LEPTON ACCELERATION BY RELATIVISTIC COLLISIONLESS MAGNETIC RECONNECTION

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ABSTRACT

We have calculated self-consistent equilibria of a collisionless relativistic electron-positron gas in the vicinity of a magnetic X-point. For the considered conditions, pertinent to extra-galactic jets, we find that leptons are accelerated up to Lorentz factors \( \Gamma_0 = \kappa e B_0 L E^2/mc^2 \gg 1 \), where \( B_0 \) is the typical magnetic field strength, \( E \equiv E_0/B_0 \), with \( E_0 \) the reconnection electric field, \( L \) is the length scale of the magnetic field, and \( \kappa \approx 12 \). The acceleration is due to the dominance of the electric field over the magnetic field in a region around the X-point. The distribution function of the accelerated leptons is found to be approximately \( \frac{dn}{d\gamma} \propto \gamma^{-1} \) for \( \gamma \lesssim \Gamma_0 \). The apparent distribution function may be steeper than \( \gamma^{-1} \) due to the distribution of \( \Gamma_0 \) values and/or the radiative losses. Self-consistent equilibria are found only for plasma inflow rates to the X-point less than a critical value.

Subject headings: acceleration of particles: magnetic fields: MHD : plasmas: jets—galaxies

1. INTRODUCTION

The observed radiation of most large scale extragalactic jets is due to incoherent synchrotron radiation. In a number of sources, M87 and 3C 273 for instance, the radiation lifetime of the electrons (and possibly positrons) is much less than the transit time from the central source (Felten 1968). Thus, there must be mechanisms for the electron “reacceleration.”

Several mechanisms have been proposed to account for the reacceleration of the electrons. These include Fermi mechanism of particle acceleration (Christiansen, Pacholczyk, & Scott 1976), Fermi acceleration in shock waves (Krimsky 1977; Axford, Leer, & Skadron 1978; Bell 1977, 1978; Blandford & Ostriker 1978), stochastic electric field acceleration (Eilek & Hughes 1990) and whistler wave acceleration (Melrose 1974). The effectiveness of the Fermi mechanism and stochastic fields in accelerating electrons is unknown (Blandford & Eichler 1987; Eilek & Hughes 1990; Jones & Ellison 1991). Resistive tearing has been discussed as a means of producing neutral layers which can accelerate electrons (Königl & Choudhuri 1985; Choudhuri & Königl 1986). Reconnection as a mechanism for accelerating electrons has been discussed by a number of authors (Blandford 1983; Begelman, Blandford, & Rees 1984; Browne 1985; Ferrari 1984, 1985; Norman 1985; Kirchner 1988; Lesch 1991; Romanova & Lovelace 1992). The acceleration of electrons by long wavelength electromagnetic waves trapped in the boundary layer of a jet was discussed by Bisnovatyi-Kogan & Lovelace (1995). The reconnection of magnetic fields at a neutral point as described by magnetohydrodynamics (MHD) has been studied extensively. It has been shown that the neutral lines can evolve into current sheets (Syrovatskii 1971). Resistive MHD simulations of reconnection by Biskamp (1986) show a strong tendency to form a current sheets.

Collisionless reconnection has been studied by following particle orbits, assuming nonrelativistic particles and prescribed fields (Galeev 1984; Zelenyi et al. 1984; Burkhart, Drake, & Chen 1990; Deeg 1991). The particles were launched away from the X-point and given an initial velocity that is appropriate for a given thermal velocity distribution. Acceleration of the particles near the X-point was observed. This work was extended to include the self-consistent fields of the particle flows (Burkhart, Drake, & Chen 1991). For sufficiently high inflow rates they found an outflow shock where the flow velocity exceeded the Alfvén velocity. In addition they found that sufficiently large inflow rates caused the width of the current layer to “collapsed to zero,” and concluded that there was a maximum inflow rate into the reconnection region. Nonrelativistic particle trajectories have been studied in reconnection regions as part of an investigation of the Earth’s magnetosphere (Speiser 1965). This work has been extended in an effort to define an effective conductivity in the absence of particle collisions (Speiser 1970).

Direct acceleration of particles in reconnection layers was proposed by Alfvén (1968). Direct relativistic acceleration of leptons near the X-line of a magnetic field configuration was studied by Romanova and Lovelace (1992). The static electric field parallel to this X-line can cause the acceleration of leptons to the very high Lorentz factors observed. Romanova and Lovelace (1992) analyzed the particle orbits in the current layer structure proposed by Syrovatskii (1971). For the case of an electron-positron plasma they obtained an energy distribution function \( \frac{dn}{d\gamma} \propto \gamma^{-1.5} \). Recently, Zenitani and Hoshino (2001) have done relativistic particle-in-cell simulations of a reconnection layer in an electron-positron plasma and have found a spectrum \( \gamma^{-1} \) out to \( \gamma \sim 20 \).

In this work we solve the relativistic equations of motions for an electron-positron plasma moving in the electric and magnetic fields of a reconnection layer indicated in Figure 1. In turn, the particle motion is used to calculate the current-density and the associated self magnetic field. The particle orbits and field calculations are done iteratively to give a self-consistent equilibrium for the re-
connection layer. Once we have this equilibrium we can determine the distribution function of the accelerated leptons. Section 2 of this paper develops the theory, and Section 3 discusses the methods and results. Section 4 gives the conclusions of this work.

2. THEORY

2.1. Physical Picture

The geometry of the stationary ($\partial / \partial t = 0$) reconnection region is shown in Figure 1. The magnetic field $\mathbf{B} = B_x(x, y)\mathbf{x} + B_y(x, y)\mathbf{y}$ has an $X$–point in the $(x, y)$ plane at $x = 0, y = 0$. A uniform electric field exists in the $z$–direction, $\mathbf{E} = E_0\mathbf{z}$ with $\nabla \times \mathbf{E} = 0$ and $E_0 < 0$. Therefore, the lepton gas drifts with velocity $v_{dy} = cE_z B_z/B^2$ in the $y$–direction towards the magnetic $X$–point from above and below. Because the gas is electrically neutral there is no net current due to this drift (both electrons and positrons drift with the same velocity in the same direction).

The $\mathbf{B}$ field vanishes at the $X$–point, and consequently in the vicinity of this point the particle motion is dominated by the electric field. The electric field accelerates electrons in say the $+z$ direction and positrons in the $-z$ direction. This gives a current-density in the $\pm x$ directions by the $(\mathbf{v} \times \mathbf{B}/c)_x$ force. At a large enough distance $|x|$ from the $X$–point the magnetic field becomes dominant and the particles exhibit their $z$–motions cancels, and there is no $y$–direction current.

We make the simplifying approximation of neglecting the $y$–thickness of the current layer. In this case, the particles move in the $y = 0$ plane. We therefore treat equilibria where the particles drift from above and below into the $y = 0$ plane where they are accelerated in the $\pm z$ directions and then expelled in the $\pm x$ directions. Thus we calculate the particle orbits $[x(t), 0, z(t)]$. Knowing the particle motion allows us to calculate the surface current-density $J_x(z)$. From this we calculate the self-magnetic field. We then use the self field to recalculate the particle orbits. We iterate on this process so that we get a self-consistent solution of the self magnetic field and the particle orbits. The self-consistent orbits of the leptons can then be used to derive the energy spectrum of the accelerated leptons.

2.2. Single Particle Motion

The magnetic field can be written as $\mathbf{B} = \nabla \times [A_z(x, y)\mathbf{z}]$, where $A_z$ is the total vector potential. For specificity we consider an $X$–type null point of the $\mathbf{B}$ so that $A_z(x, y)$ is an even function of both arguments. The magnetic field consists of a “external” component due to distant currents and the “self field” due to local currents. The external component $A^\text{ext}_z$ is divergence and curl free. We take the leading terms of a Taylor expansion of this field,

$$A^\text{ext}_z = \frac{B_0}{2L}(x^2 - y^2),$$

so that

$$B^\text{ext}_x = \frac{B_0}{L} y, \quad B^\text{ext}_y = \frac{B_0 x}{L},$$

where $B_0 > 0$ without loss of generality. The total field is given by

$$A^\text{tot}(x, y) = A^\text{ext} + A^\text{self}.$$  

$A^\text{tot}$ is an even function of both arguments but its dependence on $(x, y)$ is changed by the self-field.

We consider quasi-stationary conditions so that $\nabla \times \mathbf{E} = 0$ and thus $\mathbf{E} = -\nabla \Phi$, which is the “external” electric field. The relevant solution is $\mathbf{E} = E_0\mathbf{z}$ and $\Phi = -E_0 z$, where $E_0 = \text{const} < 0$. This corresponds to plasma flowing into the $X$ point from above and below the $y = 0$ plane. Because we consider an electron-positron plasma the “self electric field” is zero, and the total electric field is $E_0\mathbf{z}$.

The single particle motion is described by the Lagrangian

$$\mathcal{L} = -mc^2\sqrt{1 - \beta^2} + \frac{q}{c} A_z v_z - q\Phi,$$  

where $\beta \equiv |\mathbf{v}|/c$ and the electrostatic potential $\Phi = -E_0 z$. The corresponding Hamiltonian is

$$\mathcal{H} = \left[ (P - \frac{q}{c} A)^2 + m^2 c^4 \right]^{1/2} + q\Phi,$$

where $\mathbf{P}$ is the canonical momentum. Because $\partial \mathcal{L}/\partial t = 0$, the $\mathcal{H}$ = const which is the single particle energy.

A further constant of the motion is the canonical momentum in the $z$–direction, $P_0$. Because $A_z$ is independent of $z$,

$$\frac{dP_z}{dt} = -\frac{\partial \mathcal{H}}{\partial z} = qE_0, \quad \text{or} \quad \frac{d}{dt}(P_z - qE_0t) = 0,$$

so that

$$P_0 = P_z - qE_0t = m\gamma \frac{dz}{dt} + \frac{q}{c} A_z - qE_0t = \text{const}.$$  

The $z$–equation of motion is simply $dP_0/dt = 0$ which gives

$$\frac{d}{dt} \left( m\gamma \frac{dz}{dt} \right) + \frac{q}{c} B_y \frac{dx}{dt} - \frac{q}{c} B_x \frac{dy}{dt} - qE_0 = 0,$$  

where $\gamma = (1 - \beta^2)^{-1/2}$. For the limit where the $z$–acceleration is small, equation (5) gives

$$\frac{q}{c} B_y \frac{dx}{dt} - \frac{q}{c} B_x \frac{dy}{dt} - qE_0 = 0,$$
\[ \mathbf{v} = c \mathbf{E} \times \mathbf{B} / B^2 \]  

which is the well-known \( \mathbf{E} \times \mathbf{B} \) drift velocity (Northrop 1963).

We assume that away from the \( X \)-point the leptons are relatively ‘cold’ in the sense that their thermal velocity spread is less than their \( \mathbf{E} \times \mathbf{B} \) drift velocity which is less than \( c \). Thus, leptons which are initially above (below) the \( (x, z) \) plane drift down (up) towards this plane as sketched in Figure 1. A restricted class of solutions has the form \( x(t) = [x(t), 0, z(t)] \). A particle “launched” from a position \( (x_0, 0, 0) \) in the \( y = 0 \) plane with \( v_y = 0 \) stays in this plane. Of course, a particle launched away from this plane (or one that has \( v_y \neq 0 \)), oscillates about the \( y = 0 \) plane.

In this work we neglect the thickness of the current layer in the \( y \) direction. Therefore, we use the particle orbits in the \( y = 0 \) plane. We consider a set of particles entering the \( y = 0 \) plane from above and below with initial locations given by \((x_0, 0, 0)\).

Dimensionless variables are introduced as

\[ \hat{x} \equiv x/L \quad , \quad \hat{t} \equiv \omega_0 t \quad , \quad \text{where} \quad \omega_0 \equiv |q|B_0/mc \]  

is the non-relativistic cyclotron frequency in the reference field \( B_0 \). Dimensionless electric and magnetic fields are naturally measured in units of \( B_0 \). In particular, we let

\[ E \equiv |E_0|/B_0 . \]  

A characteristic Lorentz factor can be defined as

\[ \gamma_0 \equiv \frac{|q|B_0 L}{mc^2} = \frac{\omega_0 L}{c} . \]  

We can rewrite this as

\[ \gamma_0 \approx 5.9 \times 10^3 \left( \frac{B_0}{10^{-6} \text{G}} \right) \left( \frac{L}{10^{13} \text{cm}} \right) . \]  

Note that \( v_x/c = \gamma_0 (d\hat{x}/d\hat{t}) \), for example. We let \( n_\infty \) be the number density of leptons (electrons + positrons) at a large distance from the \( X \)-point. An important dimensionless quantity is

\[ \zeta = \frac{4\pi n_\infty \gamma_0 mc^2}{B_0^2} . \]  

The role of this parameter is described in §3.

### 2.3. High Energy Approximation

A particle launched at \( x_0 \) in the \( y = 0 \) plane is accelerated in the \( z \)-direction by the \( E_0 \hat{z} \) field and it drifts in the \( x \)-direction because of the \( \mathbf{v} \times \mathbf{B} \) force. The closer to the \( X \)-point a particle starts (the smaller \( x_0 \)) the higher the energy it is accelerated to. An approximate solution for the particle motion in the external magnetic field \( \mathbf{B}^{\text{ext}} \) is possible for sufficiently small \( x_0 \) where there is a large acceleration in the \( z \)-direction.

Assuming that the particle is already highly relativistic we let \( p_x(t) = m\gamma c \sin(\theta) \) and \( p_z(t) = m\gamma c \cos(\theta) \). For specificity we consider the acceleration of electrons. The equations of motion become

\[ \frac{dp_x}{dt} = -qB_y^{\text{ext}}(x) \cos(\theta) \quad , \quad \frac{dp_z}{dt} = qB_y^{\text{ext}}(x) \sin(\theta) + qE_0 . \]

We assume \( \theta^2 \ll 1 \) and discuss the conditions for this later. The energy equation gives \( m\gamma c^2 = qE_0 z + \text{const} \) and the small angle approximate gives \( t \approx z/c \).

The \( x \)-equation of motion in dimensionless variables becomes

\[ \frac{d}{dz} \left( \hat{z} \frac{d\hat{z}}{dz} \right) = \frac{\hat{x}}{E} . \]  

The relevant solution to equation (11) is

\[ \hat{x} = \hat{x}_0 I_0 \left( 2\sqrt{\gamma /E} \right) , \]  

where \( I_0 \) is the usual modified Bessel function. This dependence is valid for small angles, that is, for \((d\hat{x}/d\hat{z})^2 \ll 1\), or \( \hat{x}_0 I_1(2\sqrt{\gamma /E})/\sqrt{\gamma E}^2 \ll 1 \). A necessary condition for this is that \( \hat{x}_0 \ll E \). The geometry of the orbits for highly relativistic motion depends only on \( \hat{x}_0 \) and \( E \) (in fact, on just \( E/\hat{x}_0 \)) and not for example on \( \gamma_0 \).

Figure 2 shows orbits in the external magnetic field calculated without approximation. The approximation of equation (12) is also shown.

### 2.4. Current-Density from Single Particle Motion

Far from the \( X \)-point, ideal magnetohydrodynamics applies, and the fluid particles are confined to flux surfaces where \( A_z = \text{const} \). Even though this is a time independent situation, the flux surfaces drift inward towards the neutral layer with a velocity equal to \( v_\phi = c \mathbf{E} \times \mathbf{B} / B^2 \). Since the \( \mathbf{B} \) field is tangent to surfaces of constant \( A_z \) and the electric field is in the \( \hat{z} \) direction, the field lines drift inward towards the \( X \)-point at the same speed the particles do. Near the \( X \) point, where \( |\mathbf{E}| > |\mathbf{B}| \), ideal magnetohydrodynamics breaks down.

We assume that after a particle inflows (from above or below) to the \( y = 0 \) plane it gets launched on an orbit in this plane, \( [x(t), 0, z(t)] \). The initial position of the particle in the \( y = 0 \) plane is \( x_0 \) and some value of \( z \). We denote the number flux density of particles (electrons + positrons) inflowing from above and below the \( y = 0 \) plane as \( \mathcal{F}(x_0) \) (number per cm² per second). The simplest case is that of a uniform density of inflowing plasma where

\[ \mathcal{F} = 2n_\infty c E , \]  

where \( n_\infty \) is the number density (1/cm³) of electron plus positron density at large distances and where the factor of two comes from the two sides of the current layer. The motion of a single particle starting from position \( x_0 \) is described by its position \([x(t|x_0), 0, z(t|x_0)]\) and its velocity \([v_x(t|x_0), 0, v_z(t|x_0)]\).

The surface current-density (charge per cm per second) for each initial \( x_0 \) is then given by

\[ J_z(x|x_0)dx_0 = qN(x|x_0)\langle v_z(x|x_0) \rangle dx_0 , \]  

where \( N(x|x_0) \) (with units 1/cm³) is the surface number density (of electrons and positrons) at \( x \) launched between \( x_0 \) and \( x_0 + dx_0 \). In equation (14), \( v_z \) is the electron velocity, \( q \) is the electron charge, and the angular brackets indicate a time average. In the considered stationary state
which is uniform in \( z \), all averages are independent of \( t \) and \( z \).

Consider particles moving in the \( +x \) direction (the \( -x \) direction case follows by symmetry), then conservation of particles implies that

\[
\mathcal{N}(x|x_0)\langle v_x(x|x_0) \rangle = \text{const} = \mathcal{F}(x_0), \tag{15}
\]

where \( v_x(x|x_0) \) is the velocity of an electron or a positron. Thus,

\[
\mathcal{J}_z(x) = \int_0^{x_0} dx_0 \mathcal{J}_z(x|x_0),
\]

\[
= q \int_0^{x_0} dx_0 \mathcal{F}(x_0) \frac{\langle v_x(x|x_0) \rangle}{\langle v_x(x|x_0) \rangle}, \tag{16}
\]

where \( x_0 \) is the maximum of \( x_0 \) assumed to be \( \mathcal{O}(L) \). The total current between \( x \) and \( x + \delta x \) can be written as

\[
\delta x \mathcal{J}_z(x) = q \int_0^{x_0} dx_0 \mathcal{F}(x_0) \int_x^{x+\delta x} dx \frac{\langle v_z \rangle}{\langle v_x \rangle}.
\]

For a particular value of \( x_0 \) a particle orbit passes between \( x \) and \( x + \delta x \) for time between \( t_1 = t(x|x_0) \) and \( t_2 = t(x + \delta x|x_0) \) (and possibly for time between \( t_3 \) and \( t_4 \) and \( t_5 \) and \( t_6 \), or an odd number of intervals). This allows us to replace the spatial integration by a temporal integration over the particle orbit. Notice that \( \langle v_x \rangle = \sum \delta t_j v_{xj} / \sum \delta t_j \), where \( j = 1 \) or \( j = 1, 2, 3 \), etc. represents the different traversals of the interval \( \delta x \) at the distance \( x \), and where \( \delta t_j = \delta x / |v_{xj}| \). We have for example \( \int dt v_x = \sum \delta t_j v_{xj} = \delta x \). Therefore, we find

\[
\delta x \mathcal{J}_z(x) = q \int_0^{x_0} dx_0 \mathcal{F}(x_0) \int_{t_1}^{t_2} dt \ v_x(t, z(t)|x_0). \tag{16a}
\]

The time integral needs to be done for all values of \( t \) for which the particle is between \( x \) and \( x + \delta x \). We can put equation (16a) into dimensionless form using equations (7) - (10) and \( \mathcal{J}_z = \mathcal{J}_z / (cB_0) \). For the case where equation (13) applies this gives

\[
\delta \tilde{z} \mathcal{J}_z = \tilde{\zeta} \int d\tilde{x}_0 \int dt \frac{d\tilde{x}}{dt} \tag{16b}
\]

Here,

\[
\tilde{\zeta} = \frac{\mathcal{E} \zeta}{2\pi}, \tag{17}
\]

with \( \zeta \) defined by equation (10), is a dimensionless measure of the rate of inflow of plasma to the neutral layer.

### 2.5. Self-Magnetic Field

For the considered thin current layer with surface current-density \( J(x) \), we can express the self-magnetic field of this current as

\[
B_{y}^{\text{self}}(x) = \frac{2}{c} \mathcal{P} \int dx' \frac{\mathcal{J}_z(x')}{x' - x}, \tag{18}
\]

where \( \mathcal{P} \) is the principle value. In dimensionless variables this becomes

\[
\tilde{B}_{y}^{\text{self}}(x) = 2 \mathcal{P} \int d\tilde{x}' \frac{\mathcal{J}_z(x')}{\tilde{x}' - \tilde{x}},
\]

The total magnetic field is

\[
B_{y}^{\text{tot}}(x) = B_{y}^{\text{ext}}(x) + B_{y}^{\text{self}}(x). \tag{19}
\]

The dimensionless form of this equation is the same but with hats over the magnetic fields.

In the following we simplify the notation by dropping the hats and dropping the \( y \) and \( z \) subscripts on \( B_y \) and \( \mathcal{J}_z \).

### 3. Methods and Results

The self-consistent magnetic field of the current layer is calculated iteratively. For a relatively weak self field - sufficiently small \( \tilde{\zeta} \) compared with unity - a direct iteration scheme has been found to converge. This scheme may be described as

\[
\begin{align*}
B_{n}^{\text{tot}} & \xrightarrow{\text{orbits}} J_{n+1} \xrightarrow{\text{HT}} B_{n+1}^{\text{ext}} \xrightarrow{\text{orbits}} J_{n+2} \ldots, \tag{20}
\end{align*}
\]

where

\[
B_{n}^{\text{tot}} = x + B_{n}^{\text{self}}, \quad B_{n}^{\text{self}} \equiv 2 \mathcal{P} \int dx' \frac{J_n(x')}{x' - x}. \tag{21a}
\]

Here, “HT” stands for the Hilbert transform, and “orbits” indicates the evaluation of equation (16b) using an \( x \)-grid of 100–200 points and the calculation of 100–200 electron orbits. A stretched \( x \)-grid is used with higher resolution at small \( x \). The distribution of inflowing plasma \( \mathcal{F}(x_0) \) is assumed uniform in \( x_0 \) as given by equation (13). All cases considered here have \( \gamma_0 \gg 1 \) so that the initial, non-relativistic or modestly relativistic conditions \( (\gamma < 2) \) of particles at their starting points \( x_0 \) do not affect the reported results. In the iteration sequence we have found it advantageous to make analytical fits to the self fields \( B_{n}^{\text{self}} \) which are used in the subsequent orbit calculations.

For larger self-fields we use the iteration (20) with equation (21a) replaced by

\[
B_{n}^{\text{tot}} = x + \alpha B_{n}^{\text{self}} + (1 - \alpha)B_{n-1}^{\text{self}}, \tag{21b}
\]

where \( 0 < \alpha < 1 \) is a numerical factor, and \( B_{n}^{\text{self}} \) is still given by equation (21a). This iteration scheme is known to

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**Fig. 2.** Sample electron orbits in the \( y = 0 \) plane for the external magnetic field \( B_{y}^{\text{ext}} = B_0 x/L \) for the case \( \mathcal{E} = 0.5 \), \( \gamma_0 = 10^3 \), and \( \delta \approx 0.1, 0.2, \) and \( 0.3 \). For large \( x \), where the orbits are “looping,” the drift in the \( +x \)–direction is the \( E_x \times B_y \) drift, and the drift in the \( +z \) direction is the gradient \( B \) drift \( \propto B_y \times \nabla B_y(x,0) \). The positron orbits are mirror images for \( z \rightarrow -z \).
allow the calculation of self-consistent particle ring equilibria with large self-fields (see for example Larrabee et al. 1979, 1982).

The key dimensionless parameters which determine the self-consistent current layers are

\[ \mathcal{E}, \quad \zeta, \quad \text{and} \quad \gamma_0. \]  

(22)

For the considered limit \( \gamma_0 \gg 1 \) and fixed \( \mathcal{E} \), we find that the current density and self-field are almost independent of \( \gamma_0 \). However, the distribution function of accelerated leptons does depend on \( \gamma_0 \) and \( \mathcal{E} \) as discussed below.

### 3.1. Weak Self-Field

Figure 3 shows the current density profiles \( J(x) \) for a weak field case \( \zeta = 0.05 \) where the convergence of the direct iteration, equations (20) and (21a), is quite rapid. The main peak of the current density is due to the direct acceleration of electrons and positrons in the region where \( |E|/|B| > 1 \). The “shoulder” of the current density profile is due to the gradient \( B \) drift in the \( z \)-direction. Figure 4 shows the final profiles of the total and self-magnetic fields. The self-field acts to reduce the slope of the total field \( B_{\text{tot}} \) near the origin. For vanishing self-field the slope is unity. For the case of Figure 4, the slope is \( dB_{\text{tot}}/dx|_{0} = 0.680 \).

The energy distribution or spectrum of the leptons accelerated in the current layer \( dn/d\gamma \) is obtained from the calculation of the particle orbits. The particles inflowing to the current layer are uniformly distributed in \( x_0 \) and initially they are assumed to have \( \gamma < 2 \). The Lorentz factor of a particle as it exits the current layer (say, \( x > x_{\text{max}} \)) is therefore a function of \( x_0 \), \( \gamma_m = \gamma_m(x_0) \). As part of our orbit calculations we determine \( \gamma_m(x_0) \). We find that \( \gamma_m(x_0) \) is a monotonically decreasing function of \( x_0 \). With \( dn/d\gamma \) constant the number of particles launched between \( x_0 \) and \( x_0 + dx_0 \), we have

\[ \frac{dn}{d\gamma} = \frac{dn/dx_0}{dx/d\gamma}|_{x_0}, \]  

(23)

for \( \gamma > 2 \), where we have dropped the \( m \)-subscript on \( \gamma \).

Figure 5 shows lepton distribution for the same case as Figures 3 and 4. As mentioned earlier the magnetic field of the current layer is almost independent of \( \gamma_0 \). For these values of \( \zeta \) and \( \mathcal{E} \), we have run a range of values of \( \gamma_0 = 10^2 - 10^5 \), and find that the distributions are roughly fitted by

\[ \frac{dn}{d\gamma} \approx \frac{K}{\gamma \Gamma_0} \exp \left( -\frac{\gamma}{\Gamma_0} \right), \]  

(24)

where \( \Gamma_0 = \kappa \mathcal{E}^2 \gamma_0 \) and \( \kappa \approx 12 \). With \( K \approx 1/\ln(0.5\Gamma_0) \) the distribution is normalized to unity. The distribution (24) is not expected to be the same as that observed because of the influence of the distribution of \( \gamma_0 \) values and/or the affect of radiative losses. These affects on the spectrum are discussed in §3.4.

### 3.2. Strong Self-Field

For larger self-field strengths \( \zeta \), we use the iteration indicated by equations (20) and (21b) usually with \( \alpha = 0.5 \). For \( \zeta = 0.1 \) we have not succeeded in getting the iterations to converge. We find that the iterations lead to negative values of \( B_{\text{tot}}^2(x) \) for \( x > 0 \) which would cause trapping of particles in the \( x \)-direction. Such a configuration is inconsistent with our assumption that the accelerated particles are expelled in the \( x \)-direction. The existence of a maximum of \( \zeta \), which measures the rate of plasma inflow to the neutral layer, is consistent with the finding of a maximum plasma inflow rate by Burkhart et al. (1991). For \( \zeta = 0.075 \), equations (20) and (21b) converge after five iterations. We find that the maximum of the current-density \( J(0) \) and the half-width at half-maximum of the current-density \( \Delta x \) increases as \( \zeta \) increases. For this case we find \( dB_{\text{tot}}/dx \approx 0.412 \). The energy spectrum of the accelerated leptons is similar to equation (24).

### 3.3. Scalings

In more detail, we find the scaling relations

\[ J(0) \approx 1.53 \zeta \mathcal{E}/B', \]  

(25a)

and

\[ \Delta x \approx 1.48\mathcal{E}/\sqrt{B'}, \]  

(25b)

where \( B' \equiv dB_{\text{tot}}/dx|_{x=0} \). The first relation can be derived from the analytic orbits of §2.3. For the external magnetic field \( B_{\text{tot}} = x \), we have from §2.4,

\[ \lim_{x \to 0} J(x) = \lim_{\zeta} \int_{0}^{x} \frac{dx_0}{dx/dz} = \lim_{\zeta} \int_{0}^{x} \frac{dx_0 \sqrt{2z}}{x_0 I_1(2\sqrt{z}/\mathcal{E})} \]

\[ = \zeta \mathcal{E} \int_{0}^{\infty} dz' \frac{1}{I_0(2\sqrt{z}/\mathcal{E})} \approx 1.53\zeta \mathcal{E}, \]

(26)

where the relation between \( x_0, z_0, \) and \( z \) is given by equation (12). In the magnetic field \( B_{\text{tot}} = B'x \), equation (26) is modified by the replacement \( \mathcal{E} \to \mathcal{E}/B' \). Equation (25b) is empirical.

We can use equations (25a) and (25b) to obtain an approximate constitutive equation for the current layer. Note that \( dB_{\text{tot}}/dx|_{0} = 1 + dB_{\text{self}}/dx|_{0} \), and that \( dB_{\text{self}}/dx|_{0} = -2 \int dx [J(0) - J(x)]/x^2 \). Notice in turn that this integral is proportional to \( J(0)/\Delta x \). From this we obtain the relation

\[ \zeta \approx 0.205 \left[ (B')^{1/2} - (B')^{3/2} \right]. \]

(27)

This dependence is shown in Figure 6 along with the calculated equilibria. The left-hand part of the curve is dashed because we have not found the corresponding self-consistent equilibria. The maximum of \( \zeta \) is thus \( \zeta_{\text{max}} \approx 0.08 \). This maximum represents a maximum inflow rate to the reconnection region analogous to that found by Burkhart et al. (1991).

### 3.4. Apparent Spectra

The observed lepton distribution \( f(\gamma) \) deduced from the synchrotron spectrum of a radio source will in general be different from the distribution of accelerated particles at a given reconnection site \( (dn/d\gamma) \). For a typical radio source \( f \propto \gamma^{-2.5} \). One effect is due to the distribution of \( \Gamma_0 \) values owing mainly to the distribution of \( L \) for different reconnection sites. Note that \( k = 2\pi/L \) is the wavenumber of the spatial power spectrum of the B field. With the distribution \( W(\Gamma_0) \), the average lepton distribution function \( f(\gamma) \) for many reconnection sites is

\[ f(\gamma) = \int d\Gamma_0 W(\Gamma_0) \frac{dn(\gamma|\Gamma_0)}{d\gamma}. \]

(28)