1. INTRODUCTION

$|V_{cb}|$ and $|V_{ub}|$ are two of the fundamental constants of the Standard Model. They need to be determined by experimental measurements. These can be done with leptonic, semileptonic, or nonleptonic decays of $B$ mesons. In this talk I will discuss the determinations of $|V_{cb}|$ and $|V_{ub}|$ from inclusive semileptonic $B$ decays. An observable in these processes is schematically related to $|V_{cb}|$ or $|V_{ub}|$ in the form

$$\text{observable} = |V_{c(u)b}|^2 \cdot T,$$

where $T$ is a quantity derived from theory. Theory may also be involved in the experimental analysis to obtain the observable. With the theoretical relationship between the observable and $|V_{cb}|$ or $|V_{ub}|$, the measured quantity can be converted into a value of $|V_{cb}|$ or $|V_{ub}|$. The main obstacle in the theoretical description of the weak decay processes is long-distance strong interaction effects. There are two classes of observables: (1) routine observables, such as branching fractions and lifetimes, and (2) theory-motivated observables, such as the differential decay rate $d\Gamma/dw$ at zero recoil ($w = 1$) in the exclusive semileptonic decay $B \rightarrow D^{(*)}l\nu$ for determining $|V_{cb}|$. The latter is motivated from theory because its relationship with $|V_{cb}|$ or $|V_{ub}|$ is theoretically clean and model-independent in some sense.

2. THEORY OF INCLUSIVE SEMILEPTONIC $B$ DECAYS

Strong interaction effects on inclusive semileptonic $B$ decays are contained in the hadronic tensor

$$W_{\mu\nu} = -\frac{1}{2\pi} \int d^3y e^{i\mu\nu}(B | [j_{\mu}(y), j_\nu^{(0)}]|B)$$

$$= -g_{\mu\nu}W_1 + \frac{P_{\mu}P_{\nu}}{M_B^2}W_2 - i\epsilon_{\mu\nu\alpha\beta}\frac{P^{\alpha}q^\beta}{M_B^2}W_3$$

$$+ \frac{q_\mu q_\nu}{M_B^5}W_4 + \frac{P_\mu q_\nu + q_\mu P_\nu}{M_B^5}W_5. \quad (1)$$

where the charged current $j_{\mu}(y) = q(y)\gamma_{\mu}(1 - \gamma_5)b(y)$ with $q = c, u$. $P$ and $q$ denote the momenta of the $B$ meson and the virtual $W$ boson, respectively. The central issue in the theory of inclusive semileptonic $B$ decays is to compute the structure functions $W_{1-5}$, which are a priori independent. In the last several years, theoretical tools have been developed to address the issue.

2.1. Light-cone (LC) approach

The starting point of the LC approach [1–5] to inclusive B decays is the light-cone expansion. Since the $B$ meson is heavy, $M_B \gg \Lambda_{\text{QCD}}$, light-cone separations between the currents dominate the decay dynamics. In analog to the analysis of deep inelastic scattering, the light-cone expansion of the matrix element in the neighborhood of the light-cone, $y^2 = 0$, provides a systematic way of calculating nonperturbative QCD effects:

$$\langle B|[j^{(u)}_{\mu}(y), j^{(\nu\dagger)}_{\nu}(0)]|B\rangle = \sum_n C_n^{\mu\nu}(y, P)(y^2)^n. \quad (2)$$
At leading twist, the structure functions are related to a single universal distribution function:

\begin{align}
W_1 &= 2[f(\xi_+) + f(\xi_-)], \\
W_2 &= \frac{8}{\xi_+ - \xi_-}[\xi_+ f(\xi_+) - \xi_- f(\xi_-)], \\
W_3 &= -\frac{4}{\xi_+ - \xi_-}[f(\xi_+) - f(\xi_-)], \\
W_4 &= 0, \\
W_5 &= W_3,
\end{align}

where \( \xi_\pm = (\not{q}^0 \pm \sqrt{|\not{q}|^2 + m_b^2})/M_B \). The distribution function is defined as

\[ f(\xi) = \frac{1}{4\pi} \int \frac{d(y \cdot P)}{y \cdot P} e^{iy \cdot P} \]
\[ \times |B|\bar{b}(0)y \cdot \gamma U(0, y|b(y)|B)|_{y^2=0}. \]

It gives the probability of finding a b-quark with momentum \( \xi P \) inside the \( B \) meson with momentum \( P \) [6,7].

The leading nonperturbative QCD effect is encoded in the distribution function. What is known about the distribution function? First, the normalization of it is exactly known to be \( \int_0^1 d\xi f(\xi) = 1 \), because of the \( b \)-quark number conservation in strong interactions. Second, the gross shape of it is determined. In the free quark limit, \( f(\xi) = \delta(\xi - m_b/M_B) \). The mean value and variance of the distribution function have been calculated using the heavy quark effective theory. Consequently, the distribution function is known to be sharply peaked around \( m_b/M_B \) with a narrow width of order \( \Lambda_{QCD}/M_B \).

\subsection{2.2. Comparison with the heavy quark expansion (HQE) approach}

Another approach to inclusive \( B \) decays is the heavy quark expansion approach [8–15]. The starting point of the HQE approach is the local operator product expansion. It exploits the large \( b \)-quark mass to perform a local operator product expansion \( (y \to 0) \) of the time-ordered product of currents:

\[ T[j_\mu(y), j_-^{\prime \mu}(0)] = \sum_{\alpha} C_\mu^{\alpha}(y) O_\alpha(0). \]

The starting point of the HQE approach is different from that of the LC approach. The HQE approach is based on a short-distance \( (y \to 0) \) expansion in local operators of increasing dimension, while the LC approach is based on a non-local light-cone \( (y^2 \to 0) \) expansion in matrix elements of increasing twist. The light-cone expansion includes not only local contributions \( (y = 0) \), but also non-local contributions \( (y \neq 0) \). Thus, the LC approach contains more dynamic effects.

In addition, the HQE approach assumes quark-hadron duality and has to use the quark phase space, while the LC approach does not rely on quark-hadron duality and uses the physical hadron phase space. The singularities appear at the endpoints of the lepton energy spectra in both \( B \to X_u l\nu \) and \( B \to X_c l\nu \) calculated in the HQE approach, implying the breakdown of the operator product expansion near the boundaries of phase space. There are no endpoint singularities in the lepton energy spectra calculated in the LC approach.

The partial resummation of the heavy quark expansion introduces a different distribution function (“shape function”) [11,14,15]. It should be noted that the correction to the leading contribution given in terms of the shape function in the HQE approach is of order \( \Lambda_{QCD}/m_b \), while the correction to the leading contribution given in terms of the distribution function in the LC approach is of order \( (\Lambda_{QCD}/M_B)^2 \).

\subsection{2.3. A manifestation of quark-hadron duality violation}

The physical phase space at the hadron level is larger than the phase space at the quark level. Therefore, calculations assuming quark-hadron duality cannot account for the rate due to the extension of phase space from the quark level to the hadron level. This rate missing is a clear manifestation of the violation of quark-hadron duality. This is the case for the HQE approach.

Since the LC approach does not rely on quark-hadron duality and uses the hadron phase space, a comparison of the decay rates calculated in the LC and HQE approaches can quantitatively demonstrate how large duality is violated. It has been found that the total rate for the inclusive semileptonic decay \( B \to X_u l\nu \) \( (B \to X_c l\nu) \) calculated in the LC approach is about 14% (12%)
larger than the HQE approach [2,16]. The non-perturbative QCD correction changes sign: It increases the total rate in the LC approach, while it decreases the total rate in the HQE approach. Therefore, there is significant duality violation in the HQE approach. In particular, it cannot include the phase space effect. From the measured inclusive semileptonic branching fractions and the lifetime of the $B$ meson, the LC approach would give rise to smaller values of $|V_{cb}|$ and $|V_{ub}|$ than the HQE approach, as I will discuss in more detail below.

3. CONVENTIONAL METHODS

Conventionally, the routine observables, the branching fractions and the lifetime of the $B$ meson, have been used to determine $|V_{cb}|$ and $|V_{ub}|$ from inclusive semileptonic $B$ decays. These observables are related to $|V_{cb}|$ or $|V_{ub}|$ in the form

$$\frac{B(B \rightarrow X_c l\nu)}{\tau_B} = |V_{cb}|^2 \cdot \gamma_c, \quad (10)$$

$$\frac{B(B \rightarrow X_u l\nu)}{\tau_B} = |V_{ub}|^2 \cdot \gamma_u. \quad (11)$$

Theory is needed to compute $\gamma_c$ and $\gamma_u$. Besides parametric uncertainties, additional uncertainties in the theoretical calculations of $\gamma_c$ and $\gamma_u$ stem from the assumption of quark-hadron duality in the HQE approach and the detailed shape of the distribution function, which is unknown at present, in the LC approach.

The uncertainty due to the assumption of quark-hadron duality is irreducible. The errors usually quoted in the determinations of $|V_{cb}|$ and $|V_{ub}|$ from inclusive semileptonic $B$ decays using the HQE calculations should be interpreted with caution. The significant uncertainty from the assumption of quark-hadron duality has not been included in the errors. In contrast, the uncertainty due to the shape of the distribution function in the LC approach can be reduced. The distribution function is universal: It incorporates bound state effects in inclusive $B$ decays, including semileptonic [1–4], radiative [5], and nonleptonic [17,18] inclusive $B$ decays. Measurements of decay distributions and moments in these processes will impose constraints on the distribution function. Especially, the $\xi_u$ spectrum $[\xi_u = (q^2 + |q|)/M_B]$ in $B \rightarrow X_u l\nu$ and the photon energy spectrum in $B \rightarrow X_c \gamma$ are directly proportional to the distribution function. These spectra are most sensitive to the shape of the distribution function. Measurements of these spectra may thus provide the most stringent constraint on the distribution function. Moreover, other non-perturbative QCD methods, such as lattice QCD, may help to determine the shape of the distribution function, too.

Including the uncertainties from the input parameters and the shape of the distribution function, the calculations in the LC approach yield [2,16]

$$\gamma_c = 49 \pm 9 \text{ ps}^{-1}, \quad \gamma_u = 76 \pm 16 \text{ ps}^{-1}.$$ 

These results allow reliable determinations of $|V_{cb}|$ and $|V_{ub}|$ without the assumption of quark-hadron duality. The accuracies could be further improved, as discussed above.

A particular problem exists in the $|V_{ub}|$ determination, namely a very large $B \rightarrow X_c l\nu$ background. Various kinematic cuts can be used to suppress the background. The fractions of events with the different cuts are listed in Table 1.

<table>
<thead>
<tr>
<th>Kinematic cut</th>
<th>Fraction of events</th>
</tr>
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<tbody>
<tr>
<td>$E_l &gt; (M_B^2 - M_{D}^2)/(2M_B)$</td>
<td>$\sim 10%$</td>
</tr>
<tr>
<td>$q^2 &gt; (M_B - M_D)^2$</td>
<td>$\sim 20%$</td>
</tr>
<tr>
<td>$M_X &lt; M_D$</td>
<td>$\sim 80%$</td>
</tr>
</tbody>
</table>

Imposing kinematic cuts causes additional theoretical uncertainties, since the extrapolation in phase space to obtain the desired observable involves theory. The hadronic invariant mass ($M_X$) cut is most efficient. It retains a vast majority of phase space, so that only a small extrapolation to the entire phase space is needed. The other kinematic cuts are less efficient and would cause larger theoretical uncertainties because of a considerable dependence on dynamic subtlety.
4. A NEW METHOD FOR THE PRECISE DETERMINATION OF $|V_{ub}|$

In the light-cone limit, $y^2 \to 0$, the $b$-quark number conservation leads to the sum rule [19,20]

$$S \equiv \int_0^1 d\xi_u \frac{1}{\xi_u} \frac{d\Gamma}{d\xi_u} = |V_{ub}|^2 \frac{G_F^2 M_B^5}{192\pi^3}$$  \hfill (12)

for the charmless inclusive semileptonic decay $B \to X_u l\nu$. The sum rule establishes a theoretically clean relationship between the observable $S$ and $|V_{ub}|$. The sum rule has the following advantages: (1) it is model-independent (in particular, independent of the distribution function), (2) it has no reliance on quark-hadron duality, (3) no free parameters (such as $m_b$) except for $|V_{ub}|$ enter the sum rule, and (4) it receives no perturbative QCD corrections. Therefore, by using the sum rule, a more reliable and precise value of $|V_{ub}|$ can be determined than by using the conventional method.

Of course, to obtain the theory-motivated observable $S$ in $B \to X_u l\nu$ one must deal with the large $B \to X\ell\ell\nu$ background. The strategy appears to be: (1) apply the kinematic cut $M_X < M_D$ (or other realistic cuts), (2) measure the weighted spectrum $\xi_u^{-5}d\Gamma/d\xi_u$, (3) extrapolate it to entire phase space to obtain the integral $S$, and (4) determine $|V_{ub}|$ from the observable $S$ by using the theoretically clean relationship (the sum rule) between them.

This method for determining $|V_{ub}|$ is analogous to the determination of $|V_{cb}|$ from the observable, $d\Gamma/dw$ at zero recoil, in the exclusive semileptonic decay $B \to D^\ast\ell\nu$. Both use a theoretically clean relationship, which is derived from QCD symmetries, between the observable and $|V_{ub}|$ or $|V_{cb}|$. Both require an extrapolation in phase space to obtain the theory-motivated observable.

There are two sources of theoretical uncertainties in this method for the $|V_{ub}|$ determination, one from higher twist corrections to the sum rule and another from the extrapolation to obtain $S$. In principle, the light-cone expansion systematically takes into account higher twist corrections. The quantitative estimate using the heavy quark effective theory has shown that the error on $|V_{ub}|$ from higher twist corrections to the sum rule amounts to around 1% [21]. The error on $|V_{ub}|$ due to the shape of the distribution function used for the extrapolation has been assessed at the 6% level [20]. With improved knowledge of the shape of the distribution function, as discussed before, one could reduce this error.

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REFERENCES