We show that an observable fraction of the measured elliptic flow may originate in classical gluon fields at the initial stage of a peripheral high-energy nuclear collision. This mechanism complements the contribution of late stage mechanisms, such as those described by hydrodynamics, to the observed elliptic flow.

The elliptic flow $v_2$, both integral and differential, is a sensitive measure of collectivity of the excited and dense matter produced in ultra-relativistic heavy ion collisions [1]. The first measurements of $v_2$ from RHIC, at center of mass energies of 130 and 200 GeV, have been reported recently [3]. Hydrodynamic (HD) analysis, based on the assumption of local thermal equilibrium, matches the data for the integral $v_2$ at large centralities, but the agreement gets worse for peripheral events [5,6]. HD models also reproduce the differential $v_2$ up to momenta of 1.5 GeV/c at mid-rapidity. However, above 1.5 GeV, the experimental $v_2$ appears to saturate, while the HD model $v_2$ still grows [5].

It is natural to expect $v_2$ to be sensitive to the early evolution of the system [2], when the energy density of the produced matter is at its highest, and before the system has equilibrated. Here we compute the contribution to $v_2$ at mid-rapidity from the strong fields generated shortly after the collision. These fields originate in a Color Glass Condensate (CGC) [8], formed in a nucleus by low-$x$ partons as their distributions saturate [7]. The CGC is characterized by the color charge per unit area $\Lambda_s$ which grows with energy, centrality and the size of the nuclei. Estimates for RHIC give $\Lambda_s \sim 1.4 - 2$ GeV. Since the gluon multiplicities in CGC are large, $\sim 1/\alpha_s(\Lambda_s^2) > 1$, CGC admits a classical description. In a collision, gluon production results from overlapping CGCs of the incident nuclei [9]. Our numerical work [10,11] confirmed that strong color fields of order $1/\alpha_s$ emerge in a proper time $\tau \sim 1/\Lambda_s$ after the collision.

As before, we assume strict boost invariance, i.e., the dimensionality of the
problem is 2+1. For a numerical solution we use lattice discretization. Our original setup, suitable for central collisions of very large nuclei, must be adapted for the task at hand. To study effects of anisotropy and inhomogeneity, we consider finite nuclei. We also impose suitable neutrality conditions on the color sources [13] to prevent gluon production far outside the nucleus.

We model a nucleus as a sphere of radius $R$, filled with randomly distributed nucleons. Within each nucleon we first generate, throughout the transverse plane, a spatially uncorrelated Gaussian color charge distribution of the width $\Lambda_n$. Next, we remove the monopole and dipole components of the distribution by subtracting the appropriate uniform densities. Since the color charges of the nucleons are uncorrelated, $\Lambda_s$ becomes position-dependent, peaking at the center and vanishing at the periphery of a nucleus. We adjust $\Lambda_n$ to ensure a desired value of $\Lambda_{x0}$, i.e., $\Lambda_s$ at the center. Next, we use our standard methods [10] and determine the classical fields as a function of $\tau$.

The calculation of $v_2$ involves determining the gluon number $N$, a quantity whose meaning is ambiguous outside a free theory. We resolve this ambiguity by computing the number in two different ways; directly in Coulomb Gauge (CG) and by solving a system of relaxation (cooling) equations for the fields [11]. Both definitions give the usual particle number in a free theory. We expect the two to be in good agreement for a weakly coupled theory. If the two disagree strongly, we should not trust either. Details of the cooling method, as applied to $v_2$, are presented in our recent paper [12].

The cooling and the CG results should converge at late times, when the system is weakly coupled. The two methods agree for $N$ at fairly early times. For $v_2$, this convergence occurs at much later $\tau$, because, as explained below, $v_2$ is dominated by soft modes with momenta $p_T < \Lambda_{x0}$. Following the evolution of the system to very late $\tau$ is computationally taxing. We therefore only compute $v_2$ at late $\tau$ for a selected value of $\Lambda_{x0}R$ and centrality and extrapolate $v_2$ from the early to the late $\tau$ for the remaining values. The results, for different values of $\Lambda_{x0}R$, are shown in Fig. 1 as a function of $n_{ch}/n_{tot}$. Clearly, our asymptotic values of $v_2$ undershoot the data. Nevertheless, we see that an observable amount of $v_2$ is produced classically in the pre-equilibrium stage of a collision.

Our differential $v_2$, shown in Fig. 2 for $b/2R = 0.75$ and $\Lambda_{x0}R = 74$, grows rapidly and is peaked for $p_T \sim \Lambda_{x0}/4$. A related analytical result [14] is that for $p_T \gg \Lambda_{x0}$, $v_2(p_T) \sim \Lambda_{x0}^2/p_T^2$, consistent with our numerical data. The dominance of $v_2$ by very soft modes helps explain the persistent difference between the cooling and the CG values: these modes remain strongly coupled and cannot be described within a free theory until very late $\tau$. Concomitantly, the soft modes contain many gluons and may be described classically even at the late $\tau$ considered. Our $v_2(p_T)$ clearly disagrees with experiment [3].

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Fig. 1. The centrality dependence of $v_2$ at early times from cooling (open symbols) and CG (filled symbols). The values of $\Lambda_{s0}R$ span the RHIC-LHC range: 18.5 (squares), 37 (triangles), and 74 (stars). Full circles are preliminary STAR data [4]. The band shows the range of $v_2$ extrapolated to late times. “Corrected values” denote the late time cooling and CG result for $\Lambda_{s0}R = 18.5$ at one centrality value.

Fig. 2. Differential $v_2$ as a function of $p_T$ in units of $\Lambda_{s0}$ for $\Lambda_{s0}R = 74$.

Note that experimental $v_2$ is found indirectly, in particular, from multiparticle cumulants [15]. It has been argued recently that non-flow correlations explain much of the measured $v_2$ [16]. We plan a numerical study of non-flow effects.

We thank K. Filiminov, U. Heinz, D. Kharzeev, R. Lacey, Z. Lin, L. McLerran, A. Mueller, J.-Y. Ollitrault, K. Rajagopal, J. Rak, S. Voloshin, Nu Xu, E. Shuryak and D. Teaney for comments; Sourendu Gupta for contributing to early stages of this work. We acknowledge support from: DOE Contract No. DE-AC02-98CH10886 (R.V.), the Portuguese FCT under grants CERN/P/FIS/40108/2000 and CERN/FIS/43717/2001 (A.K., R.V.), RIKEN-BNL (R.V., Y.N.), and NSF Grant No. PHY99-07949. A.K. thanks the BNL NTG for hospitality.
References


[4] We thank R. J. Snellings for providing the preliminary STAR data on $v_2$.


