In some models of quantum gravity, space-time is thought to have a foamy structure with non-trivial optical properties. We probe the possibility that photons propagating in vacuum may exhibit a non-trivial refractive index, by analyzing the times of flight of radiation from gamma-ray bursters (GRBs) with known redshifts. We use a wavelet shrinkage procedure for noise removal and a wavelet ‘zoom’ technique to define with high accuracy the timings of sharp transitions in GRB light curves, thereby optimizing the sensitivity of experimental probes of any energy dependence of the velocity of light. We apply these wavelet techniques to 64 ms and TTE data from BATSE, and also to OSSE data. A search for time lags between sharp transients in GRB light curves in different energy bands yields the lower limit $M \geq 6.9 \cdot 10^{15}$ GeV on the quantum-gravity scale in any model with a linear dependence of the velocity of light $E/M$. We also present a limit on any quadratic dependence.

**Introduction**

In standard relativistic quantum field theory, space–time is considered as a fixed arena in which physical processes take place. The characteristics of the propagation of light are considered as a classical input to the theory. In particular, the special and general theories of relativity postulate a single universal velocity of light $c$. However, starting in the early 1960s (Wheller 1963), efforts to find a synthesis of general relativity and quantum mechanics, called quantum gravity, have suggested a need for greater sophistication in discussing the propagation of light in vacuum.

A satisfactory theory of quantum gravity is likely to require a drastic modification of our deterministic representation of space–time, endowing it with structure on characteristic scales approaching the Planck length $\ell_P \simeq m_P^{-1}$. There is at present no complete mathematical model for quantum gravity, and there are many different approaches to the modelling of space-time foam. Several of these approaches suggest that the vacuum acquires non-trivial optical properties, because of gravitational recoil effects induced by the motion of energetic particles. In particular, it has been suggested that these may induce a non-trivial refractive index, with photons of different energies travelling at different velocities. Such an apparent violation of Lorentz invariance can be explored by studying the propagation of particles through the vacuum, in particular photons emitted by distant astrophysical sources (Amelino-Camelia et al. 1998). In some quantum-gravity models, light propagation may also depend on the photon polarization (Gambini & Pullin 1999), inducing birefringence. Stochastic effects are also possible, giving rise to an energy-dependent diffusive spread in the velocities of different photons with the same energy (Ford 1995; Ellis et al. 2000a).

One may discuss the effects of space–time foam on the phase velocity, group velocity or wave-front velocity of light. In this paper, we discuss only the signature of a modification of the group velocity, related to a non-trivial refractive index $n(E)$: $v(E) = c/n(E)$. This may be derived theoretically from a (renormalized) effective Maxwell action $\Gamma_{\text{eff}}[E, B]$, where $E$ and $B$ are the electric and magnetic field strengths of the propagating wave, in the background metric induced by the quantum gravity model under consideration. Once the effective Maxwell action is known, at least in a suitable approximation, one can analyze the photon dispersion using the effective Maxwell equations (Ellis et al. 2000b).

One generally considers the propagation of photons with energies $E$ much smaller than the mass scale $M$ characterizing the quantum gravity model, which may be of the same order as the Planck mass $M_P$, or perhaps smaller in models with large extra dimensions. In the approximation $E \ll M$, the distortion of the standard photon dispersion relation may be represented as an expansion in $E/M$:

$$E^2 = k^2(1 + \xi_1(k/M) + \xi_2(k/M)^2 + \ldots),$$

**Equation**