Constraining dark energy from the abundance of weak gravitational lenses

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ABSTRACT
We examine the prospect of using the observed abundance of weak gravitational lenses to constrain the equation-of-state parameter \( w = p/\rho \) of the dark energy. Dark energy modifies the distance-redshift relation, the amplitude of the matter power spectrum, and the rate of structure growth. As a result, it affects the efficiency with which dark-matter concentrations produce detectable weak-lensing signals. Here we solve the spherical-collapse model with dark energy, clarifying some ambiguities found in the literature. We also provide fitting formulas for the non-linear overdensity at virialization and the linear-theory overdensity at collapse. We then compute the variation in the predicted weak-lens abundance with \( w \). We find that the predicted redshift distribution and number count of weak lenses are highly degenerate in \( w \) and the present matter density \( \Omega_0 \). If we fix \( \Omega_0 \) the number count of weak lenses for \( w = -2/3 \) is a factor of \( \sim 2 \) smaller than for the \( \Lambda \)CDM model \( w = -1 \). However, if we allow \( \Omega_0 \) to vary with \( w \) such that the amplitude of the matter power spectrum as measured by the Cosmic Background Explorer (COBE) matches that obtained from the X-ray cluster abundance, the decrease in the predicted lens abundance is less than 25\% for \(-1 \leq w < -0.4 \). We show that a more promising method for constraining the dark energy—one that is largely unaffected by the \( \Omega_0 - w \) degeneracy as well as uncertainties in observational noise—is to compare the relative abundance of virialized X-ray lensing clusters with the abundance of non-virialized, X-ray underluminous, lensing halos. For aperture sizes of \( \sim 15 \) arcmin, the predicted ratio of the non-virialized to virialized lenses is greater than 40\% and varies by \( \sim 20\% \) between \( w = -1 \) and \( w = -0.6 \). Overall, we find that if all other weak lensing parameters are fixed, a survey must cover at least \( \sim 40 \) square degrees in order for the weak lens number count to differentiate a \( \Lambda \)CDM cosmology from a dark-energy model with \( w = -0.9 \) at the 3\( \sigma \) level. If, on the other hand, we take into account uncertainties in the lensing parameters, then the non-virialized lens fraction provides the most robust constraint on \( w \), requiring \( \sim 50 \) square degrees of sky coverage in order to differentiate a \( \Lambda \)CDM model from a \( w = -0.6 \) model to 3\( \sigma \).

Key words: galaxy clusters—weak gravitational lensing—cosmology

1 INTRODUCTION
Observations of distant type Ia supernovae (SNIa) indicate that the universe is undergoing a phase of accelerated expansion (Perlmutter et al. 1999, Riess 1998). This, combined with the flat geometry favored by the cosmic microwave background (CMB) measurements (Miller et al. 1999, de Bernardis et al. 2002, Halverson et al. 2002, Sievers et al. 2002, Lee et al. 2001) and the evidence for a low matter-density with \( \Omega_0 \sim 0.3 \) (Peacock 2001, Percival et al. 2001), suggests that the bulk of the total energy density of the universe is in the form of some exotic dark energy with a negative equation of state. One of the primary objectives of cosmology today is to uncover the origin and nature of this dark energy.

A possible candidate for the dark energy is a cosmological constant \( \Lambda \), with an equation of state \( w = p/\rho \) (where \( p \) is the pressure and \( \rho \) is the energy density of the dark energy) strictly equal to \(-1 \). Another possibility, and one that may find favor from a particle-physics point of view, is a dynamical scalar field, termed quintessence, \( Q \). Unlike the cosmological constant, the \( Q \)-component is both time-dependent and spatially inhomogeneous with an equation of state \( w > -1 \) that is likely to be redshift dependent. Determining the value of \( w \) and how it changes with time are key to constraining the nature of the dark energy.
While the accelerating expansion implies only that $w < -1/3$, combinations of CMB data, SNIa data, and large-scale-structure data suggest that $w$ is most likely in the range $-1 < w < -0.6$ (Wang et al. 2000, Huterer & Turner 2001, Bean & Melchiorri 2002, Baccigalupi et al. 2002). Though combining these different data sets have provided some constraint on $w$, how $w$ should vary with redshift is largely unknown. Particle physics offer several possible functional forms for the quintessence field’s potential $V(Q)$ and hence possible scenarios for the time history of $w$. Nonetheless, determining $w$’s redshift evolution observationally is likely to be very challenging (Barger & Marfatia 2001, Maor et al. 2001, Weller & Albrecht 2001).

Strengthening the measured constraint on $w$ and perhaps excluding the cosmological constant as the source of the dark energy appear, however, to be attainable goals within the near future. Since the dark-energy dynamics influences both the evolution of the background cosmology and the growth of structure, it directly affects many observables. Its modification of the angular-diameter distance, the luminosity distance, and the amplitude of the matter power spectrum, are the primary sources of dark-energy constraint in measurements of CMB anisotropies, SNIa, and local cluster abundances, respectively.

In this paper we consider another possible means of constraining $w$: measurement of weak gravitational-lens abundances. Weak lensing—the weak distortion of background-galaxy images due to the deep gravitational potential of an intervening overdensity—provides a powerful technique for mapping the distribution of matter in the universe (see reviews by Bartelmann & Schneider 2001, Mellier 1999). Here we study the impact of the dark energy on the predicted redshift distortion and sky density of weak lenses. Dark energy affects the abundance of weak lenses by not only modifying the distance-redshift relation and the matter power spectrum but also by altering the rate of structure growth. In particular, the larger $w$ is the faster and earlier objects collapse. An interesting consequences of this is that if we separate weak lenses into the two observational classes—those that have collapsed and reached virial equilibrium and are therefore X-ray luminous and those that are non-virialized and hence X-ray underluminous (Weinberg & Kamionkowski 2002; hereafter WK02)—the abundance of one class evolves slightly differently from the other. Therefore the relative fraction of these two types of lenses varies with $w$. This observable is especially promising as compared to measurements of absolute abundances because it is less sensitive to uncertainties in both the cosmological parameters and the noise in the lensing map.

This paper is organized as follows. In Section 2 we briefly summarize the weak-lensing signal-to-noise estimator and discuss how we determine the mass- and redshift-dependent minimum overdensity required to produce a detectable weak-lensing signal. Section 3 is devoted to the spherical-collapse model in quintessence cosmologies. We provide fitting formulas for the non-linear overdensity at virialization and the linear-theory density at collapse and describe our approach to normalizing the matter power spectrum. In Section 4 we show the resulting effect the dark energy has on the weak-lens abundances and in Section 5 we present our conclusions.

Finally, we note that a similar analysis has recently been performed by Bartelmann, Perrotta & Baccigalupi (2002), although not for the case of non-virialized lenses. Although we agree with their general conclusion that the weak lens abundance is a potentially sensitive probe of the dark energy, our results differ from their results in important details. We discuss these differences in Section 4.2.

## 2 MINIMUM OVERDENSITY NEEDED TO PRODUCE DETECTABLE LENSING SIGNAL

In order to compute the abundance of weak gravitational lenses for dark-energy cosmologies we must first determine the necessary conditions for a halo of a given density profile and redshift to produce a detectable weak-lensing signal. Of course the more overdense a halo is relative to the background density the more it coherently distorts the nearby background galaxies and hence the stronger its lensing signal. The detectability of this signal is hampered, however, by noise in the weak-lensing map, primary of which is the intrinsic ellipticity distribution of the background galaxies. The goal is therefore to determine the minimum overdensity a halo must have such that it produces a sufficiently large signal relative to the noise so as to be detectable. A convenient method for computing this minimum overdensity is provided by Schneider’s (1996) aperture-mass technique.

Consider a lens at redshift $z_l$ of surface mass density $\Sigma(\vartheta)$ within an angular radius $\vartheta$. For a source at redshift $z_s$ the convergence $\kappa$ is given by,

$$\kappa(\vartheta) = \frac{\Sigma(\vartheta)}{2\Sigma_{\text{crit}}} \Sigma_{\text{crit}} = \frac{c^2}{4\pi G} \frac{D_s}{D_l D_{ls}},$$  \hspace{1cm} (1)

where $D_l$, $D_s$ and $D_{ls}$ are the angular-diameter distances between the lens and the observer, the source galaxy and the observer, and the lens and the source, respectively. Following Schneider (1996), define a spatially-filtered mass inside a circular aperture of angular radius $\theta$,

$$M_{ap} (\theta) \equiv \int d^2\vartheta \kappa(\vartheta) U(|\vartheta|),$$ \hspace{1cm} (2)

where $U(\vartheta)$ is a continuous weight function that vanishes for $\vartheta > \theta$. If $U(\vartheta)$ is a compensated filter function,

$$\int_{0}^{\theta} d\theta \vartheta U(\vartheta) = 0,$$ \hspace{1cm} (3)

then $M_{ap}$ can be expressed in terms of the tangential component of the observable shear, $\gamma_t$,

$$M_{ap} (\theta) = \int d^2\vartheta \gamma_t(\vartheta) Q(|\vartheta|),$$ \hspace{1cm} (4)

where the function $Q$ is related to $U$ by

$$Q(\vartheta) = \frac{2}{\sigma^2} \int_{0}^{\theta} d\vartheta' \vartheta' U(\vartheta') - U(\vartheta).$$ \hspace{1cm} (5)

In this paper we use the $l = 1$ radial filter function from the family given in Schneider et al. (1998):

$$U(\vartheta) = \frac{9}{\pi \sigma^2} (1 - x^2)^2 \left(1 - x^2\right)^2, \hspace{1cm} Q(\vartheta) = \frac{6}{\pi \sigma^2} x^2 (1 - x^2),$$ \hspace{1cm} (6)

where $x = \theta / \theta$. Taking the expectation value over galaxy positions and taking into account the redshift distribution of source galaxies then gives,
\[ M_{\text{sp}}(\theta) = \langle Z \rangle \int d^2 \vartheta \langle \gamma_1(\vartheta) \rangle Q(|\vartheta|), \]  
where \( \langle \gamma_1(\vartheta) \rangle \) is the mean tangential shear on a circle of angular radius \( \vartheta \). The function \( \langle Z \rangle \), given by,

\[ \langle Z \rangle = \int dz_s \frac{p_s(z_s)Z(z_s; z_d)}{Z(z_s; z_d)}, \]

where \( p_s(z_s) \) is the redshift distribution of source galaxies and (Seitz & Schneider 1997)

\[ Z(z_s; z_d) \equiv \lim_{\Delta z_s \to \infty} \frac{\sum_{\delta>\delta_{\text{crit}}}(z_s)}{\sum_{\delta>\delta_{\text{crit}}}(z_s)} \]

allows a source with a known redshift distribution to be collapsed onto a single redshift \( z_s \) satisfying \( Z(z_s) = \langle Z \rangle \) (Seitz & Schneider 1997; Bartelmann & Schneider 2001). The source-redshift distribution is taken to be,

\[ p_s(z_s) = \frac{\beta z_s^2}{\Gamma(3/\beta)z_0^3} \exp \left[-(z_s/z_0)^\beta \right], \]

with \( \beta = 1.5 \) and mean redshift \( \langle z_s \rangle \approx 1.5z_0 = 1.2 \) (cf., Smail et al. 1995; Brainerd et al. 1996; Cohen et al. 2000). Finally, assuming the ellipticities of different images are uncorrelated it can be shown (cf., Kruse & Schneider 1999) that the dispersion \( \sigma(\theta) \) of \( M_{\text{sp}} \) is

\[ \sigma^2(\theta) = \frac{\pi \sigma^2}{n} \int \, d\vartheta \, \vartheta Q^2(\vartheta), \]  
where \( n \) is the number density of galaxy images and \( \sigma_\gamma \) is the dispersion in the galaxies’ intrinsic ellipticity. In this paper we assume \( n = 30 \) arcmin\(^{-2} \) and \( \sigma_\gamma = 0.2 \). The signal-to-noise ratio \( S \) within an aperture radius \( \theta \) is then given by,

\[ S = \frac{M_{\text{sp}}}{\sigma_M} = \frac{2 \langle Z \rangle \sqrt{\pi n} \int \, d\vartheta \, \vartheta \langle \gamma_1(\vartheta) \rangle Q(\vartheta)}{\sigma_\gamma} \sqrt{\int \, d\vartheta \, \vartheta Q^2(\vartheta)}. \]

The tangential shear at \( \vartheta, \langle \gamma_1(\vartheta) \rangle \), depends on the amplitude and shape of the lensing halo’s density profile. Bartelmann (1995) showed that \( \langle \gamma_1(\vartheta) \rangle = \kappa(\vartheta) - \langle \kappa(\vartheta) \rangle \), where \( \langle \kappa(\vartheta) \rangle \) is the dimensionless mean surface mass density on a circle of radius \( \vartheta \) and \( \kappa(\vartheta) \) is the dimensionless mean surface mass density within a circle of radius \( \vartheta \). In this paper we describe the mass density of lensing halos with the universal density profile introduced by Navarro, Frenk & White (1996; 1997; hereafter NFW). Thus, for an NFW halo at a given redshift with a given mass and mean overdensity relative to the background (\( \Delta \equiv \langle \rho_{\text{pert}} \rangle/\rho_0 \)), we can solve for the parameters of the profile (i.e., the scale radius and the scale density) and obtain an estimate of \( \langle \gamma_1(\vartheta) \rangle \). Details of how we solve for the NFW-profile parameters are given in the Appendix of WK02. With the density profile we can determine, using equation (12), the expected value of \( S \). The minimum mean overdensity, \( \Delta_{\text{min}} \), needed to produce a detectable lens is then given by that overdensity for which \( S > S_{\text{min}} \). In this paper we assume \( S_{\text{min}} = 5 \) and \( \theta = 5' \), unless stated otherwise.

3 SPHERICAL COLLAPSE IN DARK ENERGY COSMOLOGIES

According to the spherical model of gravitational collapse a density perturbation with a nonlinear overdensity \( \Delta \) corresponds to a particular position along the linear-theory evolutionary cycle. Thus the minimum nonlinear overdensity \( \Delta_{\text{min}} \) described above corresponds to a minimum linear-theory overdensity \( \delta_{\text{min}} \); if an object of mass \( M \) at redshift \( z \) has a linear-theory overdensity \( \delta > \delta_{\text{min}} = \delta_{\text{min}}(M,z) \), then it is sufficiently overdense to produce a detectable weak-lensing signal. By determining \( \delta_{\text{min}} \) from the computed \( \Delta_{\text{min}} \) we can apply the Press-Schechter (1974) theory to calculate the number of halos per unit mass and redshift with \( \delta > \delta_{\text{min}} \) and hence \( S > S_{\text{min}} \). We can then find the redshift distribution and sky density of weak lenses and how these observables vary with \( w \). We will show that for a broad range of dark-energy cosmologies a substantial fraction of detectable weak gravitational lenses have \( \delta_{\text{min}} < \delta_c \approx 1.69 \), where \( \delta_c \) is the critical density threshold for collapse. Those objects with \( \delta < \delta_c \) are commonly thought to be density perturbations that have not yet reached virialization and are therefore expected to have observational properties that are very different from typical virialized lensing clusters.

In this Section, we present the approach used to map the minimum nonlinear overdensity \( \Delta_{\text{min}} \) to a minimum linear-theory overdensity \( \delta_{\text{min}} \) for quintessence models (QCDM). We describe the dynamical equations of gravitational collapse in QCDM and give fitting formulas for the nonlinear overdensity at virialization, \( \Delta_{\text{vir}}(z) \), and the critical density \( \delta_c \). We then discuss how we calculate the abundances of weak gravitational lenses, both those with \( \delta < \delta_c \) and those with \( \delta > \delta_c \). Below we assume a flat cosmology with a Hubble pa-
rameter \( h = 0.65 \), a spectral index \( n_s = 1 \), a baryon density \( \Omega_b h^2 = 0.02 \), and \( \Omega_0 = 0.3 \), unless stated otherwise.

### 3.1 Dynamics

In quintessence the dark energy is a dynamical, time-dependent component, \( Q \), with an equation of state parametrized by \( w \equiv p_Q/\rho_Q \), the pressure divided by the energy density. The evolution of the energy density with the cosmological scale factor goes as \( \rho_Q \propto a^{-3(1+w)} \), so that for \( w = -1 \) the standard cosmological-constant model, \( \Lambda \)CDM, is recovered. Current observational evidence cannot yet rule out a \( w \) in the range \(-1 \leq w \leq 0.5\).

In order to relate a nonlinear overdensity to a linear-theory overdensity in QCDM we must first solve for the evolution of the overdensity’s radius, \( R \), with time. For a spherical overdensity patch with uniform matter density \( \rho_{\text{pert}} = 3M/4\pi R^3 \) the evolution is described by the momentum component of the Einstein equations (Wang & Steinhardt 1998; hereafter WS98):

\[
\frac{\dot{R}}{R} = -\frac{4\pi G}{3} \left[ \rho_{\text{pert}} + (1+3w)\rho_Q \right].
\]

(13)

As WS98 pointed out, for \( w \neq -1 \) the space curvature \( k_{\text{pert}} \) inside the overdensity patch is time dependent. Physically, this is because the evolution of the energy density in the \( Q \) component is evolving independently of the change in radius of the overdensity patch. As a result, one cannot assume that within the collapsing overdensity the rate of change of the internal energy in the \( Q \)-component, \( u_Q \), equals the rate of work done by the \( Q \)-component. That is, because \( dp_Q/dt \) is nonzero unless the \( Q \)-component is the cosmological constant,

\[
\frac{du_Q}{dt} = \frac{d}{dt} (\rho_Q V) \\
\neq -p_Q \frac{dV}{dt},
\]

(14)

where \( V \propto R^3 \) is the volume of the overdensity patch. Therefore equation (13) cannot be cast in the form of a first order differential equation as is often done when going from an acceleration equation to a Friedman-like energy equation. Assuming a constant \( k_{\text{pert}} \), as was done in the version 1 preprint of Lokas & Hoffman (2001), yields significantly different solutions for the evolution of the radius, \( R(t) \), and hence for \( \Delta_{\text{vir}}(z) \) and \( \delta_c \).

If we combine equation (13) with the Friedman equation for the background,

\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \left( \rho_v + p_Q \right),
\]

(15)

and impose the boundary conditions \( dR/da|_{z=\alpha} = 0 \) and \( R|_{z=0} = 0 \), where \( a_\alpha \) is the scale factor at turn-around, then for a spherical density perturbation with a given \( \Delta \) and redshift \( z \), the unique temporal evolution of the overdensity, from linearity to nonlinearity, can be solved (cf., Appendix A in WS98). We then have a one-to-one map from \( \Delta(z) \) to \( \delta(z) \), as shown in Figure 1 for the cases \( w = -1, -2/3 \) and \( -1/3 \). The map has a mild \( w \) dependence, with a given \( \delta \) corresponding to a slightly larger \( \Delta \) as \( w \) increases. This is a consequence of the earlier formation of structure in QCDM models relative to \( \Lambda \)CDM models; overdensities col-
collapse faster and are therefore more concentrated for $w > -1$. This point is well illustrated in Figure 2 where we show the growth of a spherical perturbation for the same quintessence models. As expected, the larger $w$ is, the earlier structures reach turnaround and collapse.

It can be shown that in the limit $\delta \to \delta_c$ the spherical-collapse model predicts that the radius, $R$, of the overdensity goes to zero and hence $\Delta \to \infty$. Of course well before reaching the singular solution an actual overdensity will virialize, thereby halting its collapse. To account for this fact we invoke a simple smoothing scheme in which the radius of the matter perturbation is constant with time upon reaching the virialized overdensity (see Figure 2). We refer the reader to WK02 for details of the smoothing method.

As described in WS98, the value of $\Delta_{vir}(z)$ for quintessence models, needed here in order to implement the smoothing scheme, can be obtained via the virial theorem, energy conservation, and solving equations (13) and (15) for the overdensity at turnaround. In Figure 3 we show the resulting numerical solution to $\Delta_{vir}(z)$. We find that an accurate fitting function to $\Delta_{vir}(z)$ for $-1 \leq w \leq -0.3$, modeled after the approximation given in Kitayama & Suto (1996) for a $\Lambda$CDM cosmology, is

$$\Delta_{vir}(z) = 18\pi^2 \left[ 1 + a \Theta^b(z) \right],$$

where

$$a = 0.399 - 1.300([w^{0.426} - 1],$$

$$b = 0.941 - 0.205([w^{0.938} - 1],$$

and $\Theta(z) = 1/\Omega_0(z) - 1 = (1/\Omega_0 - 1)(1 + z)^3w$. Since structures start to form earlier the larger $w$ is, the mean gas temperature in collapsing objects is higher in larger-$w$ models. As a result, a greater overdensity is required in order for such objects to become bound and virialized, explaining why $\Delta_{vir}$ rises with increasing $w$. Note, however, that for $\Delta(z) < \Delta_{vir}(z)$ the map from nonlinear to linear overdensity has a weak dependence on not only $w$ but on $\Omega_0$ and redshift as well. The critical threshold for collapse today $\delta_c(z = 0) \approx \delta_c(z) D(z, \Omega_0, w)/D(z, \Omega_0, w)$, where $D(z, \Omega_0, w)$ is the linear growth factor (see WS98), also has a weak dependence on $\Omega_0$ and $w$, as shown in Figure 4. For $0.1 \leq \Omega_0 \leq 1$ and $-1 \leq w \leq -0.3$, we find that an accurate fitting function to $\delta_c(z)$, also modeled after the approximation given in Kitayama & Suto (1996) for a $\Lambda$CDM cosmology, is

$$\delta_c(z) = \frac{3(12\pi^2)^{3/2}}{20} [1 + \alpha \log_{10} \Omega_m(z)],$$

$$\alpha = 0.353w^4 + 1.044w^3 + 1.128w^2 + 0.555w + 0.131.$$

Incorrectly assuming that $k_{pert}$ is constant, however, yields a $\delta_c(z = 0)$ with a much stronger dependence on these parameters, with inferred values for $\Omega_0 = 0.3$ of $\delta_c(z = 0) \sim 1.5$ and $\sim 1.0$ for $w = -2/3$ and $w = -1/3$, respectively (Lokas & Hoffman 2001).

3.2 Abundances

Since we are interested in computing the abundances of both virialized weak lenses and non-virialized weak lenses we consider two ranges of overdensity in our lens-abundance calculations: (1) $\delta_{min} < \delta < \delta_c$, the non-virialized lenses, and, (2) $\delta > \delta_c \geq \delta_{min}$, the virialized lenses. As we showed in WK02, both lens types are cluster-mass overdensities. However, while the virialized lenses are typically virialized clusters that form at rare (e.g., $> 3\sigma$) high-density peaks of a Gaussian primordial distribution, the non-virialized lenses correspond to proto-clusters (e.g., $2\sigma - 3\sigma$ peaks)—mass overdensities that have not yet undergone gravitational collapse and virialized, but which have begun to break away from the cosmological expansion. These proto-clusters should contain galaxies and perhaps a few groups that later merge to form the cluster (cf., White et al. 2002). The timescale for collapse of cluster-mass objects is large, and the overdensities can be very large even before they have virialized. It should therefore not be too surprising that proto-clusters produce a weak-lensing signal that resembles that from virialized clusters.

Though the lensing signals may be similar, the two lens types are expected to have different observational features. In particular, since the X-ray luminosity is a very rapidly varying function of the virialized mass, the summed X-ray emission from a non-virialized lens should be much smaller than that from a fully virialized lensing cluster of the same mass. In referring to these proto-clusters as “dark”, we thus mean that they should be X-ray underluminous. Although the mass-to-light ratio of these clusters should be comparable to those for ordinary clusters, since (1) high-redshift clusters may be difficult to pick out in galaxy surveys and, (2) proto-clusters will typically have a sky density a few times smaller than ordinary clusters, it would also not be surprising if these dark lenses had no readily apparent corresponding galaxy overdensity. Observational evidence of such dark lenses has been reported in detections by Erben et al. (2000), Umetsu & Futamase (2000), Miralles et al. (2002), Dahle et al. (2002), and Koopmans et al. (2000), the lat-

![Figure 4. The linear-theory critical threshold for collapse, $\delta_c$, at $z = 0$ as a function of $\Omega_0$ for three constant-$w$ models. $\delta_c$ does not vary significantly over a wide range in $w$ or $\Omega_0$.](image-url)
ter involving a detection through strong, rather than weak, lensing. A more detailed discussion of the features that may distinguish dark and virialized weak lenses is given in WK02.

In order to compute the abundances of virialized and dark lenses we need to know the probability that an object of a given mass at a given redshift is in one of the above mentioned ranges in overdensity. If we assume Gaussian statistics for the initial linear-theory density field, then the probability that an object’s overdensity is in the range \( \delta_1 < \delta < \delta_2 \) is

\[
P(\delta_1 < \delta < \delta_2) = \text{erf} \left( \frac{\nu_2}{\sqrt{2}} \right) - \text{erf} \left( \frac{\nu_1}{\sqrt{2}} \right),
\]

where ‘erf’ is the error function, \( \nu = \delta/\sigma \), and \( \sigma = \sigma(M, z) \) is the rms density fluctuation of an object of mass \( M \) at redshift \( z \). From Press-Schechter theory, we know that the comoving number density of virialized objects (those with \( \delta > \delta_c \) of mass \( M = 4\pi R^3 \rho_0/3 \) in the interval \( dM \) that are at redshift \( z \) in a Universe with comoving background density \( \rho_0 \) is,

\[
\frac{dN}{dM}(M, z) = \frac{\sqrt{2} \rho_0}{\pi M^2 \sigma(M, z)} \frac{d\ln \sigma(M, z)}{d\ln M} \left[ \frac{\delta_c(z)}{\sigma(M, z)} - \frac{\delta_c(z)^2}{2\sigma^2(M, z)} \right].
\]

We can therefore compute the abundance of objects in the overdensity range \( \delta_1 < \delta < \delta_2 \) by convolving the above mass function of virialized objects with \( P(\delta_1 < \delta < \delta_2)/P(\delta > \delta_c) \). Specifically, the fraction of objects that can lens relative to those that are virialized is, for dark lenses,

\[
f_{\text{dark}}(M, z) = \begin{cases} 
\frac{P(\delta < \delta_c)}{P(\delta > \delta_c)}, & \delta < \delta_c; \\
0, & \text{otherwise};
\end{cases}
\]

and for virialized lenses,

\[
f_{\text{vir}}(M, z) = \begin{cases} 
\frac{P(\delta > \delta_c)}{P(\delta > \delta_c)}, & \delta > \delta_c; \\
1, & \text{otherwise}.
\end{cases}
\]

As noted in WK02, the lower the mass of the object the larger the minimum overdensity needed to produce a detectable weak-lensing signal. For low enough masses the minimum overdensity becomes so large that both \( f_{\text{dark}} \) and \( f_{\text{vir}} \) approach zero, thereby imposing an effective weak-lensing mass threshold. Given \( f \) and equation (20) we can compute the total comoving number density of weak lenses of a particular type. Multiplying by the comoving volume element \( dV_c/dz \) \( d\Omega(w) \) then gives the differential number count of lensing objects per steradian, per unit redshift interval:

\[
\frac{dN}{dz d\Omega} = \frac{dV_c}{dz d\Omega} \int_0^\infty f(M) \frac{dN}{dM}(M) dM.
\]

By integrating over redshift we can then compute the number of dark and virialized lenses we expect to see per unit area of sky for a given QCDM model.

### 3.3 Normalizing the power spectrum

In equation (23) the volume term and the two terms within the integrand are all functions of \( w \). While the predicted

![Figure 5](image-url)  

**Figure 5.** The dependence of \( \sigma_8 \) on \( w \) as obtained using three different approaches: fixing \( \Omega_0 = 0.3 \) and normalizing to the observed X-ray cluster abundance (solid line), fixing \( \Omega_0 = 0.3 \) and normalizing to COBE (dotted line), and allowing \( \Omega_0 \) to vary with \( w \) such that the cluster abundance constraint matches the COBE constraint (solid line).

![Figure 6](image-url)  

**Figure 6.** The region in the \( \Omega_0 - w \) plane where the X-ray cluster abundance constraint of \( \sigma_8 \), at 95% confidence, overlaps the COBE constraint of \( \sigma_8 \). The gray scale gives the corresponding \( \sigma_8 \) values and the solid line shows where the central values match.
abundance of weak lenses will therefore vary with \(w\), the degree to which it will vary depends on the shape and normalization of the power spectrum of density fluctuations. In particular, to compute the abundance of weak-lenses we need to know \(\sigma(M,z)\).

For the shape of the power spectrum we use the fitting formulas given in Ma et al. (1999) for QCDM models with the transfer function and shape parameter for ΛCDM models given by Bardeen et al. (1986) and Hu & Sugiyama (1996, eqs. [D-28] and [E-12]), respectively. Since the Q-component does not cluster on scales less than \(\sim 100\) Mpc (Caldwell, Dave & Steinhardt 1998), at the weak-lensing scales the shape of the spectrum does not differ significantly from the well studied ΛCDM shape.

The normalization of the power spectrum, often expressed in terms of \(\sigma_s\), the rms fluctuation today at a scale of \(8\ h^{-1}\) Mpc, is not as well constrained as its shape and will in general be a function of \(w\). There are two different methods commonly used to obtain the normalization: to fix it by the observed X-ray cluster abundance or to fix it by the CMB large-scale anisotropies observed by the COBE satellite. Both approaches have comparable uncertainties; the cluster abundance constraint on \(\sigma_s\) has a 20% uncertainty at the 2\(\sigma\) level (WS98) while the COBE constraint has a 7% uncertainty at the 1\(\sigma\) level (Bunn & White 1997). To obtain an estimate of how \(\sigma_s\) varies with \(w\) so that we may, in turn, determine how \(dN/dzdM\) varies with \(w\) for dark and virialized lenses, we will consider three possible approaches. The first two involve fixing the cosmological parameters (e.g., \(\Omega_0, h, \Omega_b, n_s\)) and using either the cluster-abundance constrained \(\sigma_s(w)\) or the COBE constrained \(\sigma_s(w)\). For the former we will use the fit given in WS98 and for the latter the fit given by Ma et al. (1999); see Figure 5. The third approach is to allow the cosmological parameters to be free parameters and then jointly match the cluster-abundance constraint with the COBE constraint so that each gives the same \(\sigma_s(w)\). Since measurements of \(\sigma_s\) are most degenerate with \(\Omega_b\), we will let \(\Omega_b\) be the parameter that varies. In Figure 6 we show the region in the \(\Omega_0-w\) plane where the X-ray cluster-abundance constraint, at the 95% confidence level, overlaps the COBE constraint. The solid curve shows where the central values match, with the resulting range in \(\Omega_0 (0.3 \leq \Omega_0 \leq 0.4\) for \(-1 < w < -0.4)\) within observational uncertainties (Wang et al. 2000). The corresponding \(\sigma_s(w)\) curve is shown in Figure 5. As we will show, the predicted weak-lens abundances and how they vary with \(w\) strongly depend on which \(\sigma_s(w)\) normalization approach is chosen.

4 RESULTS

We are interested in determining whether the number count and redshift distribution of both dark and virialized weak lenses have the potential to constrain \(w\). Another possibly useful observable for this purpose is the number count of dark lenses relative to virialized lenses. Since dark lenses are at an earlier stage of their dynamical evolution as compared to virialized lenses, those cosmologies that favor a faster growth of structure (i.e., QCDM models with larger \(w\)) will, for a given \(\sigma_s\), have fewer dark lenses and more virialized lenses. The ratio of the two is therefore expected to vary with \(w\). A priori, this latter observable seems particularly promising. As discussed in WK02, the ratio of dark to virialized lenses is not very sensitive to observational noise in the weak lensing maps since observational noise equally affects the detectability of both types of lenses. Contrastingly, uncertainties in observational noise will make it difficult to constrain \(w\) by simply comparing predicted weak-lens number counts with observed weak-lens number counts.

Before presenting how the above observables are mod-
Figure 8. The fraction $\chi$ of objects with overdensities in the range $\Delta_{ta} < \Delta < \Delta_{vir}$ (upper panels) and $0 < \Delta < \Delta_{ta}$ (lower panels) relative to those objects that are virialized, $\Delta > \Delta_{vir}$, as a function of mass. The left panels correspond to the COBE normalization of $\sigma_8$ with $\Omega_0 = 0.3$ and the right panels correspond to the X-ray cluster abundance normalization of $\sigma_8$ with $\Omega_0 = 0.3$. For each constant-$w$ model we show the fraction $\chi$ at $z = 0$ (bottom curve), $z = 1/2$ (middle curve), and $z = 1$ (top curve).

As noted above, the predicted abundance of weak lenses will vary with $w$ on account of three factors: the comoving volume element, the Press-Schechter comoving number density of virialized objects, and the value of $f_{\text{dark}}/f_{\text{vir}}$ [equations (21) and (22)]. The degree to which each varies depends on the chosen $\sigma_8(w)$ normalization. As Figure 7 shows, $dV_c/dz d\Omega$ decreases monotonically with increasing $w$ for both fixed $\Omega_0$ and $\Omega_0 = \Omega_0(w)$ as given by jointly normalizing $\sigma_8$ to COBE and the cluster abundance. However, because the joint normalization yields a larger $\Omega_0$ with $w$ and a less significant decline in $\sigma_8$ for $w \geq -1$ as compared to the COBE normalization with $\Omega_0$ fixed, the former approach predicts a nearly constant virialized object number density with increasing $w$ while the latter predicts a significant decrease in the number density.

A similar trend is seen in the functions $f_{\text{vir}}$ and $f_{\text{dark}}$, as Figure 8 demonstrates. Here we plot the fraction of objects that have not yet reached turnaround ($0 < \Delta < \Delta_{ta}$) and the fraction of objects that are between turnaround and virialization ($\Delta_{ta} < \Delta < \Delta_{vir}$) relative to those objects that are virialized ($\Delta > \Delta_{vir}$). The figure illustrates several key elements of structure formation according to the spherical-collapse model for dark-energy cosmologies. First, the fraction, $\chi$, of objects in both of these lower-overdensity ranges increases with mass in accordance with the hierarchical growth of structure. The fraction also increases with redshift since objects are collapsing and evolving toward virialization. It is also interesting to note that objects with $\Delta_{ta} < \Delta < \Delta_{vir}$ evolve more rapidly as compared to objects with $0 < \Delta < \Delta_{ta}$. This is demonstrated by the fact that at $z = 1$ the fraction of both types of objects is nearly the same though by $z = 0$ there are more objects that have not reached turnaround. Furthermore, the larger $w$ is, the greater the difference between the rates of evolution. These effects are a consequence of the suppression of structure growth in cosmologies with dark energy; namely, growth slows down earlier for larger $w$ and those objects that are less overdense at a given redshift have greater difficulty overcoming the repulsive effects of the dark energy and collapsing. Finally, the plots show how strongly the fraction depends...
on the chosen $\sigma_8$ normalization, with a significant variation with $w$ for the COBE normalization and a fairly small variation for the cluster-abundance normalization. This, in turn, means that the degree to which the functions $f_{\text{vir}}$ and $f_{\text{dark}}$ vary with $w$ is highly dependent on the assumed normalization approach.

4.2 Weak lens abundances

In Figure 9 we show the predicted redshift distribution of virialized lenses and dark lenses for three constant-$w$ models. For the COBE normalized $\sigma_8$ with fixed $\Omega_0$ the distributions show a fairly strong sensitivity to $w$. As $w$ increases from $-1$ to $-1/3$ the peak of the distributions shift toward lower redshifts. Although one might expect the trend to be in the opposite direction given that structures form faster for larger $w$ models, the effect is counteracted by the decrease in $\sigma_8$ with increasing $w$. That the decrease in $\sigma_8$ so overwhelms any tendency for structure to form faster for $w > -1$ is not surprising given the weak $w$ dependence in the $\Delta - \delta$ map (Figure 1) and in the function $\delta_c(z)$ (Figure 4). Note, however, that the shift in the distributions with $w$ becomes much less significant if a joint COBE – cluster abundance normalization is assumed. Finally, given that dark lenses are likely progenitors of virialized clusters, it is not surprising that both normalization approaches predict that the dark lenses have a larger mean redshift than the virialized lenses.

To determine how well the weak lens redshift distributions can constrain $w$ we generated mock redshift data and determined (using the Kolmogorov-Smirnov test) the probability of differentiating two different constant-$w$ models as a function of the number of lenses detected. We found that to differentiate a $\Lambda$CDM model from both a $w = -0.6$ model and a $w = -0.9$ model at the 3$\sigma$ level required, on average, approximately 200 weak lenses and 2000 weak lenses, respectively. As we show below, this corresponds to a survey coverage of $\sim 15$ and $\sim 150$ square degrees. Note, however, that for sufficiently wide surveys systematic uncertainties such as mass-redshift selection effects and lens density profiles might dominate the errors.

By integrating over the redshift distribution we obtain the total number of virialized and dark lenses expected per square degree on the sky. As Figure 10 shows, the COBE normalization with $\Omega_0 = 0.3$ shows a significant decline in the number count as $w$ increases. By $w = -2/3$ the number count of both virialized and dark lenses has dropped by a factor of two from the $\Lambda$CDM value. The joint normalization, in which we allow $\Omega_0$ to vary with $w$, predicts a much more mild dependence on $w$ with the number count dropping by only $\sim 20\%$ from $w = -1$ to $w = -2/3$ for both lens types. Therefore, while the COBE-only normalization approach predicts that the sky coverage needed to distinguish the $\Lambda$CDM model from a $w = -0.6$ model to 3$\sigma$ is only $\sim 2$ degree$^2$, the joint approach requires $\sim 15$ degree$^2$. Similarly, to distinguish the $\Lambda$CDM model from $w = -0.9$ requires $\sim 40$ degree$^2$ and $\sim 100$ degree$^2$, respectively. The systematic uncertainties affecting absolute sky density measurements, such as noise in the lensing maps and uncertainties in the lens density profiles, are expected to add further complications. This suggests that it will be very difficult to
Figure 10. The total number of virialized lenses (dashed curves) and non-virialized lenses (solid curves) per square degree as a function of \( w \). Thin lines correspond to the COBE normalized \( \sigma_8 \) with \( \Omega_0 = 0.3 \) and thick lines to the joint COBE–cluster abundance normalized \( \sigma_8 \) with \( \Omega_0 = \Omega_0(w) \). While the number count drops by a factor of two between \( w = -1 \) and \( w = -2/3 \) for the COBE-only normalization, the drop is much less significant for the joint normalization.

4.3 Fraction of lenses that are dark

As mentioned above, the number-count ratio of dark to virialized lenses is an observable that is much less sensitive to observational noise than is the redshift distribution and number count of weak lenses. Unfortunately, for aperture sizes \( \theta \) (defined in Section 2) less than 10' in radius the ratio is fairly constant over a broad range in \( w \), as we show in Figure 11. The ratio varies more strongly if the aperture size is increased to 15'. In particular, for \( \theta = 15' \) there is a \( \sim 20\% \) difference between the \( \Lambda \)CDM model and \( w = -0.6 \), so that differentiating the two models to a 3\( \sigma \) significance requires the detection of \( \sim 600 \) virialized lenses or equivalently a sky coverage of \( \sim 50 \) degree\(^2\). Although using the non-virialized lens fraction requires large survey coverage for modest constraints on \( w \), its principal advantage (in addition to being relatively insensitive to observational noise) is that it is not very sensitive to the chosen method of normalization; for any aperture size both the joint normalization and the COBE normalization with fixed \( \Omega_0 \) yield similar dependences on \( w \). Therefore, unlike the case for weak-lens sky-density or redshift distribution predictions, uncertainties in \( \sigma_8 \) and \( \Omega_0 \) do not strongly affect the predicted ratio of dark to virialized lenses. Incidentally, although aperture sizes greater than \( \sim 15' \) yield ratios with even stronger \( w \) dependences, noise contributions from large-scale structure become significant at such large angular distances from the

constrain \( w \) using just the number count of either virialized or dark lenses without, at the very least, a tighter constraint on \( \Omega_0 \).

We also note that our results do not agree with the results found by Bartelmann, Perrotta & Baccigalupi (2002; hereafter BPB). They found that from \( w = -1 \) to \( w \approx -0.6 \), the number of virialized weak lenses per square degree increases by nearly a factor of two. The increase is roughly linear up to the maximum after which the number count declines steeply. In obtaining these results, however, they use the formulas for \( \Delta_{\text{vir}} \) and \( \delta_c \) given in Lokas & Hoffman (2001) who assume that the space curvature within a collapsing overdensity patch is time-independent. As we showed in Section 3.1, this assumption is invalid for \( w \neq -1 \) and leads to incorrect values for \( \Delta_{\text{vir}} \) and \( \delta_c \). To confirm that this is the source of our differences, we recomputed the number count of weak lenses as a function of \( w \) using the algorithm described in BPB (which differs somewhat from ours because we are interested in separating lenses into virialized and non-virialized types). When we assume the incorrect Lokas & Hoffman (2001) values for \( \Delta_{\text{vir}} \) and \( \delta_c \), we recover the results found by BPB; however, when we assume the values for \( \Delta_{\text{vir}} \) and \( \delta_c \) predicted by solving the spherical-collapse equations of Section 3.1, we obtain results very similar to those described in the preceding paragraphs.
lens center (Hoeskstra 2002). It is therefore not practical to make measurements at radii well beyond 15′.

As an aside, while the ratio of dark to virialized lenses does not have a particularly strong w dependence, it does have a strong θ dependence; only ~ 5% of lenses are dark when θ = 3′ but ~ 50% when θ = 15′. In Figure 12 we plot the number of virialized and dark lenses as a function of θ for the ΛCDM model. As θ increases from 3′ to 15′ the sky density of dark lenses increases from zero to 5 per square degree while the sky density of virialized lenses peaks at θ = 5′ and gradually declines for larger aperture sizes. Figure 13 explains this trend. For an overdensity of mass M = 5 × 10^{14} M⊙ we plot, as a function of redshift, θ_{vir}, the projected angular size of the virialization radius, and θ_{max}, the projected angular size of the maximum radius that produces a detectable lens (i.e., θ_{max} = R_{max}(z)/D_{A}(z) where R_{max}^{3} = 3M/4πΔ_{min}(z)). For θ_{max} > θ_{vir} an overdensity can be non-virialized and still produce a detectable lensing signal (i.e., a dark lens). However, since θ defines the maximum observable angular scale, for sufficiently small θ there is no range in redshift such that θ > θ_{max} > θ_{vir}, in which case non-virialized overdensities cannot produce a detectable lens. In general we find that the minimum aperture size needed to detect dark lenses is ~ 3′. For larger θ, the area below θ_{max} and above θ_{vir} has a substantial relative increase while the area below θ_{vir} has just a mild relative increase. After taking into account the fact that the aperture mass M_{ap}(θ) decreases with increased θ, this translates to an increase in the sky density of dark lenses and a decrease in the sky density of virialized lenses for θ > 5′. The fraction of lenses that are dark therefore increases with aperture size.

5 DISCUSSION AND CONCLUSIONS

We have examined the possibility of using the measured abundance of weak gravitational lenses to constrain a principal property of the dark energy, its equation-of-state parameter w. Since dark energy modifies both the background cosmology of the universe and the growth of structure it will necessarily have an effect on the efficiency of weak lensing. The goal of this paper was to determine the nature and strength of the effect.

The change in the background cosmology with w influences the predicted weak lens abundance in essentially three ways. First, the size of comoving volume elements shrink with increasing w. Second, the distance-redshift relation is modified, thereby shifting the location of the lensing-kernel maximum (i.e., where the combination of angular diameter distances D_{L}/D_{A}/D_{S} peaks). Third, since the evolution of the background matter density is modified by the dark energy, the density of a given halo relative to the background density changes with w. This, in turn, affects the strength of a halo’s lensing signal; the larger the overdensity the stronger the signal. While the volume term is explicitly factored into the expression for the weak-lens sky density [equation (22)], the latter two effects are incorporated into the signal-to-noise estimator for which we use the aperture-mass technique introduced by Schneider (1996).

The change in the growth of structure with w is somewhat more subtle. The dark energy modifies both the rate of structure growth and the amplitude of the matter power spectrum. To determine the former we solved the spherical-collapse model with dark energy included. Though growth occurs more rapidly as w increases, the overall effect on the
The observed angular size of \( \theta_{\text{vir}} \) (dashed line) and \( \theta_{\text{max}} \) (solid line) as a function of redshift for an overdensity of mass \( M = 5 \times 10^{14} M_\odot \) for a \( \Lambda \)CDM cosmology. If \( \theta_{\text{max}} > \theta_{\text{vir}} \), an overdensity need not be virialized to produce a detectable lensing signal. However, the range in redshift over which a non-virialized lens can be detected is limited by the aperture size \( \theta \) (e.g., thin, dotted lines), which defines the maximum observable angular scale. For \( \theta \lesssim 3 \) arcmin virtually no dark lenses can be detected.

The observed angular size of \( \theta_{\text{vir}} \) (dashed line) and \( \theta_{\text{max}} \) (solid line) as a function of redshift for an overdensity of mass \( M = 5 \times 10^{14} M_\odot \) for a \( \Lambda \)CDM cosmology. If \( \theta_{\text{max}} > \theta_{\text{vir}} \), an overdensity need not be virialized to produce a detectable lensing signal. However, the range in redshift over which a non-virialized lens can be detected is limited by the aperture size \( \theta \) (e.g., thin, dotted lines), which defines the maximum observable angular scale. For \( \theta \lesssim 3 \) arcmin virtually no dark lenses can be detected.

\( \Delta - \delta \) map, needed to relate the minimum overdensity required to produce a detectable lens, \( \Delta_{\min} \), to a corresponding linear-theory overdensity \( \delta_{\min} \), is fairly small. Similarly, the linear-theory overdensity at collapse \( \delta_c \) does not vary much with \( w \). The effect on \( \Delta_{\min} \) is more significant, however. As \( w \) increases, structures require substantially greater overdensities in order to reach virial equilibrium because they collapse sooner, when the universe was younger and hotter.

To determine how the power-spectrum amplitude, \( \sigma_8 \), varies with \( w \) we considered three possible approaches. One was to normalize to the X-ray cluster abundance as was done in WS98. Another was to normalize to the COBE measurements of CMB anisotropies on large angular scales. These two approaches predict similar values of \( \sigma_8 \) for the \( \Lambda \)CDM model. However, if all cosmological parameters are held fixed as \( w \) varies, the values of \( \sigma_8 \) are no longer in accordance. This is because the cluster abundance approach is accounting for the earlier forming, and hence hotter, galaxy clusters in models with \( w > -1 \). The COBE normalization, on the other hand, is accounting for the increase in the Integrated Sachs-Wolfe (ISW) effect as \( w \) increases (c.f., BPB). Given these differing influences, the two approaches are not expected to yield the same \( \sigma_8 \) when all the cosmological parameters are held fixed to those of the \( \Lambda \)CDM model while \( w \) is varied. This suggests a third approach to normalizing the power spectrum; namely, let the parameters vary with \( w \) such that the cluster abundance normalization matches the COBE normalization. In practice we accomplished this by letting just \( \Omega_0 \) vary with \( w \), as it is the parameter most degenerate with \( \sigma_8 \). The resulting range in \( \Omega_0 \) for \( -1 < w < -0.4 \) was found to be \( 0.3 < \Omega_0 < 0.4 \) and hence within observational uncertainties. Though all three normalization approaches predict that \( \sigma_8 \) decreases with \( w \), the difference in the magnitude of the decrease between the approaches is significant. As a result, each predicts substantially different variations in the weak-lens abundance with \( w \).

Having determined all the dark energy effects, we computed the redshift distribution and sky density of weak lenses as a function of \( w \). As in WK02, we distinguished between two classes of lenses, those that have collapsed and virialized and those that have not. This distinction is based on the expectation that the virialized lenses, being in a relaxed state, are X-ray and/or optically luminous. The non-virialized lenses, being as at earlier stage in the overdensity evolutionary cycle, are expected to be X-ray underluminous because the observed X-ray luminosity function has a steep dependence on the total virialized mass within a halo. Furthermore, though the typical mass of both lens types is \( \sim \) few \( 10^{14} M_\odot \), the sky density of galaxies within the non-virialized lenses is expected to be smaller than in the virialized lenses because they have not yet collapsed and hence have larger radii (see WK02 for more details).

We found that the variation in the redshift distribution and the sky density of both lens types with \( w \) depends strongly on the power-spectrum-normalization approach. If \( \Omega_0 \) is fixed and \( \sigma_8 \) is normalized to the COBE measurements, there is a significant variation in the abundances with \( w \). In particular, the sky density of both virialized lenses and non-virialized lenses drops by a factor of two from \( w = -1 \) to \( w = -2/3 \). This decline, a result of the significant decrease in \( \sigma_8 \) with \( w \), occurs despite the faster formation of structure for \( w > -1 \). If, on the other hand, \( \Omega_0 \) is allowed to vary with \( w \) such that the COBE normalization matches the cluster-abundance normalization, the redshift distributions and sky density change very little with \( w \); between \( w = -1 \) and \( w = -2/3 \) the sky density of both lens types varies by just \( \sim 20\% \). This insubstantial variation is the result of an increase in \( \Omega_0 \) with \( w \) and a less significant drop in \( \sigma_8 \) with \( w \) as compared to the COBE normalization with \( \Omega_0 \) fixed. Obtaining a strong constraint on \( w \) from the sky density or redshift distribution of weak lenses therefore appears to be contingent on improved measurements of \( \Omega_0 \) from independent observations.

Perhaps more promising is the possibility of utilizing the observed ratio of dark lenses to virialized lenses. Unlike measurements of the absolute sky density of weak lenses, the ratio is not very sensitive to the amount of observational noise in the weak-lensing maps since the abundance of both dark lenses and virialized lenses are equally affected by noise. Similarly, the ratio does not vary significantly over a wide range in cosmological parameters so that uncertainties due to the \( \Omega_0 - w \) degeneracy are minimized. We found that for aperture sizes of \( \sim 15' \) the ratio varies by about 20\%, dropping from 0.5 to 0.4, between the \( \Lambda \)CDM model and \( w = -0.6 \). We also showed that the ratio of dark to virialized lenses increases with aperture size, in effect because larger apertures enable the detection of the more extended radii of the non-virialized lenses.

Weak lensing has already been shown to be a powerful tool...
probe of the matter distribution in the universe (see e.g., Bartelmann & Schneider 2001). It also has the potential to help constrain the amount and nature of the dark energy. Huterer (2002) showed that given reasonable prior information on other cosmological parameters, the weak-lensing convergence power spectrum can impose constraints on the dark energy comparable to those of upcoming type Ia supernova and number-count surveys of galaxies and galaxy clusters. Constraining the dark energy from absolute measurements of weak-lens abundances will likely prove difficult, however. The variation in the weak lens sky density with $w$ is sufficiently small that modest uncertainties in $\Omega_0$ (and observational noise) can mask the effect of the dark energy. More auspicious is the possibility of utilizing the relative abundance of dark lenses to virialized lenses to constrain $w$. Future weak-lensing projects such as the VISTA survey, the SNAP mission, and LSST (see Tyson et al. 2002 for a discussion of its great promise as a probe of dark energy) are expected to provide the wide-field surveys needed for this technique to be viable.

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