated in Ref.[17]; Frigerio, Smirnov and other authors [18] evaluated the effects of various neutrino-mass-matrix structures on the neutrinoless double beta decay. The authors of Ref.[19, 20, 21, 22, 23, 24] discussed neutrinoless double beta decay in different possible models. In Ref.[25], the effects of the leptonic CP violation phase on the neutrinoless double decay have been investigated. Meanwhile, discussions on the neutrinoless double decay in nuclei and other authors [18] evaluated the effects of various neutrino-mass-matrix structures on the neutrinoless double beta decay process.

By contraries, several groups suspect if the present data can definitely indicate a non-zero rate of (ββ)0ν. Aalseth et al. [27] pointed out that extraction of those signals depends on the choice of window, and some preset conditions, such as the absence of a flat background and the relative strength of the 214Bi peaks etc. The authors of Ref.[28] also criticize the claim of "evidence" which depends on the data-set choice. The most important point is that a previous analysis[29] does not find any hint towards (ββ)0ν with the same data. Even so, we are inclined to believe that the conclusion about non-zero rate of (ββ)0ν is positive and then we can extract constraint on the parameter space of "new physics" models from the obtained experimental data.

As it turns out[30], we cannot explain the solar neutrino, atmospheric neutrino together with LSND collaboration observes with three flavors of neutrinos. In the literature, the puzzle is addressed through the following approaches. First, one of the three observations is simply discarded. In the second approach a "sterile" neutrino[31] is introduced and its existence does not affect the decay width of Z boson. In the last approach there exists a mass-discrepancy between neutrino and anti-neutrino which is realized by the CPT violation terms in the neutrino sector[32]. Following a more common point (which is set without any justification), we adopt the first approach while ignoring the LSND observation due to its relatively large experimental error (more than 3 σ) here.

If we believe the announcement of the non-zero rate of (ββ)0ν and assume the light-neutrino-exchange is the dominant mechanism for the process, by considering matrix elements of Ref.[33], which include the contributions from higher order terms of the nucleon current, one finds that to fit the HEIDELBERG-MOSCOW experimental results the effective neutrino mass ⟨mν⟩ = (0.11 – 0.56)eV (95% c.l.), with the best value of 0.39eV. Though there are a few neutrino mixing schemes which can simultaneously explain the data of the solar and atmospheric neutrino experiments within the three generation model[34], the most favorable scenarios of neutrino mixing lead to ⟨mν⟩ < 0.01eV[35]. This discrepancy implies that another mechanism might be responsible for the neutrinoless double beta decay process.

In this work, we will discuss the loop-induced (ββ)0ν in the minimal supersymmetric extension of the standard model with right-handed neutrinos (MSSMRN). Although the seesaw mechanism may lead to the non-zero Majorana masses for three light neutrinos[36], the contributions from the virtual Majorana neutrino-mediated diagrams are suppressed by a small factor mν/Fp\text{FF}[37], where ⟨mν⟩ is the "effective Majorana mass parameter": |

\begin{align*}
\langle m_\nu \rangle &= |U_{e1}^2m_{\nu_1} + U_{e2}^2m_{\nu_2} + U_{e3}^2m_{\nu_3}| \quad \text{and} \quad m_{\nu_j} \quad \text{denotes the mass of the Majorana neutrino } \nu_j \quad \text{and} \quad U_{ej} \quad \text{the element of the neutrino mixing matrix.} \\
p_F &\approx 100\text{MeV is the nucleon Fermi momentum. If assuming the constraint on } \langle m_\nu \rangle \quad \text{as } \langle m_\nu \rangle \leq 0.01\text{eV (95% C.L.)}[38], \quad \text{we find } \frac{\langle m_\nu \rangle}{p_F} \leq 10^{-10}. \quad \text{On the other hand, the loop diagram contributions may play an important role in the neutrinoless double decay if we have a relatively light Majorana-like sneutrino. A point should be noted that a similar computation has been performed by Hirsch et al.[39] in the supersymmetric model with non-universal soft breaking terms and mass terms of the Majorana-type neutrino. The model we adopted here is a concrete realization of the general case discussed in Ref.[39].}
\end{align*}

The paper is organized as follows. In section 2, we review the minimal supersymmetric extension of the standard model with right-handed neutrinos and give the notations adopted in our analysis. In Section 3 we derive the supersymmetric contributions to the ΔL = 2 effective lagrangian at the quark level. The derivation
of \((\beta \bar{\beta})_{0.5}\) transition operators and nuclear matrix elements are given in section 4. Under some assumptions for the supersymmetric parameters, we give the numerical analysis on \((\beta \bar{\beta})_{0.5}\) decay of \(^{76}\text{Ge}\) in section 5. Our conclusions and discussions are made in section 6. Some complicate and tedious formulas are collected in the appendices.

2 The supersymmetric extension of the standard model with right-handed neutrinos (MSSMRN)

As the right-handed neutrinos are introduced into the game, the superpotential is written as\cite{40}

\[
\mathcal{W}_{RN} = \mu \epsilon_{i j} \hat{H}_i^1 \hat{H}_j^2 + h^{e}_{i, j} \epsilon_{i j} \hat{H}_i^1 \hat{L}_j^I \hat{R}_j^J + h^{\nu}_{i, j} \epsilon_{i j} \hat{H}_i^2 \hat{N}_j^I \hat{N}_j^J + \frac{1}{2} \hat{N}_i^I m^{n}_{i j} \hat{N}_j^I + h^{\nu}_{i, j} \epsilon_{i j} \hat{H}_i^2 \hat{Q}_j^I \hat{U}_j^J - (2) \nu
\]

Here, \(\hat{H}_i^1, \hat{H}_j^2\) are the Higgs superfields, \(\hat{L}_j^I, \hat{Q}_j^I\) are the superfields in doublets of the weak SU(2) group, where \(I=1, 2, 3\) are the indices of generations, the rest superfields \(\hat{U}_j^I, \hat{D}_j^I, \hat{N}_j^I\) and \(\hat{R}_j^I\) are in singlets of the weak SU(2). Indices \(i, j\) are contracted for the SU(2) group, and \(h^e, h^d, h^\nu\) are the Yukawa coupling constants. To break supersymmetry, non-universal soft breaking terms are introduced as

\[
\mathcal{L}^S_{RN} = -m^2_{h^1} H^1_i H^1_i - m^2_{h^2} H^2_i H^2_i - m^2_{l^1} L^I_i L^I_i - m^2_{r^1} R^I_i R^I_i - m^2_{n^1} N^I_i N^I_i - m^2_{q^1} Q^I_i Q^I_i
\]

\[
- m^2_{l^{12}} \tilde{L}^I_i \tilde{L}^I_i - m^2_{d^{12}} \tilde{D}^I_i \tilde{D}^I_i + (m_1 \lambda_B \lambda_B + m_2 \lambda_A \lambda_A + m_3 \lambda_G \lambda_G + h.c.) + \left[ m^2_{h^{12}} \epsilon_{i j} \hat{H}_i^1 H_j^2 \right]
\]

\[
+ A^e_{i j} \epsilon_{i j} \hat{H}_i^1 \hat{L}_j^I \hat{R}_j^J + A^\nu_{i j} \epsilon_{i j} \hat{H}_i^2 \hat{N}_j^I \hat{N}_j^J + \frac{1}{2} \bar{N}^I \tilde{B}^n_{i j} \tilde{N}^J + A^d_{i j} \epsilon_{i j} \hat{H}_i^1 \hat{Q}_j^I \hat{D}_j^J + A^u_{i j} \epsilon_{i j} \hat{H}_i^2 \hat{Q}_j^I \hat{U}_j^J + h.c.
\]

(2)

where \(m^2_{h^1}, m^2_{h^2}, m^2_{h^{12}}, m^2_{l^1}, m^2_{d^1}, m^2_{r^1}, m^2_{n^1}, m^2_{q^1}\) are parameters in unit of mass square, \(m_3, m_2, m_1, m_1\) denote the masses of \(\lambda_G^a\) \((a = 1, 2, \ldots, 8)\), \(\lambda_A^i\) \((i = 1, 2, 3)\) \(\lambda_B\), which are the \(SU(3) \times SU(2) \times U(1)\) gauginos respectively. \(B^n\) are free parameters in unit of mass square. \(A^e_{i j}, A^\nu_{i j}, A^d_{i j}, A^u_{i j}\) \((I, J = 1, 2, 3)\) are the soft breaking parameters that result in mass splitting between standard particles and their supersymmetric partners. Taking into account the soft breaking terms in Eq.(2), we can study the phenomenology within the minimal supersymmetric extension of the standard model with right-handed neutrinos(MSSMRN). The gauge symmetry \(SU(2) \times U(1)\) breaks down into \(U(1)\) through the nonzero vacua of two Higgs fields. Different from the MSSM, neutrinos of three generations obtain nonzero Majorana masses through the seesaw mechanism \cite{40}

\[
\hat{m}_\nu = \frac{2m^2_{W}}{g^2_{2}} U^\dagger \left( \begin{array}{c} h^\nu_1^I \\ h^\nu_2^I \\ h^\nu_3^I \end{array} \right) \left( m^n \right)^{-1} \left( \begin{array}{c} h^\nu_1^I \\ h^\nu_2^I \\ h^\nu_3^I \end{array} \right) U,
\]

(3)

where the right-handed neutrino mass matrix \(m^n\) is introduced in Eq.1 and the unitary matrix \(U\) is used to diagonalize the neutrino Yukawa coupling matrix \(h^\nu_{i j}\).

In general, the mass matrix \(\hat{m}_\nu\) is still not diagonal in the weak basis, so that we need another unitary matrix to diagonalize \(\hat{m}_\nu\).

\[
U_M^\dagger \hat{m}_\nu U_M = \frac{2m^2_{W}}{g^2_{2}} (UU_M)^\dagger \left( \begin{array}{c} h^\nu_1^I \\ h^\nu_2^I \\ h^\nu_3^I \end{array} \right) \left( m^n \right)^{-1} \left( \begin{array}{c} h^\nu_1^I \\ h^\nu_2^I \\ h^\nu_3^I \end{array} \right) UU_M
\]
neutrino oscillation among different flavors which is a target of current and future experiments. The mixing

Thus as far as $UU_M$ is not trivially a unity matrix $I$, the mass eigenvalues are non-degenerate, one can expect neutrino oscillation among different flavors which is a target of current and future experiments. The mixing for $\tilde{L}_I^\dagger$ and $\tilde{N}^{I*}$ results in nine scalar neutrinos. With the basis $\Phi^T = (\tilde{L}_I^\dagger, \tilde{N}^I, \tilde{N}^{I*})$, the resultant mass matrix of the scalar neutrinos is written as

$$
\begin{pmatrix}
\xi_{ij} & \rho_{ij} & \rho_{ij}^* \\
\rho_{ij} & -\frac{1}{2}m_\nu^2 C^2 + S^2 \delta_{L_{ij}} & +m_\nu^2 S_{L_{ij}} \\
\rho_{ij}^* & +\frac{1}{2}m_\nu^2 S_{L_{ij}} & -\frac{1}{2}m_\nu^2 C^2 - S^2 \delta_{L_{ij}}
\end{pmatrix}
\begin{pmatrix}
\frac{1}{2}(m_\nu^2 m_\nu^*)_{1J} + h_I^2 v_2^2 \delta_{L_{ij}} \\
\frac{1}{2}(m_\nu^2 m_\nu^*)_{2J} + h_I^2 v_2^2 \delta_{L_{ij}} \\
\frac{1}{2}(m_\nu^2 m_\nu^*)_{3J} + h_I^2 v_2^2 \delta_{L_{ij}}
\end{pmatrix},
$$

(5)

with

$$
\xi_{ij} = -\frac{1}{2}m_\nu^2 C^2 - S^2 \delta_{L_{ij}} + m_\nu^2 \delta_{L_{ij}},
$$

$$
\rho_{ij} = -\frac{1}{2\sqrt{2}}(h_I^v \mu v_1 \delta_{L_{ij}} + v_2 (h_I^v m_\nu^*)_{1J} - A^v_{1J} v_2),
$$

(6)

where $C_w = \cos \theta_w$, $S_w = \sin \theta_w$ and $\theta_w$ is the Weinberg angle; $v_1, v_2$ are the nonzero vacuum expectation values of the two Higgs sectors and $\tan \beta = \frac{v_2}{v_1}$ is the ratio of the two vacuum expectation values (hereafter we employ the symbols $C_\beta = \cos \beta$, $S_\beta = \sin \beta$). The eigenvectors $\tilde{L}_I^\dagger, \tilde{N}^I, \tilde{N}^{I*}$ of the weak interaction form nine neutrino mass eigenvalues $\tilde{\nu}^i (i = 1, 2, \ldots, 9)$ by diagonalizing the matrix $\hat{m}_\nu^2$:

$$
\tilde{\nu}^i = (Z^v_{1i})^* \tilde{L}_1^I + (Z^v_{2i})^* (3+1) \tilde{N}^I + (Z^v_{3i})^* (6+1) \tilde{N}^{I*},
$$

(7)

where the $\tilde{L}_1^I$ are the Dirac-type sfermions and the Lagrangian containing $\tilde{L}_1^I$ conserves the lepton number, whereas that containing the Majorana-like scalar $\tilde{N}^I$ violates the lepton number. As for the charged sleptons, fields $\tilde{L}_2^I$ and $\tilde{R}_I$ mix to produce six mass eigenvectors $\tilde{L}_I^- (i = 1, 2, \ldots, 6)$ of the squared-mass matrix:

$$
\begin{pmatrix}
\frac{1}{\sqrt{2}}(h_1^e \nu \delta_{L_{ij}} + A_{1J}^e v_1) \\
\frac{1}{\sqrt{2}}(h_2^e \nu \delta_{L_{ij}} + A_{2J}^e v_1) \\
\frac{1}{\sqrt{2}}(h_3^e \nu \delta_{L_{ij}} + A_{3J}^e v_1)
\end{pmatrix},
$$

$$
Z_L = diag(m_{\nu_1}^2, m_{\nu_2}^2, \ldots, m_{\nu_6}^2).
$$

(8)

For the gaugino and scalar quark sectors, we take the notations of Ref.[41]. Now let us turn to the phenomenology of the supersymmetric model with right-handed neutrinos.

## 3 $(\beta\beta)_{0\nu}$ decay in MSSMRN: the effective Lagrangian at quark level

In this section, considering the contributions of the model discussed above to $(\beta\beta)_{0\nu}$, we derive the relevant effective Lagrangian in terms of color-singlet currents.
Within the framework of MSSMRN, there are two sources of lepton number violation, the contributions from the Majorana neutrinos mediated processes and the Majorana-like sneutrinos mediated processes to the \( (\beta \beta)_{0\nu} \) [8]. At the quark level, \( (\beta \beta)_{0\nu} \) decay is induced by the transition of two \( d \)–quarks into two \( u \)–quarks plus two electrons. All possible Feynman diagrams that induce the \( (\beta \beta)_{0\nu} \) decay are shown in Fig.1. As stated in the introduction, the contributions of Fig.1(a) to \( (\beta \beta)_{0\nu} \) decay is suppressed by a factor \( \frac{m_{\nu}}{m_P} \). Besides this suppression factor, the contributions of Fig.1(b,c,d) to \( (\beta \beta)_{0\nu} \) still suffer from the typical loop integration suppression. In contrast to the SM case, the supersymmetric contributions are different. Three classes of Feynman diagrams are given in Fig.1(e,f,g). It is noted that the supersymmetric contributions only receive the loop integration suppression which indeed is unavoidable. Therefore, we can ignore the SM sector in our later calculations, but only keep the supersymmetric contributions, namely in the following section, we only consider the three classes of contributions (Fig.1(e,f,g)) to the \( (\beta \beta)_{0\nu} \) decay. The explicit Feynman diagrams of the three classes at one-loop level are drawn in Fig.2 and Fig.3. In those figures, Fig.2(a,b) belong to the class shown in Fig.1(e); Fig.2(c,d,e) and Fig.3(h,i) belong to the class shown in Fig.1(f) and Fig.2(f,g) and Fig.3(j,k,l,m) belong to the class shown in Fig.1(g).

After integrating out those heavy internal particles, the effective Lagrangian for \( dd \to uu + e^- e^- \) is written as

\[
\mathcal{L}_{\text{eff}}^{\Delta L_{\text{e=2}}} = \frac{G_F^2}{2m_P} \left\{ C_1 (\mu_W) \bar{u}_\alpha \gamma_\mu (1 - \gamma_5) d_\alpha \bar{u}_\beta \gamma_\nu (1 - \gamma_5) d_\beta \bar{e} (1 + \gamma_5) e^c + C_2 (\mu_W) \bar{u}_\alpha \gamma_\mu (1 - \gamma_5) d_\alpha \bar{u}_\beta \gamma_\nu (1 - \gamma_5) d_\beta \bar{e} (1 + \gamma_5) e^c \\
+ C_3 (\mu_W) \bar{u}_\alpha (1 + \gamma_5) d_\alpha \bar{u}_\beta \gamma_\mu (1 - \gamma_5) d_\beta \bar{e} \gamma^\nu (1 - \gamma_5) e^c + C_4 (\mu_W) \bar{u}_\alpha (1 - \gamma_5) d_\alpha \bar{u}_\beta \gamma_\mu (1 - \gamma_5) d_\beta \bar{e} \gamma^\nu (1 - \gamma_5) e^c \\
+ C_5 (\mu_W) \bar{u}_\alpha (1 + \gamma_5) d_\alpha \bar{u}_\beta \gamma_\mu (1 - \gamma_5) d_\beta \bar{e} \gamma^\nu (1 + \gamma_5) e^c \\
+ C_6 (\mu_W) \left( \bar{u}_\alpha \omega - d_\alpha \bar{u}_\beta \omega - d_\beta + \frac{1}{4} \bar{u} \sigma_{\mu \nu} \omega - d_\alpha \bar{u}_\beta \sigma^{\mu \nu} \omega - d_\beta \right) \bar{e} \omega + e^c \\
+ C_7 (\mu_W) \left( \bar{u}_\alpha \omega + d_\alpha \bar{u}_\beta \omega + d_\beta + \frac{1}{4} \bar{u} \sigma_{\mu \nu} \omega + d_\alpha \bar{u}_\beta \sigma^{\mu \nu} \omega + d_\beta \right) \bar{e} \omega - e^c \right\} \\
+ \left\{ C_8 (\mu_W) \bar{u}_\alpha \gamma_\mu (1 - \gamma_5) d_\alpha \bar{u}_\beta \gamma_\nu (1 - \gamma_5) d_\beta \bar{e} (i \sigma^{\rho \sigma}) (1 + \gamma_5) e^c + C_9 (\mu_W) \bar{u}_\alpha \gamma_\mu (1 - \gamma_5) d_\alpha \bar{u}_\beta \gamma_\nu (1 - \gamma_5) d_\beta \bar{e} (i \sigma^{\rho \sigma}) (1 - \gamma_5) e^c \\
+ C_{10} (\mu_W) \bar{u}_\alpha (i \sigma_{\rho \sigma}) \omega + d_\alpha \bar{u}_\beta \gamma_\mu \omega - d_\beta \bar{e} \gamma^\rho \omega + e^c \\
+ C_{11} (\mu_W) \bar{u}_\alpha (i \sigma_{\rho \sigma}) \omega + d_\alpha \bar{u}_\beta \gamma_\mu \omega - d_\beta \bar{e} \gamma^\rho \omega - e^c \\
+ C_{12} (\mu_W) \bar{u}_\alpha (i \sigma_{\rho \sigma}) \omega + d_\alpha \bar{u}_\beta \gamma_\mu \omega + d_\beta \bar{e} \gamma^\rho \omega - e^c \\
- \frac{1}{8} \eta_1 (\mu_W) \epsilon_{\mu \rho \sigma \delta} \bar{u}_\alpha \gamma^\mu \omega - d_\alpha \bar{u}_\beta \sigma^{\rho \sigma} \omega + d_\beta \bar{e} \gamma^\delta \omega + e^c \\
+ C_{13} (\mu_W) \epsilon_{\mu \rho \sigma \delta} \bar{u}_\alpha \gamma^\mu \omega - d_\alpha \bar{u}_\beta \sigma^{\rho \sigma} \omega - d_\beta \bar{e} \gamma^\delta \omega - e^c \\
+ C_{14} (\mu_W) \epsilon_{\mu \rho \sigma \delta} \bar{u}_\alpha \gamma^\mu \omega - d_\alpha \bar{u}_\beta \sigma^{\rho \sigma} \omega + d_\beta \bar{e} \gamma^\delta \omega - e^c \\
- \frac{1}{8} \eta_1 (\mu_W) \epsilon_{\mu \rho \sigma \delta} \bar{u}_\alpha \gamma^\mu \omega - d_\alpha \bar{u}_\beta \gamma^\nu \omega - d_\beta \bar{e} \sigma^{\rho \sigma} \omega - e^c \right\}.
\]
At the nuclear level, the amplitude for \( 4 \text{ Nuclear}\) singlet currents in Eq.(9) using the results of Ref.[42]. The relevant matrix elements of those currents are in the impulse approximation (NRIA)[2]. Now, we turn to the derivation of the nuclear matrix elements of the color singlet and \( Z \) of the relevant nuclear structure, i.e. the wavefunctions which describe the nuclei.

The next step of the calculations is to re-formulate the transition matrix amplitudes where all quark degrees of freedom are replaced by the corresponding nucleon degrees of freedom in nuclei and it is based on the principle so-called as "quark-hadron duality". To carry out these calculations, one needs concrete forms of the relevant nuclear structure, i.e. the wavefunctions which describe the nuclei.

4 Nuclear \( (\beta\beta)_{0\nu} \) decay

At the nuclear level, the amplitude for \( (\beta\beta)_{0\nu} \) is written as

\[
\langle (A, Z + 2), 2e^- | S - 1 | (A, Z) \rangle = \langle (A, Z + 2), 2e^- | T_{\text{eff}} \int d^4x L_{\text{eff}}^{A=2} \rangle | (A, Z) \rangle ,
\]

where the effective \( L_{\text{eff}}^{A=2} \) is given in Eq.(9). The nuclear structure is involved via the initial \((A, Z)\) and the final \((A, Z + 2)\) nuclear states which have the same atomic weight \( A \), but different electric charges \( Z \) and \( Z + 2 \). The standard framework for the calculation of the nuclear matrix elements is the nonrelativistic impulse approximation (NRIA)[2]. Now, we turn to the derivation of the nuclear matrix elements of the color singlet currents in Eq.(9) using the results of Ref.[42]. The relevant matrix elements of those currents are

\[
\begin{align*}
(P(p)|\bar{u}d|N(p')) = F_S^{(3)}(q^2)\bar{N}(p)\tau_+\bar{N}(p') ,
(P(p)|\bar{u}\gamma_5d|N(p')) = F_P^{(3)}(q^2)\bar{N}(p)\gamma_5\tau_+\bar{N}(p') ,
(P(p)|\bar{u}\gamma^\rho(1 - \gamma_5)d|N(p')) = \bar{N}(p)\left(F_V(q^2) - F_A(q^2)\gamma_5 + F_W(q^2)i\sigma^\rho q_\sigma + F_P(q^2)q_\rho\gamma_5\right)\tau_+\bar{N}(p') ,
(P(p)|\bar{u}\gamma^\rho(1 + \gamma_5)d|N(p')) = \bar{N}(p)\left(F_V(q^2) + F_A(q^2)\gamma_5 + F_W(q^2)i\sigma^\rho q_\sigma - F_P(q^2)q_\rho\gamma_5\right)\tau_+\bar{N}(p') ,
(P(p)|\bar{u}\sigma^\rho\gamma^\sigma(1 + \gamma_5)|N(p')) = \bar{N}(p)\left(J_{\sigma\rho}^\lambda + i\frac{2}{3}\epsilon^{\rho\lambda\delta}\lambda J_{\delta}\right)\tau_+\bar{N}(p') ,
(P(p)|\bar{u}\sigma^\rho\gamma^\sigma(1 - \gamma_5)|N(p')) = \bar{N}(p)\left(J_{\sigma\rho}^\lambda - i\frac{2}{3}\epsilon^{\rho\lambda\delta}\delta J_{\delta}\right)\tau_+\bar{N}(p') ,
\end{align*}
\]

where \( \bar{N} = \begin{pmatrix} P \\ N \end{pmatrix} \) is a nucleon isodoublet, and \( q = p - p' \). The tensor structure is defined as

\[
J^{\mu\nu} = T_1^{(3)}(q^2)\sigma^{\mu\nu} + \frac{iT_2^{(3)}(q^2)}{m_P}((\gamma^\mu q^\nu - \gamma^\nu q^\mu) + \frac{T_4^{(3)}(q^2)}{m_P^2}(\sigma^\mu q_\rho q^\nu - \sigma^\nu q_\rho q^\mu) .
\]

For all form factors, we take the dipole form\[43\]

\[
\frac{F_{V,A}(q^2)}{F_{V,A}} = \frac{F_{S,P,W}(q^2)}{F_{S,P,W}(0)} = \frac{T_{i}^{(3)}(q^2)}{T_{i}^{(3)}(0)} = (1 - \frac{q^2}{m_A^2})^{-2}
\]
with \( m_A = 0.85 \text{GeV} \) and \( f_V \approx 1, f_A \approx 1.261 \). The nuclear couplings obey the relations

\[
\frac{F_W(0)}{f_V} = -\frac{\mu_p - \mu_n}{2m_p} \approx \frac{3.7}{2m_p},
\]

\[
\frac{F_P(0)}{f_A} = \frac{2m_P}{m^2_\pi},
\]

(14)

where \( m_{P(\pi)} \) is the proton (pion) mass and \( \mu_{p(n)} \) is the proton (neutron) magnetic moment. With the nonrelativistic quark model, \( T^{(3)}_1(0) = 1.45, T^{(3)}_2 = -1.48, T^{(3)}_3 = -0.66 \) [42].

The transition operators are defined as

\[
\Omega_{F,N} = \sum_{a \neq b} \tau^+_a \tau^+_b \left( \frac{R_0}{r_{ab}} \right) F_N(x_A),
\]

\[
\Omega_{GT,N} = \sum_{a \neq b} \tau^+_a \tau^+_b \sigma_a \cdot \sigma_b \left( \frac{R_0}{r_{ab}} \right) F_N(x_A),
\]

\[
\Omega_{GT'} = \sum_{a \neq b} \tau^+_a \tau^+_b \sigma_a \cdot \sigma_b \left( \frac{R_0}{r_{ab}} \right) F_4(x_A),
\]

\[
\Omega_{T'} = \sum_{a \neq b} \tau^+_a \tau^+_b \left\{ 3(\sigma_a \cdot \hat{r}_{ab})(\sigma_b \cdot \hat{r}_{ab}) - \sigma_a \cdot \sigma_b \right\} \left( \frac{R_0}{r_{ab}} \right) F_5(x_A),
\]

\[
\Omega_{V,N} = \sum_{a \neq b} \tau^+_a \tau^+_b \sigma_b \left( \frac{R_0}{r_{ab}} \right) F_N(x_A),
\]

\[
\Omega_{V,1} = \sum_{a \neq b} \tau^+_a \tau^+_b \sigma_b \times \hat{r}_{ab} \left( \frac{R_0}{r_{ab}} \right) F_6(x_A),
\]

\[
\Omega_{V,2} = \sum_{a \neq b} \tau^+_a \tau^+_b \sigma_a \left( \frac{R_0}{r_{ab}} \right) F_4(x_A),
\]

\[
\Omega_{V,3} = \sum_{a \neq b} \tau^+_a \tau^+_b \left\{ 3(\sigma_a \cdot \hat{r}_{ab})\hat{r}_{ab} \right\} \left( \frac{R_0}{r_{ab}} \right) F_5(x_A).
\]

(15)

Here, \( R_0 \) is the nuclear radius being introduced to make the matrix elements dimensionless and other notations are:

\[
r_{ab} = (r_a - r_b), \quad r_{ab} = |r_{ab}|, \quad \hat{r}_{ab} = \frac{r_{ab}}{r_{ab}}, \quad x_A = m_A r_{ab},
\]

(16)

where \( r_i \) is the coordinate of the \( i \)-th nucleon. The above matrix elements have been written in the closure approximation, which is well satisfied due to the large masses of the inter-loop particles. The expressions for those structure functions \( F_i \) are given [39]

\[
F_N(x) = \frac{x}{48} (3 + 3x + x^2) e^{-x},
\]

\[
F_4(x) = \frac{x}{48} (3 + 3x - x^2) e^{-x},
\]

\[
F_5(x) = \frac{x^3}{48} e^{-x},
\]
\[ F_0(x) = \frac{x}{48}(1 + x)e^{-x}. \]

With these form factors, the reaction matrix element is obtained as
\[
\mathcal{R}^{0^+\to0^+}_{(0\beta)} = \frac{G_F^2}{\sqrt{2}m_pc_{0\nu}} \left\{ C_1(\mu_w)\Omega_{V-\mu} \left[ \bar{e}(1 + \gamma_5)e^c \right] + C_2(\mu_w)\Omega_{V-\mu} \left[ \bar{e}(1 - \gamma_5)e^c \right] + C_3(\mu_w)\Omega_1^0 \left[ \bar{e}_\gamma (1 - \gamma_5)e^c \right] + C_4(\mu_w)\Omega_2^0 \left[ \bar{e}_\gamma (1 - \gamma_5)e^c \right] + C_5(\mu_w)\Omega_0 \left[ \bar{e}_\gamma (1 + \gamma_5)e^c \right] + C_6(\mu_w)\Omega_{ST} \left[ \bar{e}(1 + \gamma_5)e^c \right] \right\},
\]

where
\[
\begin{align*}
\Omega_1^0 &= \frac{m_p}{m_e} \left\{ g^{00} \left[ \frac{F_S(0)}{f_A} \right] \Omega_{F,N} - \frac{1}{12} \left( \frac{m_A}{m_p} \right)^2 \frac{F_{ST}(0)}{f_A} \left( \Omega_{GT'} - \Omega_T \right) \right\} + g^{\rho k} \left[ \frac{F_{S}(0)}{f_A} \right] \Omega_{V,N} \\
\Omega_2^0 &= \Omega_1^0 \left( F_P(0) \right) + \left( F_P(0) \right) \right\}, \\
\Omega_{V-\mu} &= \frac{m_p}{m_e} \left\{ \left( \frac{F_{S}(0)}{f_A} \right)^2 \left\{ \left( \frac{T(0)}{f_A} \right)^2 \left( \Omega_{F} \right) - \left( \frac{T(0)}{f_A} \right)^2 \left( \Omega_{GT,N} \right) \right] + \left( \frac{m_A}{m_p} \right)^2 \left( \frac{F_{ST}(0)}{f_A} \right)^2 \left( \Omega_{F} \right) - \left( \frac{m_A}{m_p} \right)^2 \left( \frac{F_{ST}(0)}{f_A} \right)^2 \left( \Omega_{GTN} \right) \right\} \\
\Omega_{ST} &= \frac{m_p}{m_e} \left\{ \left( \frac{F_{S}(0)}{f_A} \right)^2 \left( \Omega_{F} \right) - \left( \frac{T(0)}{f_A} \right)^2 \left( \Omega_{GT} \right) \right\}.
\end{align*}
\]

The \( 0^+ \to 0^+ \) decay rate \( d\Gamma_{0\beta}^{0^+\to0^+} \) for the process \( N_i(A, Z - 2) \to N_f(A, Z) + e_1 + e_2 \) is written as
\[
d\Gamma_{0\beta}^{0^+\to0^+} = 2\pi \sum_{\text{spin}} \left[ |\mathcal{R}_{0\beta}^{0^+\to0^+}|^2 \delta(\epsilon_1 + \epsilon_2 + E_f - M_i) d\Omega_{\epsilon_1} d\Omega_{\epsilon_2} \right]
\]

\[
= \frac{G_F^4 \left( f_{AM} \right)^4}{32\pi^5 (R_{0m_p})^2} \left\{ A_0^{0\beta} + \hat{p}_1 \cdot \hat{p}_2 B_0^{0\beta \beta} + \left[ \left( \hat{p}_1 \cdot \hat{p}_2 \right)^2 - \frac{1}{3} C_0^{0\beta \beta} \right] \right\}
\]

\[
p_{12} \delta(\epsilon_1 + \epsilon_2 + E_f - M_i) d\epsilon_1 d\epsilon_2 d(\hat{p}_1 \cdot \hat{p}_2)
\]

with
\[
A_0^{(\beta)_{0\nu}} = \left| \bar{g}_{-1}(\epsilon_1) \bar{g}_{-1}(\epsilon_2) \right|^2 \left( C_1(\mu_w) - C_2(\mu_w) \right) \Omega_{V-A} + \left( C_3(\mu_w) + C_5(\mu_w) \right) \Omega_1^0 + C_4(\mu_w) \Omega_2^0
\]
entries of the quark and lepton sectors. This is just the so-called 'minimal flavor violation' scenario in the part of the Lagrangian all are flavor-conserved, the flavor changing interactions are mediated by the CKM seesaw models. In order to simplify our discussion, we assume that the Yukawa interaction and soft-breaking parameters for supersymmetry breaking, all together 10+11 scalar quark masses \( m_q \) and \( m_l \) are taken from Ref.[2].

\[
\begin{align*}
&+ \left( C_6(\mu_w) - C_7(\mu_w) \right) \Omega_{ST}^2 + \frac{1}{3} \left| \bar{f}_1(\epsilon_1) \bar{f}_1(\epsilon_2) \right|^2 \left| (C_1(\mu_w) - C_2(\mu_w) \right) \Omega_{V-A} \\
&- \left( C_3(\mu_w) + C_5(\mu_w) \right) \Omega_1^0 - C_4(\mu_w) \Omega_2^0 + \left( C_6(\mu_w) - C_7(\mu_w) \right) \Omega_{ST}^2,
\end{align*}
\]

\[
B_{0}^{(3\beta)\omega} = 2\bar{f}_1(\epsilon_1) \bar{f}_1(\epsilon_2) \bar{g}_{-1}(\epsilon_1) \bar{g}_{-1}(\epsilon_2) \left\{ \left| \left( C_3(\mu_w) + C_5(\mu_w) \right) \Omega_1^0 + C_4(\mu_w) \Omega_2^0 \right|^2 \right.
\]

\[
- \left| \left( C_1(\mu_w) - C_2(\mu_w) \right) \Omega_{V-A} + \left( C_6(\mu_w) - C_7(\mu_w) \right) \Omega_{ST}^2 \right|^2 \right\} ,
\]

\[
C_{0}^{(3\beta)\omega} = \left| \bar{f}_1(\epsilon_1) \bar{f}_1(\epsilon_2) \right|^2 \left| (C_1(\mu_w) - C_2(\mu_w) \right) \Omega_{V-A} - \left( C_3(\mu_w) + C_5(\mu_w) \right) \Omega_1^0
\]

\[
- C_4(\mu_w) \Omega_2^0 + \left( C_6(\mu_w) - C_7(\mu_w) \right) \Omega_{ST}^2 \right|^2, \tag{21}
\]

where the expressions of \( \bar{f}, \bar{g} \) are taken from Ref.[2].

## 5 The input parameters and numerical analysis

In this section, we present our numerical analysis on the neutrinoless double beta decay in the supersymmetric seesaw models. In order to simplify our discussion, we assume that the Yukawa interaction and soft-breaking part of the Lagrangian all are flavor-conserved, the flavor changing interactions are mediated by the CKM entries of the quark and lepton sectors. This is just the so-called 'minimal flavor violation' scenario in the supersymmetric models. Under the assumption, there are four couplings, \( g_1, g_2, g_3 \) and \( \mu \), the right-handed neutrino masses \( m_i^\nu \), the Yukawa couplings for neutrinos \( h_1^\nu \) \((I = 1, \ldots, N_e)\) and \( 6 + 11N_e \) free independent parameters for supersymmetry breaking, all together \( 10 + 13N_e \) parameters to be fixed besides the lepton CKM matrix elements. Among them, 11 parameters for each generation appear only in the sfermion mass matrix and the mixing. Therefore, it is more convenient to choose eight physical masses and three mixing angles for the three charged sfermions in each generation as input parameters. As for the right-handed neutrino parameters, we also set \( m_1^\nu = m_2^\nu = \cdots = m_6^\nu = m_R \) and the matrix \( U_M = I \) for simplicity. The lepton sector CKM matrix simply is \( U \) in this case. For each generation, we can specify the relevant input parameters as three scalar quark masses \( m_{q_1}, m_{q_2}, m_{q_3} \) for the squark sector, two-charged-scalar lepton masses and one mixing angle \( m_{l_1}, m_{l_2}, m_{l_3} \), the right-handed \( \theta_{\tilde{e}_1}, \theta_{\tilde{e}_2}, \theta_{\tilde{e}_3} \), three sneutrino masses \( m_{\tilde{\nu}_1}, m_{\tilde{\nu}_2}, m_{\tilde{\nu}_3} \) for the slepton sector. As well, the Yukawa couplings of neutrinos \( h_1^\nu \) are also input parameters. Another scalar down-type quark mass is obtained through the relation

\[
\cos^2 \theta_{\tilde{q}_1} m_{\tilde{q}_1}^2 + \sin^2 \theta_{\tilde{q}_1} m_{\tilde{q}_2}^2 - m_a^2 = \cos^2 \theta_{\tilde{q}_2} m_{\tilde{q}_2}^2 + \sin^2 \theta_{\tilde{q}_2} m_{\tilde{q}_3}^2 - m_a^2 + m_w^2 \cos 2\beta . \tag{22}
\]

Assuming the relations \( m_{\tilde{\nu}_1} \ll m_{\tilde{\nu}_2} < m_{\tilde{\nu}_3} \) hold among the three scalar neutrino masses, the sneutrino mixing matrix is written as (accurate to order \( \mathcal{O}(\frac{m_{\tilde{\nu}_1}}{m_{\tilde{\nu}_3}}^2) \))

\[
Z_{\tilde{\nu}_I} = \begin{pmatrix}
1 & \frac{\sqrt{2} \nu_1}{m_{\tilde{\nu}_1}} & 0 \\
-\frac{\nu_1}{m_{\tilde{\nu}_1}} & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\
-\frac{\nu_1}{m_{\tilde{\nu}_2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{pmatrix}, \tag{23}
\]

9
where
\[
\rho_1 = \frac{\sqrt{(\zeta_I - m_{\nu_I}^2)(m_{\nu_I}^2 + m_{\nu_2}^2 - \zeta_I)}}{\sqrt{2}}
\tag{24}
\]
with \( \zeta_I = \cos^2 \theta_{\nu_I} m^2_{\nu_1} + \sin^2 \theta_{\nu_I} m^2_{\nu_2} - m^2_{\nu_3} - m^2_W (C^2_{\beta\beta} - S^2_{\beta\beta}) \). A point needs to be specified that \( B^0_I \) (\( I = 1, 2, \cdots, N_G \)) are negative when we derived Eq.23. For the neutrinoless double \( \beta \) decay, the lepton number is violated, thus only the Majorana component of the neutrino or scalar neutrino is responsible for the process. Therefore the amplitude must be proportional to the mixing entry. Diagonalizing the mass-square matrix, there are three eigen-values, which are \( m_{\nu_1}^2 = \xi_I - \frac{2\rho_I}{m_{\nu_2}^2}, m_{\nu_2}^2 = \frac{1}{2}\left( m_{\nu_1}^2 + 2h_{\nu I}^2 v_2^2 + m_{\nu_3}^2 + B^0_I \right), \)
\[
m_{\nu_3}^2 = \frac{1}{4}\left( m_{\nu_1}^2 + 2h_{\nu I}^2 v_2^2 + m_{\nu_3}^2 - B^0_I \right),
\]
respectively and here \( m_{\nu_1}^2 + 2h_{\nu I}^2 v_2^2 + m_{\nu_3}^2 \) is a positive constant. The corresponding eigenvector of \( m_{\nu_1}^2 \) is
\[
\hat{\nu}_1^I = \hat{L}_I^I - \frac{2\rho_I}{m_{\nu_1}^2 + 2h_{\nu I}^2 v_2^2 + m_{\nu_3}^2 + B^0_I} \hat{N}^I = \frac{2\rho_I}{m_{\nu_1}^2 + 2h_{\nu I}^2 v_2^2 + m_{\nu_3}^2 + B^0_I} \hat{N}^{I*},
\tag{25}
\]
where \( \hat{L}_I^I, \hat{N}^I \) and \( \hat{N}^{I*} \) are one Dirac component and two Majorana components in the mass eigenvector. The mixing entry is proportional to \( \frac{2\rho_I}{m_{\nu_1}^2 + 2h_{\nu I}^2 v_2^2 + m_{\nu_3}^2 + B^0_I} \). If \( B^0_I \) is negative, obviously the mixing is larger (in this case \( m_{\nu_1}^2 < m_{\nu_2}^2 < m_{\nu_3}^2 \)) and the total amplitude is proportional to \( \frac{2\rho_I}{m_{\nu_1}^2 + 2h_{\nu I}^2 v_2^2 + m_{\nu_3}^2 + B^0_I} \sim \frac{m_{\nu_1}^2}{m_{\nu_2}^2} \). Otherwise if \( B^0_I \) is positive the amplitude is still proportional to \( \frac{m_{\nu_1}^2}{m_{\nu_2}^2} \), but the relation becomes \( m_{\nu_1}^2 < m_{\nu_2}^2 < m_{\nu_3}^2 \) instead, and the mixing would be relatively small. To meet the data, the mixing cannot be too small. In the three generation neutrino case, the contribution is suppressed by the factor \( \langle m_{\nu} \rangle / p_F \sim 10^{-10} \) in addition to the electro-weak coupling factor, so that cannot be substantial to explain the double beta decay data. In this SUSY model with right-handed neutrinos, the mixing factor is \( \frac{m_{\nu_1}^2}{m_{\nu_2}^2} \), typically, \( m_{\nu_1} \) is about several TeV, so if \( m_{\nu_2} \) is of a medium energy scale, (about \( 10^7 \) GeV), its contribution can meet the observed data on the double beta decay.

Concerning the remaining relevant parameters, \( \mu, m_1, m_2, m_3 \), it is customary to use, in place of \( \mu, m_2, m_3 \), the two chargino masses \( m_{\chi^\pm}, \) gluino mass \( m_{\tilde{g}} \) and \( m_1 \). From Eq.1 and Eq.2, one can easily express the original parameters appearing in the Lagrangian in terms of the physical input parameters (the physical masses and mixing angles of sfermions). For the charged sfermion sector, the original parameters are expressed in terms of the physical input parameters and the expressions can be found in Ref.[41]. We present the explicit expressions of the original parameters of the sneutrino sector in terms of the physical input parameters in appendix A. The consequent new relevant physical parameters are: \( \tan \beta, m_1, m_2, m_{\chi^\pm}, m_{\tilde{e}_L}^1, m_{\tilde{e}_R}^1, \theta_{\tilde{g}_1}, \theta_{\tilde{g}_3}, m_{\tilde{l}_1}, m_{\tilde{l}_2}, m_{\tilde{l}_3}, h^\prime, m_R, U_{3 \times 3} \) (\( I = 1, 2, 3 \)) plus the SM input parameters.

As for the SM parameters, we take \( m_t = 1.78 \) GeV, \( m_b = 5 \) GeV, \( m_t = 174 \) GeV, \( m_Z = 91.18 \) GeV, \( m_W = 80.33 \) GeV, \( \alpha_s(m_W) = \frac{1}{125}, \alpha_s(m_W) = 0.12 \) at the weak scale. Choosing the mass of the right-handed neutrino masses is a bit tricky. In order to explain the data of the solar and atmospheric neutrino experiments, the effective neutrino Majorana mass \( m_{\nu_M} \sim 0.01 \) eV is required in the most favored neutrino mixing scenarios. When setting \( m_{\nu} \sim 0.01 \) eV, if assuming the Yukawa coupling of neutrinos \( h_{\nu} \) is approximately equal to unity, the seesaw mechanism requires the mass of the right-handed neutrino to be \( m_{\nu_R} \sim 10^{14} \) GeV, if \( h_{\nu} \approx 0.1 \), we have \( m_R \sim 10^{12} \) GeV. For we take three right-handed neutrinos with the same mass \( m_R \), we assume that the
neutrino Yukawa couplings have the relation $h_3^\nu \gg h_{1,2}^\nu$ in order to fit the data of the atmospheric neutrino and solar neutrino experiments.

In our calculations, we take the input supersymmetric physical parameters as following

$$m_{\tilde{e}_1^1} = m_{\tilde{e}_2^1} = m_{\tilde{e}_1^2} = 2\text{TeV}, \quad \theta_{\tilde{e}_1} = \theta_{\tilde{e}_2} = \frac{\pi}{2},$$

$$m_{\tilde{e}_1^3} = m_{\tilde{e}_2^3} = m_{\tilde{e}_1^2} = 2\text{TeV}, \quad m_{\tilde{e}_2^2} = 200\text{GeV}, \quad \theta_{\tilde{e}_3} = \frac{\pi}{2} \quad (I = 1, 2, 3),$$

$$m_{\nu_1^3} = m_{\nu_2^3} = 2\text{TeV}, \quad m_{\nu_1} = m_{\nu_2} = 10^8\text{GeV}, \quad m_{\nu_3} = m_{\nu_3} = 10^{14}\text{GeV},$$

$$h_1^\nu = 0, \quad h_2^\nu = \frac{h_3^\nu}{10}, \quad m_1 = m_2 = 300\text{GeV}, \quad m_{h^+} = 200\text{GeV}, \quad m_{h^0} = 500\text{GeV}.$$

For the mixing matrix of the lepton sector, we consider two possibilities, they are given as

$$U_1 = \begin{pmatrix} 0.91 & 0.35 & 0.24 \\ -0.42 & 0.72 & 0.55 \\ 0 & -0.60 & 0.80 \end{pmatrix}, \quad U_2 = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & 0.5 \\ \frac{1}{\sqrt{2}} & 0.5 & 0.5 \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

The first mixing matrix of Eq.26 corresponds to the solution of the solar neutrino anomaly based on the MSW mechanism with larger mixing angles (LMA) and the second corresponds to the vacuum oscillation solution for very low values of the squared mass difference (Just So)[37].

In order to investigate how the HEIDELBERG-MOSCOW experimental data constrain the parameter space of the supersymmetric seesaw model, we compute the lifetime of neutrinoless double beta decay in nuclei $^{76}G_e$. In Fig.4, we plot the neutrinoless double beta decay lifetime of nuclei $^{76}G_e$ versus the lightest $\tau$-sneutrino mass $m_{\nu_3}$. For the CKM matrix of the lepton sector, we take $U_M = U_1$. Fig.4(a) corresponds to $h_3^\nu = 1$, $m_2 = 10^{14}\text{GeV}$, and Fig.4(b) to $h_3^\nu = 0.1$, $m_2 = 10^{12}\text{GeV}$. The other relevant parameters are: $m_{\nu_3} = 4 \times 10^7\text{GeV}$, $\tan\beta = 20$ (solid lines) or $\tan\beta = 30$ (dashed-lines). From Fig.4, we find that the theoretical prediction on the $(\beta\beta)_{0\nu}$ decay half life time of nuclei $^{76}G_e$ can meet the experimental data as long as the lightest $\tau$ sneutrino has a mass less than 1TeV. As the lightest $\tau$ sneutrino mass increases to a certain value, the theoretically calculated values of the $(\beta\beta)_{0\nu}$ decay half life time of nuclei $^{76}G_e$ are larger than the experimental upper limit. When we set the lepton CKM matrix as $U_M = U_2$, and the other parameters to be the same as in Fig.4, we re-calculate the relation of the half-life time versus $m_{\nu_3}$ and the results are shown in Fig.5. The trend observed in Fig.5 is similar to that in Fig.4 and the similarity is understood as a property of the model.

6 Discussion and Conclusion

In this work, we analyze the neutrinoless double beta decay of nuclei $^{76}G_e$ in the supersymmetric seesaw model. With some assumptions on the model parameter space, we can naturally explain the solar and atmospheric neutrino experiments with the oscillation scenario and simultaneously the HEIDELBERG-MOSCOW experimental result of the $^{76}G_e$ neutrinoless double decay. In the numerical analysis, we assume that the masses of the three generation right handed neutrinos are degenerate, i.e. $m_1^\nu = m_2^\nu = m_3^\nu = m_R$, the assumption can simplify our calculation to a certain extent. Meanwhile, we take $h_1^\nu = 0$, $h_2^\nu = \frac{h_3^\nu}{10}$. The choice corresponds to the hierarchical class solutions of neutrino masses in the three-generation models. As for the lepton CKM matrix, we choose two typical mixing matrices. In fact, we may have some other choices on the supersymmetric seesaw model parameter space to understand the neutrino oscillation together with the HEIDELBERG-MOSCOW neutrinoless double beta decay experiment. An important point is noted that for
each special assumption, we must have at least one sneutrino of middle energy scale, otherwise the data of
the three experiments cannot be simultaneously fitted in this supersymmetric sea saw model.

Being more explicit, in the preferred three-generation neutrino model, the contribution from the virtual
Majorana-type neutrinos is suppressed by a factor $< m_\nu > / p_F$ of about $10^{-10}$ and as well as the typical loop
suppression factor at the electro-weak scale. Thus the SM with right-handed neutrinos cannot meet the data
because of the unavoidable suppression factor. In contrast, as discussed in the text, the lightest sneutrino has
a Majorana-type component which violates the lepton number and contributes to the neutrinoless double beta
decay and it does not suffer from the strong suppression factor $< m_\nu > / p_F$. Generally, the fraction of this
component is about $10^{-6}$. In addition to the typical loop suppression, the total suppression factor is about
$10^{-8}$ at the amplitude level. This value (or just the order of magnitude) can meet the data of the solar and
atmospheric neutrino experiments and the HEIDELBERG-MOSCOW $(\beta\beta)_{0\nu}$ data simultaneously. Therefore
the result implies that the observed data in the solar and atmospheric neutrino experiments can be explained
by the oscillations mechanisms of the three-generation neutrinos, whereas the neutrinoless double beta decay
is due to the contribution from the SUSY particles and the applied model is the SUSY see-saw model with
the right-handed neutrinos. On other side, thus one can expect that the $(\beta\beta)_{0\nu}$ decay data greatly constrain
the parameter space of the model, even though there still is large free room in the space which should be
restricted by further experiments.

Our conclusion is that the chosen parameter space of the supersymmetric model with right-handed neu-
trinos can naturally explain the observed solar, atmospheric neutrino experiments and the HEIDELBERG-
MOSCOW $(\beta\beta)_{0\nu}$ data. Even though the parameter space of the model still cannot be completely determined,
the progress is noticeable.

Introducing a "sterile" neutrino or a CPT-violation term in the neutrino sector, we can also accommodate
all the lepton flavor violation processes[44]. How to distinguish between the supersymmetric seesaw model and
those models is an interesting subject. Because the "sterile" neutrino participates in the weak interaction only
via the mixing with the normal neutrinos, accurate measurements and systematic analysis on the $\tau$, $\mu$ rare
decays may indicate the difference between the supersymmetry seesaw and "sterile" neutrino models. One
plausible measurement which can distinguish the supersymmetry seesaw model from the CPT-violation model
is the detection of neutrino magnetic moments in accurate experimental measurements (if possible). Because
in the SUSY seesaw model, the neutrinos are purely Majorana-type which cannot have non-zero magnetic
moments[45], by contraries, the neutrinos which reside in the CPT violation model can possess a Dirac-type
neutrino component, thus can accommodate a non-zero magnetic moment. Obviously, such measurements
would be extremely difficult, however, on the other side, almost all experiments on neutrinos are very difficult,
so we lay our hope on the future developments of physics and technology.

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A The sneutrino mixing under MIFF assumption

With the MIFF assumption, the sneutrino mass matrix reduces to (one-generation)

\[ m_{\nu^l} = \begin{pmatrix} \xi_l & \frac{1}{2} m^2_{\nu^l} + h^2 \nu^2 v_2^2 + \frac{1}{2} m^2_{\nu^l} & \frac{1}{2} B^\nu_{\nu^l} \\ \rho^l \frac{1}{2} m^2_{\nu^l} & \frac{1}{2} B^\nu_{\nu^l} \\ \rho^l \frac{1}{2} B^\nu_{\nu^l} \end{pmatrix} \]  

with

\[ \xi_l = -\frac{1}{2} m^2_w \frac{C^2_d - S^2_d}{C^2_w} + m^2_{\nu^l}, \]

\[ \rho^l = -\frac{h^\nu \mu v_1 + h^\nu \mu v_2}{\sqrt{2}} + \frac{A^\nu_{\nu^l} v_2}{\sqrt{2}}. \]  

Defining a symbol

\[ \Delta = (\frac{1}{2} m^2_{\nu^l} + h^2 \nu^2 v_2^2 + \frac{1}{2} m^2_{\nu^l} + \frac{1}{2} B^\nu_{\nu^l} - \xi_l)^2 + 8 \rho^2, \]

the three eigenvalues of sneutrino mass matrix are obtained as

\[ m^2_{\nu^1} = \frac{1}{4} \left( m^2_{\nu^l} + 2 h^2 \nu^2 v_2^2 + m^2_{\nu^l} + B^\nu_{\nu^l} + 2 \xi_l - 2 \Delta \right), \]

\[ m^2_{\nu^2} = \frac{1}{4} \left( m^2_{\nu^l} + 2 h^2 \nu^2 v_2^2 + m^2_{\nu^l} + B^\nu_{\nu^l} + 2 \xi_l + 2 \Delta \right), \]

\[ m^2_{\nu^3} = \frac{1}{4} \left( m^2_{\nu^l} + 2 h^2 \nu^2 v_2^2 + m^2_{\nu^l} - B^\nu_{\nu^l} \right). \]  

Assuming the relations \( \xi_l, \rho^l \ll m^2_{\nu^l} + 2 h^2 \nu^2 v_2^2 + m^2_{\nu^l}, \) or \( \xi_l, \rho^l \ll B^\nu_{\nu^l}, \) Eq.29 is simplified as (up to order \( \mathcal{O}\left(\frac{\rho_l \xi_l}{m^2_{\nu^l} + 2 h^2 \nu^2 v_2^2 + m^2_{\nu^l} + B^\nu_{\nu^l}}\right)\))

\[ m^2_{\nu^1} = \xi_l - \frac{2 \rho^2}{m^2_{\nu^2}}, \]

\[ m^2_{\nu^2} = \frac{1}{2} \left( m^2_{\nu^l} + 2 h^2 \nu^2 v_2^2 + m^2_{\nu^l} + B^\nu_{\nu^l} \right), \]

\[ m^2_{\nu^3} = \frac{1}{4} \left( m^2_{\nu^l} + 2 h^2 \nu^2 v_2^2 + m^2_{\nu^l} - B^\nu_{\nu^l} \right). \]  

The corresponding unitary matrix which diagonalizes the mass matrix is also simplified as Eq.23.

B The expression for \( \eta_{\ell} \)

\[ \eta_{\ell_1} (\mu_W) = \frac{m_{\nu^1}}{m^2_w} \frac{m^2_{\nu^l}}{4 \pi S^2_w C^2_w} \frac{\alpha_e}{2} \left( V_{ud} \right)^2 U_{mn} U_{np} (Z_{\nu}^{(3+m)j} + Z_{\nu}^{(6+m)j}) (Z_{\nu}^{(3+o)j} + Z_{\nu}^{(6+o)j}) Z_{\nu}^{nk} Z_{\nu}^{pi} Z_{\nu}^{1k} Z_{\nu}^{1s} \]

\[ \left( S^2_{\nu} Z_{\nu}^{1s} + C^2_{\nu} Z_{\nu}^{2s} \right)^2 \sum_{a=L_{i,j},\bar{L}_{i,j},\nu^l} \frac{x^2_a \ln x_a}{\prod \left( x_b - x_a \right)}, \]
\[ \eta_{a2} (\mu_W) = \frac{m_{\tau} m_p}{m_W} \frac{\alpha_e}{\pi C_W^2} (V_{ud}^\ast) F_{\mu_n U_{op}} (Z^{(3-m)j}_\nu + Z^{(6+m)j}_\nu) (Z^{(3-o)j}_\nu + Z^{(6-o)j}_\nu) Z_n^k Z_p^i Z_L^4 Z_L^* Z_L^* , \]

\[ \left( \frac{Z_L^4}{Z_L^*} \right)^2 \sum_{a=L, r, L, \nu} \sum_{a \neq b} \frac{x_a^2 \ln x_a}{x_a (x_b - x_a)} , \]

\[ \eta_{b1} (\mu_W) = \frac{2 m_p}{m_W^4 \pi S_W^2} U_{em} U_{en} (Z^{(3+m)j}_\nu + Z^{(6+m)j}_\nu) (Z^{(3+n)j}_\nu + Z^{(6+n)j}_\nu) \left( Z^{li}_\nu - \frac{h_{\mu}^\nu}{g_2} Z^{2i}_\nu \right) \]

\[ \left( Z^{1k} - \frac{h_{\mu}^\nu}{g_2} Z^{2k}_\nu \right) \sum_{a=L, \nu, L, \nu} \frac{x_a \ln x_a}{x_a (x_b - x_a)} \left( m_a^2 m_{\nu} \left( Z^{2j}_\nu Z^{2i}_\nu - \frac{1}{\sqrt{2}} Z^{4i}_\nu Z^{2i}_\nu \right) \right) , \]

\[ \eta_{b2} (\mu_W) = \frac{4 m_p}{m_W^4 \pi S_W^2} U_{em} U_{en} (Z^{(3+m)j}_\nu + Z^{(6+m)j}_\nu) (Z^{(3+n)j}_\nu + Z^{(6+n)j}_\nu) \left( Z^{li}_\nu - \frac{h_{\mu}^\nu}{g_2} Z^{2i}_\nu \right) \]

\[ \left( Z^{1k} - \frac{h_{\mu}^\nu}{g_2} Z^{2k}_\nu \right) \left( Z^{2j}_\nu Z^{2i}_\nu + \frac{1}{\sqrt{2}} Z^{3i}_\nu Z^{2i}_\nu \right) \left( Z^{2j}_\nu Z^{1k}_\nu - \frac{1}{\sqrt{2}} Z^{4j}_\nu Z^{2k}_\nu \right) \]

\[ \sum_{a=L, \nu, L, \nu} \frac{x_a \ln x_a}{x_a (x_b - x_a)} \left( m_a^2 m_{\nu} \right) , \]

\[ \eta_{c1} (\mu_W) = \frac{m_p}{m_W^2 \pi S_W^2} (V_{ud}^\ast) F_{\mu_n U_{no}} (Z^{(3+m)j}_\nu + Z^{(6+m)j}_\nu) (Z^{(3+n)j}_\nu + Z^{(6+n)j}_\nu) \left( Z^{1i}_\nu - \frac{h_{\mu}^\nu}{g_2} Z^{2i}_\nu \right) \]

\[ \sum_{a=L, \nu, L, \nu} \frac{x_a^2 \ln x_a}{x_a (x_b - x_a)} \left( m^{\nu}_{\nu} \right) C_W \left( Z^{2j}_\nu Z^{1i}_\nu + \frac{1}{\sqrt{2}} Z^{3j}_\nu Z^{2i}_\nu \right) \left( S_W Z^{1i}_\nu + C_W Z^{2j}_\nu \right) , \]

\[ \eta_{c2} (\mu_W) = \frac{m_p}{m_W^2 \pi S_W^2} (V_{ud}^\ast) F_{\mu_n U_{no}} (Z^{(3+m)j}_\nu + Z^{(6+m)j}_\nu) (Z^{(3+n)j}_\nu + Z^{(6+n)j}_\nu) \left( Z^{1i}_\nu - \frac{h_{\mu}^\nu}{g_2} Z^{2i}_\nu \right) \]

\[ \sum_{a=L, \nu, L, \nu} \frac{x_a^2 \ln x_a}{x_a (x_b - x_a)} \left( m^{\nu}_{\nu} \right) C_W \left( Z^{2j}_\nu Z^{1i}_\nu - \frac{1}{\sqrt{2}} Z^{3j}_\nu Z^{2i}_\nu \right) \left( S_W Z^{1i}_\nu + C_W Z^{2j}_\nu \right) , \]

\[ \eta_{d1} (\mu_W) = -\frac{m_p m_n}{m_W^2 \pi S^2 W U_{ud}^\ast} (V_{ud}^\ast) F_{\mu_n U_{no}} Z^{1i}_\nu Z^{1i}_\nu (Z^{(3+n)k}_\nu + Z^{(6+n)k}_\nu) (Z^{(3+o)k}_\nu + Z^{(6+o)k}_\nu) \left( Z^{1i}_\nu - \frac{h_{\mu}^\nu}{g_2} Z^{2i}_\nu \right) \]

\[ \left( Z^{1i}_\nu - \frac{h_{\mu}^\nu}{g_2} Z^{2i}_\nu \right) \left( Z^{1i}_\nu - \frac{h_{\mu}^\nu}{g_2} Z^{2i}_\nu \right) \left( \frac{1}{3} S_W Z^{1m}_\nu + C_W Z^{2m}_\nu \right) \left( Z^{2m}_\nu Z^{1i}_\nu - \frac{1}{\sqrt{2}} Z^{4m}_\nu Z^{2i}_\nu \right) \]

\[ \sum_{a=L, \nu, L, \nu} \frac{x_a^2 \ln x_a}{x_a (x_b - x_a)} , \]

\[ \eta_{d2} (\mu_W) = -\frac{m_p m_n}{m_W^2 \pi S^2 W U_{ud}^\ast} (V_{ud}^\ast) F_{\mu_n U_{no}} Z^{1i}_\nu Z^{1i}_\nu (Z^{(3+n)k}_\nu + Z^{(6+n)k}_\nu) (Z^{(3+o)k}_\nu + Z^{(6+o)k}_\nu) \left( Z^{1i}_\nu - \frac{h_{\mu}^\nu}{g_2} Z^{2i}_\nu \right) \]

\[ \left( Z^{1i}_\nu - \frac{h_{\mu}^\nu}{g_2} Z^{2i}_\nu \right) \left( Z^{1i}_\nu - \frac{h_{\mu}^\nu}{g_2} Z^{2i}_\nu \right) \left( \frac{1}{3} S_W Z^{1m}_\nu + C_W Z^{2m}_\nu \right) \left( Z^{2m}_\nu Z^{1i}_\nu - \frac{1}{\sqrt{2}} Z^{4m}_\nu Z^{2i}_\nu \right) \]

\[ \sum_{a=L, \nu, L, \nu} \frac{x_a^2 \ln x_a}{x_a (x_b - x_a)} , \]
\[
\eta_{el}(\mu_W) = \frac{m_{\mu} \alpha_e}{g^2} \frac{2}{3}\alpha_s \left( V_{ud} \right)^2 U_{po} U_{en} Z_{l+}^{1s} Z_{l+}^{2s} \sum_{a=\bar{U}_i, \bar{\nu}, \bar{\kappa}_l, \bar{\kappa}_i, \bar{\kappa}_0, \bar{\alpha}_n a \neq b} x_a^2 \ln x_a \frac{1}{x_b - x_a} + \left( Z_{l+}^1 - \frac{h_\mu}{g^2} Z_{l+}^{2s} \right) \left( Z_{l+}^1 - \frac{h_\nu}{g^2} Z_{l+}^{2s} \right) \sum_{a=\bar{U}_i, \bar{\nu}, \bar{\kappa}_l, \bar{\kappa}_i, \bar{\kappa}_0, \bar{\alpha}_n a \neq b} x_a^2 \ln x_a \frac{1}{x_b - x_a} \right]
\]

\[
\eta_{e2}(\mu_W) = \frac{m_{\mu} \alpha_s}{2 \pi S_{F}^2 C_{\tau}^2} \frac{4 \alpha_e}{9 \pi S_{F}^2 C_{\tau}^2} \left( V_{ud} \right)^2 U_{po} U_{en} Z_{l+}^{1s} Z_{l+}^{2s} \sum_{a=\bar{U}_i, \bar{\nu}, \bar{\kappa}_l, \bar{\kappa}_i, \bar{\kappa}_0, \bar{\alpha}_n a \neq b} x_a^2 \ln x_a \frac{1}{x_b - x_a} + \left( Z_{l+}^1 - \frac{h_\mu}{g^2} Z_{l+}^{2s} \right) \left( Z_{l+}^1 - \frac{h_\nu}{g^2} Z_{l+}^{2s} \right) \sum_{a=\bar{U}_i, \bar{\nu}, \bar{\kappa}_l, \bar{\kappa}_i, \bar{\kappa}_0, \bar{\alpha}_n a \neq b} x_a^2 \ln x_a \frac{1}{x_b - x_a} \right]
\]

\[
\eta_{f1}(\mu_W) = \frac{8 m_{\mu} \alpha_s}{9 \pi S_{F}^2 C_{\tau}^2} \left( V_{ud} \right)^2 U_{po} U_{en} Z_{l+}^{1s} Z_{l+}^{2s} \sum_{a=\bar{U}_i, \bar{\nu}, \bar{\kappa}_l, \bar{\kappa}_i, \bar{\kappa}_0, \bar{\alpha}_n a \neq b} x_a^2 \ln x_a \frac{1}{x_b - x_a} + \left( Z_{l+}^1 - \frac{h_\mu}{g^2} Z_{l+}^{2s} \right) \left( Z_{l+}^1 - \frac{h_\nu}{g^2} Z_{l+}^{2s} \right) \sum_{a=\bar{U}_i, \bar{\nu}, \bar{\kappa}_l, \bar{\kappa}_i, \bar{\kappa}_0, \bar{\alpha}_n a \neq b} x_a^2 \ln x_a \frac{1}{x_b - x_a} \right]
\]

\[
\eta_{f2}(\mu_W) = \frac{8 m_{\mu} \alpha_s}{9 \pi S_{F}^2 C_{\tau}^2} \left( V_{ud} \right)^2 U_{po} U_{en} Z_{l+}^{1s} Z_{l+}^{2s} \sum_{a=\bar{U}_i, \bar{\nu}, \bar{\kappa}_l, \bar{\kappa}_i, \bar{\kappa}_0, \bar{\alpha}_n a \neq b} x_a^2 \ln x_a \frac{1}{x_b - x_a} + \left( Z_{l+}^1 - \frac{h_\mu}{g^2} Z_{l+}^{2s} \right) \left( Z_{l+}^1 - \frac{h_\nu}{g^2} Z_{l+}^{2s} \right) \sum_{a=\bar{U}_i, \bar{\nu}, \bar{\kappa}_l, \bar{\kappa}_i, \bar{\kappa}_0, \bar{\alpha}_n a \neq b} x_a^2 \ln x_a \frac{1}{x_b - x_a} \right]
\]

\[
\eta_{g1}(\mu_W) = \frac{8 m_{\mu} \alpha_s}{9 \pi S_{F}^2 C_{\tau}^2} \left( V_{ud} \right)^2 U_{po} U_{en} Z_{l+}^{1s} Z_{l+}^{2s} \sum_{a=\bar{U}_i, \bar{\nu}, \bar{\kappa}_l, \bar{\kappa}_i, \bar{\kappa}_0, \bar{\alpha}_n a \neq b} x_a^2 \ln x_a \frac{1}{x_b - x_a} + \left( Z_{l+}^1 - \frac{h_\mu}{g^2} Z_{l+}^{2s} \right) \left( Z_{l+}^1 - \frac{h_\nu}{g^2} Z_{l+}^{2s} \right) \sum_{a=\bar{U}_i, \bar{\nu}, \bar{\kappa}_l, \bar{\kappa}_i, \bar{\kappa}_0, \bar{\alpha}_n a \neq b} x_a^2 \ln x_a \frac{1}{x_b - x_a} \right]
\]

\[
\eta_{g2}(\mu_W) = \frac{8 m_{\mu} \alpha_s}{9 \pi S_{F}^2 C_{\tau}^2} \left( V_{ud} \right)^2 U_{po} U_{en} Z_{l+}^{1s} Z_{l+}^{2s} \sum_{a=\bar{U}_i, \bar{\nu}, \bar{\kappa}_l, \bar{\kappa}_i, \bar{\kappa}_0, \bar{\alpha}_n a \neq b} x_a^2 \ln x_a \frac{1}{x_b - x_a} + \left( Z_{l+}^1 - \frac{h_\mu}{g^2} Z_{l+}^{2s} \right) \left( Z_{l+}^1 - \frac{h_\nu}{g^2} Z_{l+}^{2s} \right) \sum_{a=\bar{U}_i, \bar{\nu}, \bar{\kappa}_l, \bar{\kappa}_i, \bar{\kappa}_0, \bar{\alpha}_n a \neq b} x_a^2 \ln x_a \frac{1}{x_b - x_a} \right]
\]

\[
\eta_{h1}(\mu_W) = \frac{8 m_{\mu} \alpha_s}{9 \pi S_{F}^2 C_{\tau}^2} \left( V_{ud} \right)^2 U_{po} U_{en} Z_{l+}^{1s} Z_{l+}^{2s} \sum_{a=\bar{U}_i, \bar{\nu}, \bar{\kappa}_l, \bar{\kappa}_i, \bar{\kappa}_0, \bar{\alpha}_n a \neq b} x_a^2 \ln x_a \frac{1}{x_b - x_a} + \left( Z_{l+}^1 - \frac{h_\mu}{g^2} Z_{l+}^{2s} \right) \left( Z_{l+}^1 - \frac{h_\nu}{g^2} Z_{l+}^{2s} \right) \sum_{a=\bar{U}_i, \bar{\nu}, \bar{\kappa}_l, \bar{\kappa}_i, \bar{\kappa}_0, \bar{\alpha}_n a \neq b} x_a^2 \ln x_a \frac{1}{x_b - x_a} \right]
\]

\[
\eta_{h2}(\mu_W) = \frac{8 m_{\mu} \alpha_s}{9 \pi S_{F}^2 C_{\tau}^2} \left( V_{ud} \right)^2 U_{po} U_{en} Z_{l+}^{1s} Z_{l+}^{2s} \sum_{a=\bar{U}_i, \bar{\nu}, \bar{\kappa}_l, \bar{\kappa}_i, \bar{\kappa}_0, \bar{\alpha}_n a \neq b} x_a^2 \ln x_a \frac{1}{x_b - x_a} + \left( Z_{l+}^1 - \frac{h_\mu}{g^2} Z_{l+}^{2s} \right) \left( Z_{l+}^1 - \frac{h_\nu}{g^2} Z_{l+}^{2s} \right) \sum_{a=\bar{U}_i, \bar{\nu}, \bar{\kappa}_l, \bar{\kappa}_i, \bar{\kappa}_0, \bar{\alpha}_n a \neq b} x_a^2 \ln x_a \frac{1}{x_b - x_a} \right]
\]
\[
\eta_1(\mu_W) = \frac{m_{\mu}m_{\gamma}^0}{m_W^2} \frac{\alpha_e}{12\pi S_W^2 C_W^2} \left( V_{ud}^* \right)^2 U_{en} U_{J \bar{J}} Z_{D}^{1is} Z_{D}^{2ik} Z_{L}^{1js} Z_{L}^{2jk} \left( Z_{\nu}^{(3+1)k} + Z_{\nu}^{(6+1)k} \right) \\
\sum_{a=\tilde{D},i,\kappa,\tilde{\kappa},\tilde{\nu},m,\kappa_m \neq a} \frac{x_a^2 \ln x_a}{(x_b - x_a)},
\]

\[
\eta_1(\mu_W) = \frac{m_{\mu}m_{\gamma}^0}{m_W^2} \frac{\alpha_e}{4\pi S_W^2 C_W^2} \left( V_{ud}^* \right)^2 U_{en} U_{J \bar{J}} Z_{D}^{1is} Z_{D}^{2ik} Z_{L}^{1js} Z_{L}^{2jk} \left( Z_{\nu}^{(3+1)k} + Z_{\nu}^{(6+1)k} \right) \\
\sum_{a=\tilde{D},i,\kappa,\tilde{\kappa},\tilde{\nu},m,\kappa_m \neq a} \frac{x_a^2 \ln x_a}{(x_b - x_a)},
\]

\[
\eta_1(\mu_W) = \frac{m_{\mu}m_{\gamma}^0}{m_W^2} \frac{\alpha_e}{2\pi S_W^2 C_W} \left( V_{ud}^* \right)^2 U_{en} U_{J \bar{J}} Z_{D}^{1is} Z_{D}^{2ik} Z_{L}^{1js} Z_{L}^{2jk} \left( Z_{\nu}^{(3+1)k} + Z_{\nu}^{(6+1)k} \right) \\
\sum_{a=\tilde{D},i,\kappa,\tilde{\kappa},\tilde{\nu},m,\kappa_m \neq a} \frac{x_a^2 \ln x_a}{(x_b - x_a)},
\]

\[
\eta_1(\mu_W) = \frac{m_{\mu}m_{\gamma}^0}{m_W^2} \frac{4\alpha_s}{9\pi} \left( V_{ud}^* \right)^2 U_{en} U_{J \bar{J}} Z_{D}^{1is} Z_{D}^{2ik} Z_{L}^{1js} Z_{L}^{2jk} \left( Z_{\nu}^{(3+1)k} + Z_{\nu}^{(6+1)k} \right) \\
\sum_{a=\tilde{D},i,\kappa,\tilde{\kappa},\tilde{\nu},m,\kappa_m \neq a} \frac{x_a^2 \ln x_a}{(x_b - x_a)},
\]

\[
\eta_1(\mu_W) = \frac{m_{\mu}m_{\gamma}^0}{m_W^2} \frac{4\alpha_s}{9\pi} \left( V_{ud}^* \right)^2 U_{en} U_{J \bar{J}} Z_{D}^{1is} Z_{D}^{2ik} Z_{L}^{1js} Z_{L}^{2jk} \left( Z_{\nu}^{(3+1)k} + Z_{\nu}^{(6+1)k} \right) \\
\sum_{a=\tilde{D},i,\kappa,\tilde{\kappa},\tilde{\nu},m,\kappa_m \neq a} \frac{x_a^2 \ln x_a}{(x_b - x_a)},
\]

\[
\eta_1(\mu_W) = \frac{m_{\mu}m_{\gamma}^0}{m_W^2} \frac{\alpha_e}{8\pi S_W^2 C_W^2} \left( V_{ud}^* \right)^2 U_{en} U_{J \bar{J}} Z_{D}^{1is} Z_{D}^{2ik} Z_{L}^{1js} Z_{L}^{2jk} \left( Z_{\nu}^{(3+1)k} + Z_{\nu}^{(6+1)k} \right) \\
\sum_{a=\tilde{D},i,\kappa,\tilde{\kappa},\tilde{\nu},m,\kappa_m \neq a} \frac{x_a^2 \ln x_a}{(x_b - x_a)},
\]
\[
\begin{align*}
(Z_\nu^{(3+p)m} + Z_\nu^{(6+p)m})& (Z_1^{1n} - \frac{h_{\nu}}{g_2} Z_2^{2n}) (Z_1^{1k*} - \frac{h_{\nu}}{g_2} Z_2^{2k*}) \left(\frac{1}{3} Z_{N^1}^1 S_W + Z_{N^2}^2 S_W\right) \\
& \left(\frac{1}{3} Z_{N^1}^1 S_W - Z_{N^2}^2 S_W\right) \sum_{a = \tilde{D}_1, \tilde{d}_3, \tilde{d}_2, \tilde{U}_1, \tilde{\nu}_m, \kappa_n} \frac{x_a^2 \ln x_a}{(x_b - x_a)}, \\
\eta_2(\mu_W) &= -\frac{m_p m_{\nu}}{m_W^2 3\pi C_W^2} \left(\frac{V_{ud}^*}{9\pi}\right)^2 U_{e\nu} U_{\nu p} \sum_{a = \tilde{D}_1, \tilde{d}_3, \tilde{d}_2, \tilde{U}_1, \tilde{\nu}_m, \kappa_n} \frac{x_a^2 \ln x_a}{(x_b - x_a)}, \\
\eta_{m1}(\mu_W) &= \frac{m_p m_{\nu}}{m_W^2 4\pi} \left(\frac{V_{ud}^*}{9}\right)^2 U_{e\nu} U_{\nu p} \sum_{a = \tilde{D}_1, \tilde{d}_3, \tilde{d}_2, \tilde{U}_1, \tilde{\nu}_m, \kappa_n} \frac{x_a^2 \ln x_a}{(x_b - x_a)}, \\
\eta_{m2}(\mu_W) &= \frac{m_p m_{\nu}}{m_W^2 4\pi} \left(\frac{V_{ud}^*}{9}\right)^2 U_{e\nu} U_{\nu p} \sum_{a = \tilde{D}_1, \tilde{d}_3, \tilde{d}_2, \tilde{U}_1, \tilde{\nu}_m, \kappa_n} \frac{x_a^2 \ln x_a}{(x_b - x_a)},
\end{align*}
\]

Where \(x_i = \frac{m_i^2}{m_W^2}\). The coefficient \(C_i(\mu_W) (i = 1, 2, \cdots, 14)\) are defined as

\[
C_1(\mu_W) = \eta_1(\mu_W) + \eta_2(\mu_W) + \eta_3(\mu_W) + \eta_4(\mu_W) + \eta_5(\mu_W) + \eta_6(\mu_W) + \eta_7(\mu_W) + \eta_8(\mu_W) + \eta_9(\mu_W),
\]

\[
C_2(\mu_W) = \eta_2(\mu_W) + \eta_2(\mu_W) + \eta_3(\mu_W) + \eta_4(\mu_W) + \eta_5(\mu_W) + \eta_6(\mu_W) + \eta_7(\mu_W) + \eta_8(\mu_W) + \eta_9(\mu_W),
\]

\[
C_3(\mu_W) = \eta_1(\mu_W) - \eta_2(\mu_W) + \eta_3(\mu_W) + \eta_4(\mu_W) + \eta_5(\mu_W) + \eta_6(\mu_W) + \eta_7(\mu_W) + \eta_8(\mu_W) + \eta_9(\mu_W),
\]

\[
C_4(\mu_W) = \eta_1(\mu_W) + \eta_2(\mu_W) + \eta_3(\mu_W) + \eta_4(\mu_W) + \eta_5(\mu_W) + \eta_6(\mu_W) + \eta_7(\mu_W) + \eta_8(\mu_W) + \eta_9(\mu_W),
\]

\[
C_5(\mu_W) = -\frac{\eta_1(\mu_W)}{2} + \eta_2(\mu_W),
\]

\[
C_6(\mu_W) = \eta_1(\mu_W) + \eta_2(\mu_W),
\]

\[
C_7(\mu_W) = \eta_2(\mu_W) + \eta_3(\mu_W),
\]

\[
C_8(\mu_W) = \eta_3(\mu_W) + \eta_2(\mu_W) - \eta_1(\mu_W) + \frac{1}{4} \eta_2(\mu_W),
\]

\[
C_9(\mu_W) = -\eta_2(\mu_W) + \frac{1}{2} \eta_1(\mu_W),
\]

\[
C_{10}(\mu_W) = \frac{7}{4} \eta_1(\mu_W) - \eta_2(\mu_W),
\]

Where \(x_i = \frac{m_i^2}{m_W^2}\). The coefficient \(C_i(\mu_W) (i = 1, 2, \cdots, 14)\) are defined as

\[
C_1(\mu_W) = \eta_1(\mu_W) + \eta_2(\mu_W) + \eta_3(\mu_W) + \eta_4(\mu_W) + \eta_5(\mu_W) + \eta_6(\mu_W) + \eta_7(\mu_W) + \eta_8(\mu_W) + \eta_9(\mu_W),
\]

\[
C_2(\mu_W) = \eta_2(\mu_W) + \eta_2(\mu_W) + \eta_3(\mu_W) + \eta_4(\mu_W) + \eta_5(\mu_W) + \eta_6(\mu_W) + \eta_7(\mu_W) + \eta_8(\mu_W) + \eta_9(\mu_W),
\]

\[
C_3(\mu_W) = \eta_1(\mu_W) - \eta_2(\mu_W) + \eta_3(\mu_W) + \eta_4(\mu_W) + \eta_5(\mu_W) + \eta_6(\mu_W) + \eta_7(\mu_W) + \eta_8(\mu_W) + \eta_9(\mu_W),
\]

\[
C_4(\mu_W) = \eta_1(\mu_W) + \eta_2(\mu_W) + \eta_3(\mu_W) + \eta_4(\mu_W) + \eta_5(\mu_W) + \eta_6(\mu_W) + \eta_7(\mu_W) + \eta_8(\mu_W) + \eta_9(\mu_W),
\]

\[
C_5(\mu_W) = -\frac{\eta_1(\mu_W)}{2} + \eta_2(\mu_W),
\]

\[
C_6(\mu_W) = \eta_1(\mu_W) + \eta_2(\mu_W),
\]

\[
C_7(\mu_W) = \eta_2(\mu_W) + \eta_3(\mu_W),
\]

\[
C_8(\mu_W) = \eta_3(\mu_W) + \eta_2(\mu_W) - \eta_1(\mu_W) + \frac{1}{4} \eta_2(\mu_W),
\]

\[
C_9(\mu_W) = -\eta_2(\mu_W) + \frac{1}{2} \eta_1(\mu_W),
\]

\[
C_{10}(\mu_W) = \frac{7}{4} \eta_1(\mu_W) - \eta_2(\mu_W),
\]

Where \(x_i = \frac{m_i^2}{m_W^2}\). The coefficient \(C_i(\mu_W) (i = 1, 2, \cdots, 14)\) are defined as

\[
C_1(\mu_W) = \eta_1(\mu_W) + \eta_2(\mu_W) + \eta_3(\mu_W) + \eta_4(\mu_W) + \eta_5(\mu_W) + \eta_6(\mu_W) + \eta_7(\mu_W) + \eta_8(\mu_W) + \eta_9(\mu_W),
\]

\[
C_2(\mu_W) = \eta_2(\mu_W) + \eta_2(\mu_W) + \eta_3(\mu_W) + \eta_4(\mu_W) + \eta_5(\mu_W) + \eta_6(\mu_W) + \eta_7(\mu_W) + \eta_8(\mu_W) + \eta_9(\mu_W),
\]

\[
C_3(\mu_W) = \eta_1(\mu_W) - \eta_2(\mu_W) + \eta_3(\mu_W) + \eta_4(\mu_W) + \eta_5(\mu_W) + \eta_6(\mu_W) + \eta_7(\mu_W) + \eta_8(\mu_W) + \eta_9(\mu_W),
\]

\[
C_4(\mu_W) = \eta_1(\mu_W) + \eta_2(\mu_W) + \eta_3(\mu_W) + \eta_4(\mu_W) + \eta_5(\mu_W) + \eta_6(\mu_W) + \eta_7(\mu_W) + \eta_8(\mu_W) + \eta_9(\mu_W),
\]

\[
C_5(\mu_W) = -\frac{\eta_1(\mu_W)}{2} + \eta_2(\mu_W),
\]

\[
C_6(\mu_W) = \eta_1(\mu_W) + \eta_2(\mu_W),
\]

\[
C_7(\mu_W) = \eta_2(\mu_W) + \eta_3(\mu_W),
\]

\[
C_8(\mu_W) = \eta_3(\mu_W) + \eta_2(\mu_W) - \eta_1(\mu_W) + \frac{1}{4} \eta_2(\mu_W),
\]

\[
C_9(\mu_W) = -\eta_2(\mu_W) + \frac{1}{2} \eta_1(\mu_W),
\]

\[
C_{10}(\mu_W) = \frac{7}{4} \eta_1(\mu_W) - \eta_2(\mu_W),
\]
\[ C_{11}(\mu_w) = -\frac{1}{2}\left(\eta_h(\mu_w) + \eta_i(\mu_w) + 3\eta_l(\mu_w) + 3\eta_m(\mu_w)\right), \]

\[ C_{12}(\mu_w) = -\left(\eta_h(\mu_w) - \frac{3}{4}\eta_i(\mu_w) + \frac{1}{4}\eta_l(\mu_w) - \frac{1}{2}\eta_i(\mu_w) - \frac{1}{2}\eta_m(\mu_w)\right), \]

\[ C_{13}(\mu_w) = \frac{1}{4}\left(\eta_h(\mu_w) + \eta_i(\mu_w) + \eta_l(\mu_w) + \eta_m(\mu_w)\right), \]

\[ C_{14}(\mu_w) = -\frac{1}{4}\left(\eta_h(\mu_w) + \eta_i(\mu_w) + \eta_l(\mu_w)\right). \]  

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References


Figure 1: The possible Feynman diagrams which contribute to \((\beta\beta)_{0\nu}\) in MSSM
Figure 2: The short-range contributions to $\left(\beta\beta\right)_{0\nu}$ in MSSMRN (Part one)
Figure 3: The short-range contributions to $(\beta\beta)_{0\nu}$ in MSSMRN (Part two)
Figure 4: The lifetime of neutrinoless double beta decay of nuclei $^{76}G_e$ versus the lightest $\tau$-sneutrino mass with leptonic CKM matrix $U_M = U_1$ and (a) $h'^{\nu}_{3} = 1$, $m_R = 10^{14}$GeV, $m_{\tilde{\nu}_3}^2 = 4 \times 10^7$GeV; (b) $h'^{\nu}_{3} = 0.1$, $m_R = 10^{12}$GeV, $m_{\tilde{\nu}_3}^2 = 4 \times 10^7$GeV. The solid line corresponds to $\tan \beta = 20$ and the dash line to $\tan \beta = 30$; the other parameters are taken as in the text.
Figure 5: The lifetime of neutrinoless double beta decay of nuclei $^{76}Ge$ versus the lightest $\tau$-sneutrino mass with leptonic CKM matrix $U_M = U_2$ and (a) $h_3^{\nu} = 1$, $m_R = 10^{14}$GeV, $m_{\tilde{\nu}_3} = 4 \times 10^7$GeV; (b) $h_3^{\nu} = 0.1$, $m_R = 10^{12}$GeV, $m_{\tilde{\nu}_3} = 4 \times 10^7$GeV. The solid line corresponds to $\tan \beta = 20$ and the dash line to $\tan \beta = 30$; the other parameters are taken as in the text.