Strong interaction effects in semileptonic $B$ decays*

Nikolai Uraltsev INFN, Sezione di Milano, Milan, Italy;
Department of Physics, University of Notre Dame du Lac, Notre Dame, IN 46556, U.S.A.;
St. Petersburg Nuclear Physics Institute, Gatchina, St. Petersburg 188300, Russia

Strong interaction effects are addressed in connection to extracting $|V_{cb}|$. A comprehensive approach is described not relying on a $1/m_c$ expansion; it allows a percent accuracy without ad hoc assumptions about higher-order effects. An alternative to the $M_X^2$ variable is proposed improving convergence. Intrinsic hardness of integrated observables with a cut on $E_\ell$ is discussed; it can be responsible for the behavior of $\langle M_X^2 \rangle$ reported by BaBar. Consequences of the proximity to the ‘BPS’ limit are considered.

The heavy quark (HQ) expansion, a first-principle QCD application of the Wilsonian OPE to heavy quarks has yielded novel insights into the dynamics of heavy flavor hadrons. (For a review, see [1] and references therein.) An important phenomenological application of the heavy quark expansion is extracting $|V_{cb}|$ and $|V_{ub}|$ from measured decay rates with high accuracy and little model dependence. This requires a genuine control over nonperturbative effects in $B$ decays.

A popular method to determine $|V_{cb}|$ uses the decay rate $B \to D^* \ell \nu$ near zero recoil. At this kinematic point the $B \to D^*$ formfactor $F_{D^*}(0)$ is unity when $m_b, m_c \to \infty$. Driven by the charm mass scale, power corrections are still significant: $F_{D^*} \simeq 0.9$ to order $1/m_Q^2$ [2], and the $1/m_c^2$ effects were estimated to be in the 3% range [3]. Relying on expansion in $1/m_c$ makes it difficult to overcome a 5% level of accuracy here without compromising reliability of theoretical predictions.

These estimates were supported by recent lattice studies which yielded, as central values, surprisingly close numbers, $F_D \simeq 0.88$ and $F_{D^*} \simeq 0.91$ to order $1/m_Q^2$ and $1/m_c^3$, respectively [4]. Since the method is based on $1/m_Q$ expansion for both $b$ and $c$, an important issue is higher-order as well as exponential in $m_c$ terms. This sophisticated lattice approach will hopefully be refined in the future. Presently a large fraction of the corrections to $F_{D^*}(0) = 1$ is still added theoretically rather than emerges directly in the lattice simulations.

On experimental side, extrapolating the decay amplitude to zero recoil introduces additional uncertainty. (A parametrization relied upon is alleged to be rigorously derived from QCD. This is not true.) It can be reduced incorporating the model-independent inequalities for the slope of the IW function stemming from the set of the HQ sum rules; however this has not yet been implemented in the experimental analyses.

**Inclusive decays and HQ expansion.** More extensive opportunities are provided by inclusive semileptonic $B$ decays. Total semileptonic decay rate $\Gamma_{\ell\ell}(B)$ is now one of the best measured quantities in $B$ physics. Theory-wise nonperturbative effects are controlled by the 10 year old QCD theorem [5] which established absence of the leading $\Lambda_{QCD}/m_b$ power corrections to total decay rates. It applies to all sufficiently inclusive decay probabilities, not only semileptonic. Moreover, the theorem relates the inclusive $B$ widths to (short-distance) quark masses and expectation values of local $b$-quark operators in actual $B$ mesons. The general expansion parameter for inclusive decays is energy release, in $b \to c \ell \nu$ it constitutes $m_b - m_c \simeq 3.5$ GeV.

Heavy quark masses are full-fledged QCD parameters entering various hadronic processes. The expectation values like $\mu_c^2$ determining the width in the OPE likewise enjoy the status of observable parameters. As pointed out shortly after, the masses and relevant nonperturbative parameters can be determined from the $B$ decay distributions themselves [6,7]. Nowadays this strategy is being implemented in a number of experimental studies.

The new generation of data provides accurate measurements of many inclusive characteristics in $B$ decays. It is also encouraging that proper theoretical formalism gradually finds its way into

their analyses. Recent theoretical findings allow to shrink theoretical uncertainties – among them constraints from the exact HQ sum rules and the consequences of the proximity to the so-called ‘BPS’ regime signified by the hierarchy \( \mu_G^2 - \mu_G^2 \ll \mu_G^2 \) suggested by experiment.

Present theory allows to aim a percent accuracy in \( |V_{cb}| \). Such a precision becomes possible due to a number of theoretical advances. The low-scale running masses \( m_b(\mu) \), \( m_c(\mu) \), the expectation values \( \mu^2(\mu) \), \( \mu^2(\mu) \) are completely defined and can be determined from experiment with an in principle unlimited accuracy. Violation of local duality potentially limiting theoretical predictability, has been scrutinized and found to be negligibly small in total semileptonic \( B \) widths \[8\]. Present-day perturbative technology makes computing \( \alpha_s \)-corrections to the Wilson coefficients of nonperturbative operators feasible. It is also understood how to treat higher-order power corrections.

High accuracy can be achieved in a comprehensive approach where many observables are measured in \( B \) decays to extract necessary ‘theoretical’ input parameters. This can be compared with early days of the heavy quark expansion when experiment aimed mainly at measuring \( \Gamma_{sl}(B) \), while the rest had to be supplied by theory. This limited the accuracy of \( |V_{cb}| \) in the mid 1990s by about 5%.

With \( b \to c \) widths depending strongly on \( m_b - m_c \), previous analyses to some extent relied on expansion in \( 1/m_c \) since employed the relation

\[
m_b - m_c = \frac{\rho_D^2}{6} \left( \frac{1}{m_c} - \frac{1}{m_b} \right) + \frac{\rho_D^3}{4} \left( \frac{1}{m_c^2} - \frac{1}{m_b^2} \right) + O \left( \frac{1}{m_Q^3} \right).
\]

Reliability of the \( 1/m_c \) expansion is however questionable. Already for the \( 1/m_Q^2 \) terms above one has \( \frac{1}{m_c^2} > 14 \frac{1}{m_b^2} \); even for the worst mass scale in the width expansion, \( \frac{1}{m_b^2} \) is at least 8 times smaller than \( \frac{1}{m_Q^2} \). On top of that there are indications \[9\] that the nonlocal correlators affecting meson masses can be particularly large – a pattern also observed in the ‘t Hooft model \[10\]. This expectation is supported by the pilot lattice study \[11\] which – if taken at face value – suggests a very large value of \( \rho_{D}^3 + \rho_{D}^2 \). On the other hand, non-local correlators are not measured in inclusive \( B \) decays.

A partial cure to this problem was suggested recently \[9\]: The proximity to the ‘BPS’ limit leads to much smaller power corrections for the analogue of the mass relation \( \mu \) applied to ground-state mass difference \( M_B - M_D \) than in the standard spin-averaged masses. Since in the conventional approach this is the major source of uncertainty, pseudoscalar meson masses should be rather used to constrain \( m_b - m_c \).

Many recent extractions of \( |V_{cb}| \) from \( \Gamma_{sl}(B) \) relied on strong – and probably unjustified – assumptions about “six hadronic \( D = 6 \) parameters” appearing in order \( 1/m_Q^2 \). This led to the lore that the uncertainties in \( |V_{cb}| \) are by far dominated by theory. In fact this depends on the perspective adopted, and could be traced to the ‘eclectic’ approach where only \( \Gamma_{sl}(B) \), \( \bar{\mu} \) and \( \mu^2_\pi \) are relegated to experiment, whereas all the remaining information must come from theory. Since theory itself warns that the mass relation \( \mu \) for charm is the weakest point, and a number of rigorous theoretical constraints are disregarded here, such an approach clearly cries for improvement.

Fortunately, there is a way totally free from relying on charm mass expansion; the validity of the latter can rather be examined a posteriori. It was put forward some time ago \[12\] to utilize the power of the comprehensive approach and makes full use of a few key facts \[7, 8\]:

- Total width to order \( 1/m_b^2 \) is affected by a single new Darwin operator (its expectation value is \( \rho^1_D \)); the moments also weakly depend on \( \rho^1_{LS} \).
- No nonlocal correlators ever enter per se.
- Deviations from the HQ limit in the expectation values are driven by the full scale \( 1/m_b^2 \) (and are additionally suppressed by proximity to the BPS limit); they are negligible in practice.
- Exact sum rules and inequalities which hold for properly defined Wilsonian parameters.

Some of the HQ parameters like \( \mu^2_G \) are known beforehand. Proper field-theoretic definition allows its accurate determination from the \( B^* - B \) mass splitting: \( \mu^2_G (1 \text{ GeV}) = 0.35^{+0.03}_{-0.02} \text{ GeV}^2 \) \[9\]. A priori less certain is \( \mu^2_\pi \). However, the inequality \( \mu^2_\pi > \mu^2_G \) valid for any definition of kinetic and chromomagnetic operators respecting the QCD commutation relation \( [D_j, D_k] = -ig_sG_{jk} \), and the corresponding sum rules essentially limit its range: \( \mu^2_\pi (1 \text{ GeV}) = 0.45 \pm 0.1 \text{ GeV}^2 \).

Running \( b \) quark mass was accurately extracted
from $\sigma (e^+ e^- \to \Upsilon(nS))$ in the end of the 1990s: $m_b(1 \text{ GeV}) = 4.57 \pm 0.06 \text{ GeV}$ for the “kinetic” $m_b(\mu)$. However, considering all available constraints, I think that 4.57 GeV is on the lower side of the $m_b$ range centered rather around 4.63 GeV.

Often extracted from the data are the “HQET parameters” ($-\lambda_1, \Lambda$) – they actually correspond to extrapolating the $\mu$-dependent quantities down to $\mu = 0$. They are ill-defined and make no sense out of the context of a concrete computation; they are meaningful only as intermediate stage entries. However, a translation can often be made into properly defined parameters. Say, in the context of the recent CLEO analyses it reads

$$\Lambda_{\text{HQET}} \simeq \Lambda(1 \text{ GeV}) - 0.255 \text{ GeV}$$

$$-\lambda_1 \simeq \mu_b^2(1 \text{ GeV}) - 0.18 \text{ GeV}^2.$$ (2)

The central values quoted by CLEO [13] thus correspond to $m_b(1 \text{ GeV}) = 4.62 \text{ GeV}$, $\mu_b^2(1 \text{ GeV}) = 0.43 \text{ GeV}^2$, surprisingly close to the theoretical expectations!

**Lepton and hadron moments.** Moments of the charged lepton energy in the semileptonic $B$ decays are traditional observables to measure heavy quark parameters. New at this conference are DELPHI results for the first three moments. Two moments with the lower cut at $E_\ell = 1.5 \text{ GeV}$ or $E_\ell = 1.7 \text{ GeV}$ are presented by CLEO, who also measured average photon energy $\langle E_\gamma \rangle$ subject to constraint $E_\gamma > 2 \text{ GeV}$.

Another useful set of observables are moments of the invariant hadronic mass squared $M^2_\chi$ in semileptonic decays. Their utility follows from the fact [7] that, at least if charm were heavy enough the first, second and third moments would more or less directly yield $\bar{\Lambda}$, $\mu_b^2$ and $\bar{\rho}_D$. The hadronic moments were measured in different settings by DELPHI, CLEO and Babar. The details can be found in the original experimental talks [13–16].

Let me now illustrate how this strategy works number-wise. Lepton energy moments, for instance, are given by the following approximate expressions ($b \to u$ decays are neglected):

$$\langle E_\ell \rangle = 1.38 \text{ GeV} + 0.38 [(m_b - 4.6 \text{ GeV}) - 0.7(m_c - 1.15 \text{ GeV})] + 0.03(\mu_e^2 - 0.4 \text{ GeV}^2)$$

$$- 0.09(\bar{\rho}_D - 0.12 \text{ GeV}^3),$$

$$\langle (E_\ell - \langle E_\ell \rangle)^2 \rangle = 0.18 \text{ GeV}^2 + 0.1 [(m_b - 4.6 \text{ GeV}) - 0.6(m_c - 1.15 \text{ GeV})] + 0.045(\mu_e^2 - 0.4 \text{ GeV}^2)$$

$$- 0.06(\bar{\rho}_D - 0.12 \text{ GeV}^3),$$

$$\langle (E_\ell - \langle E_\ell \rangle)^3 \rangle = -0.033 \text{ GeV}^3 - 0.03 [(m_b - 4.6 \text{ GeV}) - 0.8(m_c - 1.15 \text{ GeV})] + 0.024(\mu_e^2 - 0.4 \text{ GeV}^2)$$

$$- 0.035(\bar{\rho}_D - 0.12 \text{ GeV}^3).$$ (3)

The moments depend basically on one and the same combination of masses $m_b - 0.65 m_c$: dependence on $\mu_b^2$ is rather weak. To even larger extent this applies to the CLEO’s cut moments $R_1$, $R_2$ and the ratio $R_0$ – they depend practically on a single combination $m_b - 0.63 m_c + 0.3 \mu_b^2$. The effect of the spin-orbital average $\bar{\rho}_L \bar{\rho}_S$ is negligible.

Now take a look at $|V_{cb}|$. Its value extracted from $\Gamma_{sl}(B)$ has the following dependence on the HQ parameters:

$$\frac{|V_{cb}|}{0.042} = 1 - 0.65 [(m_b - 4.6 \text{ GeV}) - 0.61(m_c - 1.15 \text{ GeV})]$$

$$+ 0.013(\mu_e^2 - 0.4 \text{ GeV}^2) + 0.1(\bar{\rho}_D - 0.12 \text{ GeV}^3)$$

$$+ 0.06(\bar{\rho}_D^2 - 0.35 \text{ GeV}^2) - 0.01(\bar{\rho}_L \bar{\rho}_S + 0.15 \text{ GeV}^3) =$$

$$1 - 0.65 \times [(\mu_e^2 - 1.38 \text{ GeV}) - 0.06(m_c - 1.15 \text{ GeV})]$$

$$- 0.07(\mu_e^2 - 0.4 \text{ GeV}^2) - 0.05(\bar{\rho}_D^2 - 0.12 \text{ GeV}^3) - 0.08(\bar{\rho}_D^2 - 0.35 \text{ GeV}^2) - 0.005(\bar{\rho}_L \bar{\rho}_S + 0.15 \text{ GeV}^3);$$ (4)

a combination of the parameters has been replaced by the first lepton moment in Eq. (3), and the sensitivity to $\mu_e^2$ and $\bar{\rho}_D$ is illustrated. If we do a similar exercise for the lepton moment $R_1$ with the cut at $E_\ell > 1.5 \text{ GeV}$, the coefficients giving the remaining sensitivity of $|V_{cb}|$ to $m_c$, $\mu_e^2$ and $\bar{\rho}_D$ will become 0.02, 0.19 and 0.13, respectively. We see that the precise value of charm mass is irrelevant, but reasonable accuracy in $\mu_e^2$ and $\bar{\rho}_D$ is required.

The first hadronic moment takes the form

$$\langle M^2_\chi \rangle = 4.54 \text{ GeV}^2 - 5.0 [(m_b - 4.6 \text{ GeV}) - 0.62(m_c - 1.15 \text{ GeV})]$$

$$- 0.66(\mu_e^2 - 0.4 \text{ GeV}^2) + (\bar{\rho}_D^2 - 0.12 \text{ GeV}^3),$$ (5)

i.e., given by nearly the same combination $m_b - 0.7 m_c + 0.1 \mu_e^2 - 0.2 \bar{\rho}_D^2$ as the lepton moment. Not very constraining, it provides, however a highly nontrivial check of the HQ expansion. These two first moments together, for example verify the heavy quark sum rule for $M_B - m_b$ with the accuracy about 40 MeV! In this respect it is more elaborate than the higher lepton moments.

The dependence expectedly changes for higher hadronic moments:

$$\langle (M^2_\chi - \langle M^2_\chi \rangle)^2 \rangle = 1.2 \text{ GeV}^4 - 0.003(m_b - 4.6 \text{ GeV})$$

$$- 0.68(m_c - 1.15 \text{ GeV}) + 4.5(\mu_e^2 - 0.4 \text{ GeV}^2)$$

$$- 5.5(\bar{\rho}_D^2 - 0.12 \text{ GeV}^3),$$
\[
\langle (M_X^2 - \langle M_X^2 \rangle)^3 \rangle = 4 \text{GeV}^6 + (m_b - 4.6 \text{ GeV}) \\
- 3 (m_c - 1.15 \text{ GeV}) + 5 (\mu^2_\pi - 0.4 \text{ GeV}^2) \\
+ 13 (\rho^2_D - 0.12 \text{ GeV}^3). \tag{6}
\]

Ideally, they would measure the kinetic and Darwin expectation values separately. At the moment, however, we have only an approximate evaluation and informative upper bound on $\mu^3_D$. The current sensitivity to $\mu^2_\pi$ and $\rho^3_D$ is about 0.1 GeV$^2$ and 0.1 GeV$^3$, respectively.

We see that measuring the second and third hadronic moments is the real step in implementing the comprehensive program of extracting $|V_{cb}|$. Clearly, more work – both theoretical and experimental – is required to fully use its power. It is crucial that this extraction carries no hidden assumptions, and at no point we rely on $1/m_c$ expansion. Charm quark could be either heavy, or light as strange or up quark, without deteriorating – and even improving the accuracy!

**Experimental cuts and hardness.** There is a problem, however, which should not be underestimated. The intrinsic ‘hardness’ of the moments deteriorates when the cut on $E_\ell$ is imposed. As a result, say the extraordinary experimental accuracy of CLEO’s $R_0-R_2$ cannot be even nearly utilized by theory, whether or not the expressions we use make this explicit.\(^2\)

For total widths the effective energy scale parameter is generally $Q = m_b - m_c$. When OPE applies we can go beyond purely qualitative speculations about hardness. Then it is typically given by $Q \lesssim \omega_{\text{max}}$, with $\omega_{\text{max}}$ the threshold energy at which the decay process disappears once $m_b$ is replaced by $m_b - \omega$. With the $E_\ell > E_{\text{min}}$ cut then

\[
Q \simeq m_b - E_{\text{min}} - \sqrt{E_{\text{min}}^2 + m_c^2} \tag{7}
\]

constituting only meager 1.25 GeV for $E_{\text{min}} = 1.5$ GeV, and falls even below 1 GeV for the decays with $E_\ell > 1.7$ GeV. This may explain the unexpected behavior of the first hadronic moment with respect to the cut on $E_\ell$ reported by BaBar earlier at this session [15].

In $b \to s + \gamma$ decays one has $Q \simeq m_b - 2E_{\text{min}}$, once again a rather soft scale 1.2 GeV if the lower cut is set at $E_\gamma = 2$ GeV. Hence, the reliability of theory can be questioned when one aims for maximum precision. For higher moments the hardness further deteriorates in either decays. A high premium then should be placed for lowering the cuts [17].

On the theoretical side, the higher hadronic moments can be affected by nonperturbative physics formally scaling as powers of $1/m_b$ greater than 3. At the same time, these moments are instrumental for a truly model-independent comprehensive studies of $B$ mesons; improvement is needed already for the third moment, its expression given above is not too accurate. Considering alternative kinematic variables will help to improve the convergence. Namely, it is advantageous to trade the traditional hadronic mass $M_X^2$ for the observable more closely corresponding to the quark virtuality $\Delta$, defined as

\[
N_X^2 = M_X^2 - 2\Lambda E_X, \tag{8}
\]

where $E_X = M_B - q_0$ is the total hadronic energy in the $B$ restframe, and $\Lambda$ a fixed mass parameter. Its preferred values are about $M_B - m_B(1$ GeV$)$ and can be taken 600–700 MeV. The higher moments $\langle(N_X^2 - \langle N_X^2 \rangle)^2 \rangle, \langle(N_X^2 - \langle N_X^2 \rangle)^3 \rangle...$ should enjoy better theoretical stability.

The kinematic variable $N_X^2$ is not well constrained inclusively at LEP experiments, however can be used in the $B$ threshold production at CLEO and $B$ factories.\(^3\) This possibility should be carefully explored.

**BPS limit.** An intriguing theoretical environment opens up if $\mu^2_\pi(1$ GeV$)$ is eventually confirmed to be close enough to $\mu^2_G(1$ GeV$)$ as currently suggested by experiment, say it does not exceed 0.45 GeV$^2$. If $\mu^2_\pi - \mu^2_G \ll \mu^2_\pi$ it is advantageous to analyze strong dynamics expanding around the point $\mu^2 = \mu^2_G$ [9]. This is not just one point of a continuum in the parameter space, but a quite special ‘BPS’ limit where the heavy flavor ground state satisfies functional relations $\tilde{\sigma}(B) = 0$. This limit is remarkable in many respects, for example, saturates the bound [18] $g^2 - \frac{3}{4}$ for the slope of the IW function. In some instances like the $B \to D$ zero-recoil amplitude it extends the heavy flavor (but not spin) symmetry to higher orders in $1/m_Q$. One of its practical application has been mentioned – the robust relation for $m_b - m_c$ via $M_B - M_D$. Exclusive $B \to D^*$ decay can also benefit from the proximity to BPS. The exact spin sum rules yield a constraint on the

\(^2\) An instructive example of how naive analysis can miss such effects was given in [17], Sect. 5.

\(^3\) I am grateful to experimental colleagues for discussing this point.
IW slope
\[ \mu_{\pi}^2 - \mu_{\rho}^2 = 3\varepsilon^2(\varphi^2 - \frac{3}{2}) \leq 0.45 \text{ GeV} \leq \varepsilon \leq 1 \text{ GeV} \] (9)
thus leaving only a small room for the slope of the actual \( B \to D^* \) formfactor, excluding values of \( \varphi^2 \) exceeding \( 1.15-1.2 \). This would be a very constraining result for a number of experimental studies.

Conclusions. Experiment has entered a new era of exploring \( B \) physics at the nonperturbative level, with qualitative improvement in \( |V_{cb}| \). The comprehensive approach will allow to reach a percent level of reliable accuracy in translating \( \Gamma_{\pi}(B) \) to \( |V_{cb}| \). Recent experiments have set solid grounds for dedicated future studies at \( B \) factories. We already observe a nontrivial consistency between quite different measurements, and between experiment and QCD-based theory.

There are obvious lessons to infer. Experiment must strive to weaken the cuts in inclusive measurements used in extracting \( |V_{cb}| \). Close attention should be paid to higher moments or their special combinations, as well as exploring complementary kinematic observables.

The theory of heavy quark decays is now a mature branch of QCD – still there are a number of directions where it can be developed further. I believe that the analyses presented at the Conference should provide theorists with substantial motivation to refine it at least in the following:

- Calculating perturbative corrections to Wilson coefficients of subleading operators.
- Scrutiny of higher-order power corrections.
- A thorough study of alternative kinematic variables, for instance moments of \( N_2 \).

To fully realize the physical information in the quest for the ultimate precision, a truly comprehensive analysis must implement all theoretical constraints on HQ parameters; the suitable framework uses well-defined running parameters having physical meaning. Heavy quark sum rules appear to yield strong constraints on the parameter space; it is important to study the question of their saturation. If a low \( \mu^2 \) around \( 0.45 \text{ GeV}^2 \) is confirmed by experiment, the BPS expansion will play an important role in analyzing nonperturbative effects – in particular, guide us through higher-order corrections.

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REFERENCES

13. D. Cronin-Hennessy (CLEO collab.), HQ-1-4 (these proceedings); hep-ex/0209024.
14. M. Calvi (DELPHI collab.), HQ-2-1 (these proceedings); DELPHI 2002-070 CONF 604;
15. V. Luth (BaBar collab.), HQ-1-5 (these proceedings); hep-ex/0207084.
16. M. Battaglia, CP-2-2 (these proceedings).