On a mechanism for enhancing magnetic activity in tidally interacting binaries

T. Zaqarashvili

School of Mathematics and Statistics, University of St Andrews, St Andrews, Fife KY16 9SS, Scotland
Abastumani Astrophysical Observatory, Al. Kazbegi ave. 2a, 380060 Tbilisi, Georgia

G. Javakhishvili
Abastumani Astrophysical Observatory, Al. Kazbegi ave. 2a, 380060 Tbilisi, Georgia
and
G. Belvedere

Dipartimento di Fisica e Astronomia, Universitá di Catania, Via S.Sofia 78, I-95123 Catania, Italy

ABSTRACT

We suggest a mechanism for enhancing magnetic activity in tidally interacting binaries. We suppose that the deviation of the primary star from spherical symmetry due to the tidal influence of the companion leads to stellar pulsation in its fundamental mode. It is shown that stellar radial pulsation amplifies torsional Alfvén waves in a dipole-like magnetic field, buried in the interior, according to the recently proposed swing wave-wave interaction (Zaqarashvili 2001a). Then amplified Alfvén waves lead to the onset of large-scale torsional oscillations, and magnetic flux tubes arising towards the surface owing to magnetic buoyancy diffuse into the atmosphere producing enhanced chromospheric and coronal emission.

Subject headings: Stars:binaries – stars: activity – stars: oscillations

1. Introduction

It is widely accepted that most single stars and components of wide binary systems follow well-defined rotation-activity relationships. However some stars - particularly components of
close binaries - show larger chromospheric and coronal activity in comparison to similar single stars with the same rotation period. Young and Koniges (1977) have noticed that binary giants with circular orbits have a tendency to show enhanced CaII H and K emission, this stressing the importance of binarity in the phenomenon of enhanced chromospheric emission. Basri et al. (1985) and Simon and Fekel (1987) noticed that subgiants in synchronized binary systems were more active than single stars with the same rotation period. Rutten (1987) attributes this "overactivity" to a difference in the internal stellar structure and points out that overactive stars do not appear to deviate from the flux-flux relationships, thus suggesting that their atmospheric structure does not differ too much from that of other cool stars. Glebocki and Stawikowski (1988) noticed that MgII h and k fluxes in close binary systems correlate much better with parameters connected to the separation of the components than with rotational parameters.

Schrijver and Zwaan (1991) showed that the chromospheric, transition-region, and coronal emissions from relatively close binaries are enhanced as compared to single dwarfs and giants with the same rotation period. They concluded that a star can be called overactive if the radiative losses from its outer atmosphere are significantly over the level expected from a single cool star with the same mass, chemical composition, age and average surface rotational rate, while these radiative losses do not deviate significantly from the flux-flux relationships defined for the class of single cool stars. Overactivity appears to occur whenever a binary system containing a cool primary or two cool components is sufficiently close to induce strong tidal interaction.

De Medeiros and Mayor (1995) also showed that tidal effects play a direct role in determining the X-ray activity level in binary evolved stars. The circularisation of the orbit was a necessary property for enhanced coronal activity. Dempsey et al. (1993) concluded that synchronous binaries show a slight trend for increasing chromospheric emission with decreasing period, while the asynchronous binaries show abnormally high activity levels for their rotational periods. Recently Gunn et al. (1998) also point out the influence of unknown effects of binarity on the activity levels.

Thus the observations stress the influence of the companion’s gravity on the magnetic activity of stars in relatively close binary systems. In order to explain the phenomenon the mechanism of magnetic activity must be clearly understood. It is widely believed that magnetic activity in the Sun and late type solar-like stars can be explained in the framework of a mean-field dynamo operating in or just below the convective zone (Parker 1955a,b; Steenbeck et al. 1966; Yoshimura 1978a,b; Belvedere 1991). However alternative theories, mainly some kind of hydromagnetic oscillator (or torsional oscillations) have been developed from time to time (Walén 1949; Cowling 1953; Piddington 1971; Layzer et al. 1979; Dicke
1982; Gough 1988). Although the mean-field dynamo seems to have captured the essentials
of solar activity even if many uncertainties still remain (see, e.g., reviews by Belvedere 1985,
1997; Schmitt 1994, Brandenburg 1994), the enhanced magnetic activity in binaries can be
hardly explained by a mean field dynamo, also because dynamo requires large latitudinal
or radial angular velocity gradients. In fact observations show small latitudinal gradients,
and almost nothing is known about radial ones. Schrijver and Zwaan (1991) suggested a
mechanism for the enhancement of dynamo efficiency related to the motion of stars about
the system’s centre of gravity, which lies well outside the stars. However no clear physical
or mathematical formulations of the influence were developed after this suggestion and the
problem remained unsolved. Therefore it is natural to seek the key of the phenomenon in
a possibly alternative theory for strong magnetic activity. The main problem of oscillator
theories was the absence of an energy source to supply oscillations. If binarity can support
torsional oscillations inside the star, then this may lead to a straightforward explanation of
enhanced magnetic activity.

In this paper we develop the idea that stellar pulsation induced by the gravitational force
of the companion star in tidally interacting binaries may amplify the torsional oscillations
of a seed magnetic field, thus leading to enhanced chromospheric emission. We suggest that
the deviation of the star from spherical symmetry due to the gravitational influence of
the companion induces stellar pulsation in its fundamental frequency, as it tries to retain
its original spherical form. Then we show that the radial pulsation of the star with a
dipole-like magnetic field causes amplification of torsional Alfvén waves in the interior. The
mechanism of coupling between radial pulsation and torsional Alfvén waves is based on
the recently proposed swing wave-wave interaction Zaqarashvili (2001a), which accounts for
the coupling between longitudinal and transversal waves. The main physical meaning of
the swing interaction is that compressible waves cause periodical variation of both medium
density and magnetic field (thus, of the Alfvén speed), affecting the propagation properties of
the Alfvén waves. Then the temporal evolution of pure Alfvén waves is governed by Mathieu’s
equation and consequently the harmonics with half the frequency of compressible waves grow
exponentially in time. That means that the energy of longitudinal waves can be transferred
to transversal waves. The coupling between sound and Alfvén waves propagating along the
magnetic field (Zaqarashvili 2001a,b) as well as between fast magnetosonic waves propagating
across the magnetic field and Alfvén waves propagating along the field (Zaqarashvili and
Roberts 2002b), is studied in the simplest rectangular case. It is also supposed that stellar
radial pulsation can amplify torsional Alfvén waves, which, in certain conditions, may lead
to the onset of torsional oscillations of a dipole-like magnetic field (Zaqarashvili and Roberts
2002a). In the present paper we show in details how the pulsation induced by the tidal
distortion of the star can amplify the torsional Alfvén waves.
2. Resonant torsional Alfvén waves in the stellar interior

Consider a binary system which is well separated so that mass transfer by Roche-lobe overflow does not occur. Then we can exclude mass transfer as a mechanism for enhanced chromospheric activity. Let the primary be the primary as a main sequence star (or a subgiant) with a convective envelope. The secondary can be a different type of star (if it is also a main sequence or subgiant star, then the same mechanism can be applied to both components of the binary system). We will concentrate on the dynamics of the primary considering the gravity of the companion as an external force.

We use the ideal magnetohydrodynamic (MHD) equations:

\[
\frac{\partial \rho}{\partial t} + \nabla (\rho \mathbf{v}) = 0, \tag{1}
\]

\[
\frac{\partial \mathbf{v}}{\partial t} + \rho (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla p + \mathbf{j} \times \mathbf{B} - \rho \nabla \phi, \tag{2}
\]

\[
\nabla \times \mathbf{B} = \mu \mathbf{j}, \tag{3}
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}), \tag{4}
\]

\[
\nabla^2 \phi = 4\pi G \rho, \tag{5}
\]

\[
p = p_0 \left( \frac{\rho}{\rho_0} \right)^\gamma, \tag{6}
\]

where \(\rho\) is the medium density, \(p\) is the pressure, \(\mathbf{v}\) is the velocity, \(\mathbf{B}\) is the magnetic field, \(\mathbf{j}\) is the current, \(G\) is the gravitational constant, \(\phi\) is the gravitational potential, \(\mu\) is the magnetic permeability and \(\gamma\) is the ratio of specific heats. We consider a medium with zero viscosity, infinite conductivity and negligible displacement current. We argue that the coupling between pulsation and torsional waves occurs below the convection zone, therefore we do not consider convective effects and consequently the \(\alpha\)-term in the induction equation. Here and in remaining part of the paper we also neglect the rotational effects and consider a spherically symmetric density distribution. The gravitational influence exerted by the companion is also neglected in the equations, because we are interested to the companion’s gravity only as a source for the initial deviation from the equilibrium. Its action during the further processes studied here is not relevant, so it is neglected for simplicity.

We consider an unperturbed dipole-like seed magnetic field in the stellar interior. The existence of such stable large-scale configuration in the interior is an open question. A wide class of magnetic field topologies that are either purely toroidal or purely poloidal have been shown to be dynamically unstable (Tayler 1973; Wright 1973; Markey and Tayler 1973). However special stable configurations with poloidal and toroidal field of similar strength
might exist (Tayler 1980; Mestel 1984). The magnetic fields of magnetic A stars and the magnetic white dwarfs, which do not change on long time scales, are the observational evidence of stable configurations (Spruit 1999). In general it may be stated that curl-free magnetic configurations are stable while electrical currents leads to instability. All magnetic configurations studied by Tayler are current carrying, so they automatically lead to instability due to the Lorentz force. However if the magnetic field inside a star originates from the interstellar field during the gravitational contraction, then the Lorentz force acting on the induced currents during the contraction may redistribute the magnetic field leading to a minimum potential energy state (force-free configurations) which is relatively stable (see also Taylor (1986) for a similar phenomenon). But the process of large-scale magnetic field formation is beyond the scope of this paper and therefore for simplicity we consider a purely poloidal magnetic field which in spherical coordinates \((r, \theta, \phi)\) in the equatorial regions may be written as

\[
B = (0, B_\theta(r), 0).
\]

It must be mentioned that all the results obtained in the paper are similar also for a purely toroidal unperturbed magnetic field, therefore the physical meaning of the phenomenon does not depend on the magnetic configuration.

We also consider a relatively weak magnetic field, so that \(M/|\Upsilon| \ll 1\), where \(M\) is the magnetic and \(\Upsilon\) is gravitational potential energy. Then the star can be considered to be in hydrostatic equilibrium (if we neglect the companion’s gravity) so that its own gravity is balanced by the pressure gradient.

The slight deviation from the equilibrium leads to stellar pulsation which may be studied by the linear perturbation theory. The stellar adiabatic oscillations (without the magnetic field) can be divided into two classes (Cox 1980; Gautschy and Saio 1995): high-frequency p-modes or pressure modes, the restoring force of which is the pressure gradient, and low-frequency g-modes, where buoyancy acts as the restoring force. There are also intermediate oscillations sometimes called f-modes (Cowling 1941).

Any external action on the bounded system leads to oscillation in the first eigenfrequency which is also called the fundamental frequency of the system. The mechanical analog of the fundamental oscillation is the tuning fork in the case of an impulsive force and the musical trumpet in the case of a continuous force. Additionally a periodic external force leads to resonance when its frequency equals one of the system eigenfrequencies.

The oscillation of the star in the fundamental mode yields a wavelength comparable with the stellar radius. Therefore the fundamental frequency will be of order of \(\sqrt{GM/R^3}\), where \(R\) and \(M\) are the radius and the mass of star (Cowling 1952) (for the Sun the oscillation period is about 2 hours). A purely radial pressure mode \((l = 0, \text{ where } l\) is the spherical
degree of a mode) with one velocity node at the centre, and one antinode at the surface will have a frequency of the order of \( c_0/R (\sqrt{GM/R^3} \sim c_0/R \) because of the equilibrium between pressure gradient and gravity), where \( c_0 \) is the mean sound speed in the interior. A large-scale magnetic field inside the star probably causes the frequency splitting corresponding to fast and slow MHD waves (Roberts and Soward 1983). If the magnetic field is relatively weak then the frequency corresponding to fast waves remains almost the same \( c_0/R \), while the frequency corresponding to slow MHD waves will be of order of \( v_A/R \), where \( v_A \) is the mean Alfvén speed in the interior.

2.1. Oscillation of the primary induced by tidal distortion

The tidal force exerted by the companion leads to the deviation of the star from equilibrium. In synchronous binaries the deformation is always in the direction of the companion, because of the same value of the rotational and orbital period. Then for circular orbits a new equilibrium may be established where the tidal force plays a significant role. This is a stationary tide. However in asynchronous binaries the tidal bulk will change location permanently and a stationary state can be hardly established even for circular orbits. This is a dynamical tide. Now the tidal bulk will be "retarded" with respect to the companion. This retarded tide may lead to stellar oscillation when the star tries to return to its spherically symmetric form (due to this retarded tide exerted from the Moon, the Earth's rotation period is supposed to be increasing). Like the musical trumpet (or the tuning fork) which oscillates in its fundamental frequency under the action of an external force, the primary probably will oscillate in its fundamental mode. The stronger is the tidal action of the companion, the stronger is the oscillation.

If the reciprocal of the orbital period is close to the eigenfrequencies of the free oscillations of the star, then resonance can lead to significant enhancement of the oscillation (Cowling 1941; Zahn 1970; Smeyers et al. 1998). But the resonant conditions can be fulfilled only for low-frequency g-modes. Moreover, the observational evidence of tidally excited resonant oscillations is not well established, possibly due to the absence of any systematic study (Willems and Aerts 2002). However, a retarded tidal bulk in asynchronous binaries may lead to the oscillation of the primary star in the fundamental frequency which is higher than the orbital angular frequency.

In any case, it may be supposed that the star under consideration in tidally interacting binaries oscillates either in its fundamental frequency or in a low frequency g-mode which is in resonance with the tidal force. For simplicity (and avoiding the uncertainty connected with the velocity polarization of the waves) we consider a coupling between purely radial
axisymmetric pulsation and purely torsional Alfvén waves, thus we take \( \partial / \partial \phi = 0 \). Of course, realistic conditions are much more complicated (tidally induced oscillations probably depends on \( \phi \)), however for a better understanding of the coupling mechanism it is worth looking at the simplest case.

Then the linearized equations (1)-(6) can be split into radial and toroidal components, where the radial component corresponds to pulsations and the toroidal component to torsional waves.

The radial part of the equations is

\[
\frac{\partial b_\theta}{\partial t} = -B_\theta \frac{\partial u_r}{\partial r} - \frac{\partial B_\theta}{\partial r} u_r - \frac{B_\theta}{r} u_r, \tag{8}
\]

\[
\frac{\partial \rho}{\partial t} = -\rho_0 \frac{\partial u_r}{\partial r} - \frac{\partial \rho_0}{\partial r} u_r - 2 \frac{\rho_0}{r} u_r, \tag{9}
\]

\[
\rho_0 \frac{\partial u_r}{\partial t} = -\frac{\partial}{\partial r} \left[ \delta p + \frac{B_\theta b_\theta}{\mu} \right] - \frac{2 B_\theta b_\theta}{\mu r} - \delta \rho \frac{\partial \phi}{\partial r}, \tag{10}
\]

\[
\frac{\partial \delta p}{\partial t} + u_r \frac{\partial p_0}{\partial r} = c_0^2 \left( \frac{\partial \delta \rho}{\partial t} + u_r \frac{\partial \rho_0}{\partial r} \right), \tag{11}
\]

where \( c_0 = \sqrt{\gamma p_0 / \rho_0} \) is the sound speed, while the toroidal part is

\[
\frac{\partial b_\phi}{\partial t} = \frac{B_\phi}{r} \frac{\partial u_\phi}{\partial \theta}, \tag{12}
\]

\[
\rho_0 \frac{\partial u_\phi}{\partial t} = \frac{B_\theta b_\phi}{\mu r} \frac{\partial \phi}{\partial \theta}. \tag{13}
\]

\( u_r, u_\phi, b_\theta, b_\phi, \delta \rho \) and \( \delta p \) are the velocity, magnetic field, density and pressure perturbations respectively. Equations (8)-(11) govern the radial pulsation of the star, while equations (12)-(13) describe the torsional Alfvén waves. It must be noticed that if an unperturbed purely poloidal magnetic field (7) is replaced by a purely toroidal field \( B = (0, 0, B_\phi(r)) \), equations (8)-(13) remain unchanged. The Alfvén waves now will be described by \( b_\theta, u_\theta \) instead of \( b_\phi, u_\phi \). Therefore the phenomenon does not depend on the particular magnetic field configuration.

In general, stellar radial adiabatic pulsation can be represented as a standing spherical wave (Cox 1980; Biront et al. 1982; Roberts and Soward 1983; Christensen-Dalsgaard 1988), with a linear radial velocity field

\[
u_r = \alpha F(r) \sin(\omega_n t), \tag{14}\]

where \( \omega_n \) is the eigenfrequency, \( F(r) \) is the eigenfunction and \( \alpha \) is the pulsation amplitude. The expression of the eigenfunction \( F(r) \) depends on the spatial profiles of the unperturbed
values and may be represented by some combination of spherical Bessel functions. However, here we are not interested in specific pulsation functional forms, but our aim is to show that pulsation in general leads to amplification of torsional Alfvén waves. Therefore we retain its general form \( F(r) \), which then may be specified by choosing the spatial distribution of pressure, density and magnetic field throughout the star. However it must be mentioned that equations (8)-(11) govern only the fast mode of pulsation, because the velocity is strictly across the field lines. The equations governing the slow mode of pulsation must also concern the radial component of the magnetic field \( B_r \), which significantly complicates the problem. Therefore we describe only the coupling between the fast mode of pulsation and the torsional Alfvén waves, keeping in mind that a similar physical process occurs for the slow mode too.

Using expression (14) we find the density and magnetic field perturbations from the linearized continuity and induction equations (8)-(9):

\[
\delta \rho = \frac{\alpha}{\omega_n} \cos(\omega_n t) F_\rho(r),
\]

\[
b_\theta = \frac{\alpha}{\omega_n} \cos(\omega_n t) F_b(r),
\]

where

\[
F_\rho(r) = \rho_0(r) \frac{\partial F(r)}{\partial r} + \left( \frac{\partial \rho_0(r)}{\partial r} + \frac{2 \rho_0(r)}{r} \right) F(r),
\]

\[
F_b(r) = B_\theta(r) \frac{\partial F(r)}{\partial r} + \left( \frac{\partial B_\theta(r)}{\partial r} + \frac{B_\theta(r)}{r} \right) F(r).
\]

Equations (15)-(16) show that the radial pulsation leads to local periodical variation of density and magnetic field throughout the star. It must be mentioned that the fundamental mode of pulsation can be interpreted as a standing wave with one velocity node at the stellar centre and one antinode at the surface, which imposes the corresponding boundary conditions on the eigenfunction \( F(r) \).

The variation of magnetic field and density due to pulsation leads to periodical variation of the Alfvén speed at each level \( r \). In next section we show that it determines exponential amplification of torsional Alfvén waves with half the frequency of pulsation.

### 2.2. Swing amplification of torsional Alfvén waves due to the radial pulsation of the star

The good mechanical analogue of swing coupling between longitudinal and transversal waves is a mathematical pendulum with a stiffness spring (Zaqarashvili and Roberts 2002b).
The system includes two kinds of oscillations: the pendulum transversal oscillations due to gravity and the spring oscillation along the pendulum axis due to stiffness force. The oscillations are coupled and it is shown that in certain conditions, when the frequency of spring oscillations is twice as large as that of pendulum transversal oscillations, the energy of spring oscillations along the pendulum axis is transferred to transversal oscillations and vice versa.

In the case of a star, the pulsation corresponds to the spring oscillation and the torsional Alfvén waves correspond to the transversal oscillations. The torsional Alfvén waves are the result of the magnetic tension force action against the fluid inertia. On the other hand, the pulsation makes the work against the tension force during each compression and thus may lead to amplification of the transversal oscillations with half the frequency of the pulsation (Zaqarashvili and Roberts 2002a,b).

In order to study the influence of pulsations on Alfvén waves we must retain the expressions of \( u_r, \delta \rho \) and \( b_\theta \) in equations (12)-(13) which now take the form

\[
\frac{\partial b_\theta}{\partial t} = \frac{B_\theta + b_\theta}{r} \frac{\partial u_\phi}{\partial \theta} - \frac{u_r}{r} \frac{\partial b_\theta}{\partial \theta},
\]

(19)

\[
(\rho_0 + \delta \rho) \frac{\partial u_\phi}{\partial t} + \frac{(\rho_0 + \delta \rho) u_r}{r} u_\phi = \frac{B_\theta + b_\theta}{\mu r} \frac{\partial b_\theta}{\partial \theta}.
\]

(20)

From these equations we derive the Hill type second order differential equation with periodical coefficients

\[
\frac{\partial^2 u_\phi}{\partial t^2} + \left( \frac{\partial u_r}{\partial r} + \frac{2u_r}{r} + \frac{1}{\rho_0 + \delta \rho} \frac{\partial \delta \rho}{\partial t} - \frac{1}{B_\theta + b_\theta} \frac{\partial b_\theta}{\partial t} \right) \frac{\partial u_\phi}{\partial t} -
\]

\[
- \left[ \frac{(B_\theta + b_\theta)^2}{\mu (\rho_0 - \delta \rho)^2} \frac{\partial^2}{\partial \theta^2} + \frac{1}{\rho_0 + \delta \rho} \frac{\partial u_r}{\partial t} + \frac{1}{B_\theta + b_\theta} \frac{u_r}{r} \frac{\partial b_\theta}{\partial t} - \frac{1}{\rho_0 + \delta \rho} \frac{u_r}{r} \frac{\partial \delta \rho}{\partial t} - \frac{u_r}{r} \frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} \right] u_\phi = 0.
\]

(21)

Using the well-known transformation

\[
u_\phi = u(t) \exp \left( -\frac{1}{2} \int \left[ \frac{\partial u_r}{\partial r} + \frac{2u_r}{r} + \frac{1}{\rho_0 + \delta \rho} \frac{\partial \delta \rho}{\partial t} - \frac{1}{B_\theta + b_\theta} \frac{\partial b_\theta}{\partial t} \right] dt \right)
\]

(22)

leads to elimination of the first time derivative and after neglecting terms of order \( \alpha^2 \) we have

\[
\frac{\partial^2 u}{\partial t^2} - \frac{V_A^2(r)}{r^2} \left[ 1 + \frac{\alpha}{\omega n} \left( 2 \frac{F_b(r)}{B_\theta(r)} - \frac{F_\rho(r)}{\rho_0(r)} \right) \cos(\omega_n t) \right] \frac{\partial^2 u}{\partial \theta^2} +
\]

\[
+ \frac{\alpha \omega_n}{2} \left[ -\frac{\partial F(r)}{\partial r} + \frac{F_\rho(r)}{\rho_0(r)} - \frac{F_b(r)}{B_\theta(r)} \right] \cos(\omega_n t) u = 0,
\]

(23)
where
\[ V_A(r) = \frac{B_\theta(r)}{\sqrt{\mu \rho_0(r)}} \] (24)

is the Alfvén speed. Equation (23) has time-dependent coefficients while the radial coordinate \( r \) stands as a parameter. Then we can perform a Fourier transform with \( \theta \) dependence
\[ u = \int \hat{u}(l, t) e^{i \theta} dl, \] (25)

which leads to Mathieu’s equation
\[ \frac{\partial^2 \hat{u}}{\partial t^2} + \left[ \frac{V_A^2(r) l^2}{r^2} + \alpha \Psi(r) \cos(\omega_n t) \right] \hat{u} = 0, \] (26)

where
\[ \Psi(r) = \frac{V_A^2(r) l^2}{\omega_n r^2} \left( 2 \frac{F_b(r)}{B_\theta(r)} - \frac{F_\rho(r)}{\rho_0(r)} \right) - \frac{\omega_n}{2} \left( \frac{\partial F(r)}{\partial r} - \frac{F_\rho(r)}{\rho_0(r)} + \frac{F_b(r)}{B_\theta(r)} \right). \] (27)

This equation is well studied and its main resonant solution occurs when the frequency of the Alfvén waves is half that of the external force
\[ \omega_A = \frac{B_\theta(r) l}{r \sqrt{\mu \rho_0(r)}} = \frac{\omega_n}{2}. \] (28)

In this case equation (26) has the exponentially growing solution (Landau and Lifshitz 1988)
\[ \hat{u}(t) = \hat{u}_0 e^{\frac{\omega \Psi(r)}{2 \omega_n} t} \left[ \cos \frac{\omega_n t}{2} + \sin \frac{\omega_n t}{2} \right], \] (29)

where \( \hat{u}_0 = \hat{u}(0) \) and the phase sign depends on \( \alpha \Psi(r) \); it is + for negative \( \alpha \Psi(r) \) and − for positive \( \alpha \Psi(r) \). Note that the solution has a resonant character within the frequency interval
\[ \left| \omega_A - \frac{\omega_n}{2} \right| < \left| \frac{\alpha \Psi}{\omega_n} \right|. \] (30)

Thus pulsation leads to exponential amplification of torsional Alfvén waves at half frequency \( \frac{1}{2} \omega_n \). The growth rate depends on the amplitude \( \alpha \) and spatial structure of the pulsation eigenfunction \( F(r) \), which in turn depends on the radial structure of density \( \rho_0 \) and magnetic field \( B_\theta \). The resonant condition (28) imposes a restriction on the wave number \( l \) at each distance \( r \) from the stellar center. In other words, the pulsation “picks up” the harmonics of torsional Alfvén waves with a certain wavelength at each distance \( r \). It gives
the idea that at some distances from the stellar center, where the conditions for the onset of standing waves are satisfied

\[ l = \frac{\omega_n r}{2 \frac{\mu \rho_0 (r)}{B_\theta (r)}} = 1, 2, 3, \ldots \]  

(31)

the amplified Alfvén waves may lead to the setup of torsional oscillations. In these regions the pulsation energy can be dramatically "absorbed" by transversal oscillations.

It is clear that the energy transferred into torsional Alfvén waves is to be extracted from pulsation. In fact the term \(-b_\phi^2/\mu r - \partial(b_\phi^2/2\mu)/\partial r\) (or ponderomotive force), which describes the back reaction of Alfvén waves on pulsation, should be added in the righthand-side of the equation (10). At the initial stage, this term is of second order strength, however it becomes significant after some time because of the exponential growth of \(b_\phi\) and leads to the damping of pulsation. This process is clearly seen in the rectangular case (Zaqarashvili and Roberts 2002b).

### 3. Discussion

Observations show enhanced chromospheric and coronal activity in relatively close binaries (Schrijver and Zwaan 1991). The binary systems considered in that paper are well separated so that mass transfer by Roche-lobe overflow does not occur. The chromospheric, transition-region and coronal emissions from the binaries are enhanced in comparison to single stars with the same mass, chemical composition, age and mean surface rotation rate. This is somehow a strange phenomenon, because in the framework of dynamo theory the magnetic activity does not depend on whether the star is single or component of a binary system.

It is clear however that tidal interaction somehow causes the enhancement of magnetic activity. It leads to the idea that some mechanism, other than classical dynamo, may generate in the stellar interior the magnetic fields that give rise to the strong activity observed in the considered binary systems. Several alternative mechanisms were supposed to explain magnetic activity in the Sun and late type stars (see the review of Belvedere (1985)). Most of them are connected to the hydromagnetic oscillation of a seed magnetic field either in the core (Dicke 1982; Gough 1988) or in the radiative layers (Walén 1949; Cowling 1953; Piddington 1971; Layzer et al. 1979; Zaqarashvili 1997). However, a drawback of these mechanisms is the absence of an energy source to support the oscillations. The recently suggested mechanism (Zaqarashvili and Roberts 2002a) of transformation of pulsation energy into energy of torsional oscillations is the first step towards this direction. If this is the case,
then any energy source able to support the pulsation can be also considered as the source of torsional oscillations.

Indeed, here we suggest that torsional oscillations, amplified by tidally induced pulsation, may be responsible for enhanced magnetic activity in tidally interacting binaries. The proposed mechanism is alternative to the mean field dynamo, therefore the presence of a dynamo field is not considered. It is also clear that in our paper we suggest a self-consistent mechanism which assumes a seed field in the radiative interior, but operates differently from the alternative mechanisms suggested by the authors quoted above which, on the other hand, refer only to the Sun.

We suppose that the deviation of the star from spherical symmetry due to tidal interaction sets up its pulsation in the fundamental mode like a tuning fork. The companion’s gravity may induce stellar pulsation either due to resonance between a dynamic tide and g-mode oscillations (Cowling 1941; Zahn 1970; Smeyers et al. 1998; Willems and Aerts 2002) or to an unstable retarded tide in asynchronous systems. The conditions required for resonance between the periodic tidal force and the stellar free oscillations can be fulfilled only for low-frequency g-modes. The fundamental frequency of pulsation is usually higher (for the Sun, it corresponds to a few hours) than the orbital angular frequency of binary systems. Although observational evidence of tidally excited resonant oscillations is not well established, possibly due to the absence of any systematic study (Willems and Aerts 2002), on the other hand, a retarded tide in asynchronous binaries will be unstable and may lead to stellar pulsation in the fundamental mode.

However the existence of asynchronous orbits in relatively close binary systems is under question. The tidal theory (Zahn 1977) predicts that spin-orbit synchronization of a component of a late-type close binary system occurs before the orbit becomes circular, unless the spin angular momentum is comparable to the orbital one (Hut 1980; Savonije and Papaloizou 1984). However, several binary systems TZ For, λ And, AY Cet and α Aur include asynchronously rotating giants in circular orbits (Anderson et al. 1984; Hall 1986). A very interesting RS CVn-type system is λ And, which is asynchronous and shows enhanced activity (Donati et al. 1995). For instance, Habets and Zwaan (1989) try to explain asynchronous rotation in close binary systems with circular orbits, however the problem still remains.

Thus we suggest that the tidal force exerted by the companion on the primary in binaries may induce stellar radial pulsation either due to resonance or due to a retarded tide. The next question is how pulsation may affect magnetic activity.

Suppose the star has large-scale dipole-like seed magnetic field in the interior. As we already discussed, the various magnetic configurations, mostly purely toroidal or purely poloidal, were found to be dynamically unstable (Tayler 1973; Wright 1973; Markey and
Tayler 1973). However special stable configurations with poloidal and toroidal field of similar strength might exist (Tayler 1980; Mestel 1984). Also plasma relaxation theory (Taylor 1986) yields that instabilities may lead to a particular minimum-energy state (mainly force-free), which is relatively stable. Therefore for simplicity we consider the magnetic field to be purely poloidal. Notice that a purely toroidal magnetic field in the equatorial regions leads to the same equations, which indicates that the physics of the phenomenon does not depend on the particular field configuration. The strength of the unperturbed magnetic field in the interior cannot be evaluated since the analysis is linear. However, let us try to give an estimate of the energy which can be transferred to magnetic oscillations due to tidal interaction, and of the mean magnetic field strength. Let the binary system components have mass $M$ and radius $R$ of the order of the solar ones. The energy of tidal interaction is $GM^2R/d^2$, where $d$ is the orbital separation. Then, for an orbital separation $d \sim 100R$, this energy is of the order of $10^{45}$ ergs. The total magnetic field energy is expressed by: $B^2/2\mu (4/3)\pi R^4$. If we assume that all the tidal interaction energy goes into magnetic field energy, we get a mean equipartition magnetic field $B \sim 10^6$ G, but this is clearly an upper limit. Indeed this result is obtained in the ideal case that the whole tidal interaction energy is converted into the magnetic one. For a more realistic situation let us suppose that, due to viscous and ohmic dissipation, the conversion efficiency be only 1%. Even with this low efficiency, the magnetic field strength can attain values as high as $\sim 10^5$ G, which are comparable with those commonly believed to exist at the base of the convection zone of the Sun and active stars. These values are not very different from the surface fields measured in very active stars that may reach $10^4$ G or more.

In the presence of a large-scale magnetic field, pulsation induces a periodical variation of the local Alfvén speed at each distance from the stellar centre, which in turn leads to a crucial influence on the dynamics of torsional Alfvén waves. We found that the time behaviour of torsional Alfvén waves is governed by Mathieu’s equation (26), therefore the harmonics with half the frequency of pulsation grow exponentially in time. The growth rate of torsional Alfvén waves depends on the amplitude of pulsation (see also Zaqarashvili and Roberts (2002a)). This is a very important result, because pulsations can be easily excited by any non-electromagnetic force. Then the torsional oscillations may have a number of energy sources, which can be of importance for the study of stellar activity in general. However the discussion of these situations is beyond the scope of this paper, therefore we only consider magnetic activity in binary stars. Gravitational energy in binary systems is much larger than the energy involved in magnetic activity. Unfortunately, it is not possible to evaluate the amount of energy transferred to torsional oscillations, because the theory is basically linear. However, it is reasonable to consider the back-reaction of amplified Alfvén waves on pulsation itself. Energy conservation yields that the amount of the energy transferred into
Alfvén waves must be extracted from pulsation itself. Indeed, this is the case, as can be easily shown in rectangular geometry (Zaqarashvili and Roberts 2002b). However, if pulsation has a continuous energy source (in our case, the companion’s gravity), then Alfvén waves will grow until their amplitudes exceed a certain value, after which magnetic buoyancy probably leads to eruption of magnetic tubes at the surface. There, they may release their energy, this leading to enhanced chromospheric and coronal activity.

In our model, we expect chromospheric and coronal activity to be stronger in asynchronous binaries. Some observations seem to indicate it (Dempsey et al. 1993), however future establishment of a better link between theory and observations is needed.

Another interesting point is that the unperturbed magnetic field in the stellar interior causes the splitting of the fundamental frequency of adiabatic pulsation into a fast and a slow mode. Both modes can be interpreted as spherical standing waves with one node in the center and one antinode at the surface. The fast mode corresponds to fast magnetosonic waves, and, for weak magnetic field, their frequency is of the order of \( c_0/R \). The slow mode corresponds to slow magnetosonic waves, and their frequency is of the order of \( v_A/R \). The difference between these frequencies depends on the ratio of hydrodynamic and magnetic pressures and can be very large for weak magnetic fields. So a star with an even weak magnetic field should have two pulsation timescales. This means that resonant Alfvén waves must also have two timescales, which correspond to fast and slow modes. In this paper we consider the coupling between the fast mode of pulsation and torsional waves, because the description of the fast mode is relatively easy, while the slow mode of pulsation requires more complicated mathematical formulation. However, the physical meaning of the coupling is similar, therefore we may argue about the coupling between the slow mode of pulsation and torsional waves without giving detailed calculations (however it is essential to study the details of their coupling in future). The particular characteristic of the coupling between the slow mode and torsional Alfvén waves is that the latter have spatial scales comparable to the stellar radius, because Alfvén and slow magnetosonic waves have similar phase velocities in the case of a weak magnetic field. Therefore they may lead to the onset of large-scale torsional oscillations.

4. Conclusion

In conclusion we summarize the process which gives rise to enhanced chromospheric and coronal activity in tidally interacting binaries in the three steps:

I - tidal interaction causes deviation of the star from spherical symmetry, which leads
to stellar pulsation in its fundamental frequency;

II - pulsation amplifies torsional Alfvén waves through the mechanism of swing wave-wave interaction which leads to onset of torsional oscillations in the radiative interior;

III - magnetic flux tubes erupted at the surface by magnetic buoyancy enhance chromospheric and coronal emission.

However, the proposed mechanism needs future study. The details of the mechanism of amplified pulsation due to tidal interaction should be specified. It is also essential to give a detailed mathematical formulation of the coupling between the slow mode of pulsation and torsional Alfvén waves, which may lead to large-scale torsional oscillations of a dipole-like magnetic field.

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