Constraints on Star Formation from the Close Packing of Protostars in Clusters

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ABSTRACT

The mm-wave continuum sources (MCS) in Ophiuchus have mutual collision rates less than their collapse rates by a factor of 10 to 100, suggesting most will form stars without further interactions. However, the ratio of these rates would have exceeded unity in the past if they were only 2.5 times larger than they are now. Such a high previous ratio suggests three possible scenarios: (1) the MCS contracted from lower densities and acquired their present masses through collisional agglomeration, (2) they contracted independently from lower densities elsewhere and moved to the cluster core recently, or (3) they grew from smaller sizes at a constant high density. The third of these is most likely, implying that the MCS formed in the shocked regions of a supersonically turbulent fluid. The first scenario gives the wrong mass function and the second does not give the observed hierarchical clustering. The ratio of rates also exceeds unity today if the MCS have envelopes with smooth profiles that end in pressure balance with the ambient cloud cores; this suggests again that turbulent flows define their outer layers. Proximity constraints like this are even more important in massive clusters, including globular clusters, in which massive stars with the same or greater space density are more strongly interacting than the Ophiuchus MCS. As a result, the density contrast for MCS must be larger in massive clusters than it is in Ophiuchus or else significant coalescence will occur in the protostellar phase, possible forming massive black holes. A proportionality to the second power of the Mach number allows the MCS cores to collapse independently. These results suggest that stars in dense clusters generally form on a dynamical time by the continuous collection and rapid collapse of turbulence-shocked gas. Implications of proximity constraints on the initial stellar mass...
function are also discussed. Warm cloud cores can produce a top-heavy IMF because of a simultaneous increase in the thermal Jeans mass and the collisional destruction rate of low mass MCS.

**Key words:** stars: formation; stars: mass function; stars:clusters; ISM: clouds

## 1 INTRODUCTION

The mm-wave continuum sources (hereafter MCS) in Ophiuchus (Motte, André & Neri 1998; Johnstone et al. 2000), Serpens (Testi & Sargent 1998; Testi et al. 2000), and Orion (Johnstone et al. 2001) should reveal the process of their own formation. They appear to be the last phase in the gas before it collapses into stars. According to Motte et al., the typical density of an MCS in the Oph B2 core is $\sim 6 \times 10^7 \, \text{H}_2 \, \text{cm}^{-3}$, the size is $\sim 3000 \, \text{AU}$, the mass is $\sim 0.3 \, M_\odot$, and the projected separations are $\sim 3 \, \text{arcmin} \sim 0.1 \, \text{pc} \sim 20,000 \, \text{AU}$. This separation corresponds to a spatial density on the order of $10^3 \, \text{pc}^{-3}$, considering that the overall core size is about the same 0.1 pc. This density is comparable to that in embedded star clusters (McCaughrean & Stauffer 1994; Carpenter et al. 1997).

The relative velocities between the MCS are probably $\sim 2^{1/2}$ times the three-dimensional virial speeds in their cloud cores, which for Oph B2 is $(6GM/5R)^{1/2} \sim 2 \, \text{km s}^{-1}$ for a core mass of $M = 50 \, M_\odot$ and a core radius $R = 0.055 \, \text{pc}$ (Motte et al.). Sources outside of dense cores may have smaller velocity dispersions (Testi et al. 2000; Belloche, André, & Motte 2001). If the dispersion is small and the MCS are far apart, then they will not interact before they collapse into stars and the final stellar mass function will be about the same as the MCS mass function. However, if the dispersion and density of MCS in a cloud core are high enough, then some may collide before they collapse, and this can affect the mass function (e.g., Bonnell, Bate, & Zinnecker 1998).

The relative collision rate should have been higher in the past if the MCS passed through a lower density state. The collision rate scales approximately with the square of the MCS radius, $R^2$, and the collapse rate scales with $R^{-3/2}$, from the square root of density with mass conservation. The ratio of the collision rate to the collapse rate therefore scales with $R^{7/2}$, which is a sensitive function of MCS size. This implies that even if the MCS are not collision-dominated now, they did not have to be much larger in the past before their

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internal dynamical evolution was severely affected by interactions. If such interactions were
destructive for the MCS or detrimental to their stellar-like mass function (Motte et al. 1998),
then they could not have formed by contraction from the cloud core through a sequence of
nearly constant mass, but had to form from smaller masses by coagulation or accumulation
through a sequence of near-constant density, or they had to form elsewhere.

Here we determine the relative collision rates for the MCS in core regions B and C
of Ophiuchus. We also determine the relative rates for idealized MCS with radial density
profiles from Whitworth & Ward-Thompson (2001) and a mass distribution from the initial
stellar mass function (IMF).

The ratio of the collision rate to the collapse rate increases with the mass of an MCS.
This means that massive clusters with the same or higher densities will have more severe
crowding problems than the cores in Ophiuchus, which is making only low-mass stars. This
implies that the contrast between the gas density in an MCS and the gas density in the
ambient cloud core must increase with cluster mass or density. This effect is modelled in
Section 3.

An important simplification in this problem is that collisional destruction of MCS should
be about the same for interactions with other MCS as it is for interactions with protostars
and stars. This is because the grazing collision distance between one MCS of mass $M_i$ and
another of mass $M_j$ having the same density is equal to the tidal distance for the destruction
of the first MCS by a point source of mass $M_j$. Tidal destruction occurs when an MCS enters
a region where the average density is the same as its own density or higher. Thus our result
is unchanged if only a fraction of the final stars are present as MCS at any one time. Most
likely, there is a continuous conversion of existing MCS into stars while new MCS are forming
out of the ambient cloud core gas. The ratio of the numbers of MCS to final stars will be
about the same as the ratio of the dynamical times, which is the inverse square root of the
density contrast.

We also consider whether the turnover mass in the IMF might depend on the cluster core
density, taking higher values for denser clusters because of limitations from crowding. The
effect is probably small, but it may be observable under extreme conditions. For example,
coalescence in the protostellar phase of very dense clusters could shift a significant fraction
of low-mass MCS to higher mass. Considering how close the Salpeter IMF is to an $M^{-2}dM$
form (this form has equal total mass in equal logarithmic intervals of mass), this shift
means that an IMF which extends as a Salpeter power law to low mass but falls below
the Salpeter power law at high mass in a low density environment might shift to one that
turns over at low mass and has a Salpeter power-law slope at high mass in a high density
environment. Observations of the IMF support these changes with stellar density, although
other explanations are possible.

2 THE RELATIVE COLLISION RATE

The collision rate of an MCS is determined from the space density of its neighbors, the
relative velocity dispersion, and the gravitationally-enhanced collision cross section. The
collapse rate is determined from the MCS density. The ratio of these two rates indicates the
relative probability of collisions versus isolated collapse. If the ratio exceeds unity then colli-
sions should be important. Here we use the observed sizes and masses for MCS tabulated by
Motte, Andre & Neri (1998) and we determine the ratio of these rates to see how important
collisions are. We then consider the same ratios again for the MCS with hypothetically lower
densities. The results are then generalized to the case in which the number of MCS is given
by the initial mass function and each MCS has an internal density profile from Whitworth

2.1 Observed mm-wave continuum sources in Ophiuchus

A field of \( N \) MCS with radii \( R_j \), mass \( M_j \), and uniform velocity dispersion \( v \) inside a cloud
core with radius \( R_{\text{cloud}} \) and mass \( M_{\text{cloud}} \) has grazing (hard-sphere) collisions with a particular
MCS of radius \( R_i \), mass \( M_i \), and velocity \( v \) at the rate

\[
\omega_i = \frac{\pi}{4\pi R_{\text{cloud}}^3/3} \sum_{j=1}^{N} (R_i + R_j)^2 2^{1/2} v \left[ 1 + \frac{2G(M_i + M_j)}{2v^2(R_i + R_j)} \right].
\]  

(1)

The virial speed in the cloud core gives the three-dimensional velocity dispersion,

\[
v = (3GM/5R)^{1/2},
\]  

(2)

which is an approximation valid for a uniform cloud; the extra factors of \( 2^{1/2} \) near \( v \) and \( 2 \)
near \( v^2 \) in equation (1) are from relative motion assuming the same speed for all MCS. The
collapse rate for a uniform MCS of density \( \rho_i = 3M_i/(4\pi R_i^3) \) is

\[
\omega_{\text{collapse},i} = \left(32G\rho_i/3\pi\right)^{1/2}.
\]  

(3)
Figure 1. The ratios of collision rate to collapse rate for mm-wave continuum sources (MCS) in Ophiuchus are shown versus the MCS masses using solid symbols. The source Oph B2 is indicated by diamonds and Oph C by circular symbols. The ratios increase slightly with mass but are generally less than one. The ratios are shown again as open symbols for MCS radii larger than observed by a factor of 2.5. Even this small change in radius would cause several of the MCS to collide or coalesce before they could collapse into a star, as the plotted ratios become larger than 1. This suggests that the MCS did not pass through a lower density stage with their present masses.

The ratios of the collision rates to the collapse rates for MCS in the B and C cores of Ophiuchus are shown in Figure 1, plotted as solid symbols versus the MCS mass. The ratios are in the range from 0.001 to 0.1, increasing slightly with mass for Oph B. Such low values indicate that collisions are slower than internal dynamical processes. If the MCS are gravitationally unstable to collapse into stars, then they will do this independently, without interactions.

The same ratios are plotted again in Figure 1 as open symbols for a hypothetical previous state in which each MCS was larger than it is today by a factor of 2.5. The MCS masses are assumed to be unchanged. A small difference in MCS size corresponds to a large difference in relative collision rate, so with larger sizes, some MCS have ratios of rates larger than 1. These MCS could not have reached their present state by purely dynamical collapse.
without strongly interacting with previous neighbors. Internal evolution that is slower than the collapse rate makes previous interactions even more likely, as would the presence of a larger number of MCS in the previous cloud core.

The dynamical time scale in the present-day MCS is only \( \sim 4000 \) years at the average MCS density of \( \sim 6 \times 10^7 \) \( \text{H}_2 \) cm\(^{-3}\). The dynamical time in each whole core is \( \sim 10 \) times longer because of the \( 100 \times \) lower average core density. If the MCS and the cores around them both evolved dynamically on their own time scales, then the MCS would have had twice their present radii only several \( \times 10^4 \) years ago, when the cores as a whole were not much different than they are today. This is such a short time ago compared to the age of the whole Ophiuchus star forming region that it seems unlikely the MCS recently suffered some collision without producing tidal features such as bridges or tails. Most likely the MCS have not been in a lower density state since they were part of the ambient cloud core gas.

2.2 Protostars with Smooth Density Profiles

The MCS in Ophiuchus are denser than the average cloud cores by a factor of \( \sim 100 \), suggesting a mismatch in thermal pressure. The boundaries of the MCS must therefore have either a gradual density gradient, taking the MCS density down to the cloud core density, or a shock front at which the core thermal pressure balances an inter-MCS ram pressure from relative motions or turbulent flows. Boundaries like this are required whether or not the MCS are self-gravitating.

Dense mm-wave sources are often observed to have density gradients like the one fitted by Whitworth & Ward-Thompson (2001), which is

\[
\rho(r) = \frac{\rho_{\text{flat}}}{\left[1 + (r/R_{\text{flat}})^2\right]^2}, \tag{4}
\]

for radius \( R_{\text{flat}} \) and density \( \rho_{\text{flat}} \) in the central region where the profile flattens. The density contrast is represented by

\[
\mathcal{C} = \frac{\rho_{\text{flat}}}{\rho_s}, \tag{5}
\]

where \( \rho_s \) is the surface density of each MCS. The parameter \( \mathcal{C} \) depends on the definition of the surface. We assume that the MCS surface occurs where its density equals the average density in the cloud core (e.g., \( \mathcal{C} = 50 \) is the ratio of the average MCS density to the average cloud core density in Oph B2; \( \mathcal{C} = 187 \) is the average for Oph C – Motte et al. 1998).

Equations (4) and (5) give the radius of the MCS, \( R_s \), as a function of \( \mathcal{C} \) and \( R_{\text{flat}} \).
\[ R_s = a R_{\text{flat}} \quad \text{for} \quad a = \left( C^{1/2} - 1 \right)^{1/2}. \]  

For mass
\[ M = \int_0^{R_s} 4\pi r^2 \rho(r) dr, \]  

\[ R_{\text{flat}} = \left( \frac{M}{2\pi \rho_{\text{flat}}} \right)^{1/3} \left( \arctan(a) - \frac{a}{1 + a^2} \right)^{-1/3}. \]

The average density of an MCS is
\[ \rho_{\text{av}} = \frac{3M}{4\pi R_s^3} = \frac{3\rho_{\text{flat}}}{2a^3} \left( \arctan(a) - \frac{a}{1 + a^2} \right), \]

giving an average collapse rate
\[ \omega_{\text{collapse,a}} = \left( \frac{32G\rho_{\text{av}}}{3\pi} \right)^{1/2}. \]

The collapse rate in the flat part of the density profile is
\[ \omega_{\text{collapse,f}} = \left( \frac{32G\rho_{\text{flat}}}{3\pi} \right)^{1/2}. \]

A cluster of MCS with smooth density profiles has a higher relative collision rate than what we have just calculated for the MCS in Ophiuchus because smoothly tapered MCS are bigger and their average densities are comparable to the cloud core density, giving them a longer collapse time. For a general case, the collision rate for an MCS depends on the space density of other MCS, which is a function of mass through the IMF.

A convenient form for the IMF at low to moderate mass is
\[ n(M) dM = n_0 M^{-2.3} \left[ 1 - \exp \left( -M^2 / M_t^2 \right) \right] dM, \]

which has a Salpeter slope of -2.3 toward higher mass and a turn-over toward lower masses at \( M_t \) (see Elmegreen 1997). The turnover mass may equal the thermal Jeans mass unless crowding limitations are important; we discuss these limitations in Section 4. We introduce a high-mass modification to the IMF in Section 3 (see Fig. 3).

To get \( n_0 \), we set the lower mass limit from observations of Ophiuchus, we equate the total MCS mass in the IMF to the observed total, and we constrain the number of MCS with a mass greater than some maximum mass to be unity. In this way, the IMF has the correct total mass and it also falls off gradually at high mass to give only one most massive star. The minimum mass, \( M_{\text{min}} \), is set equal to 0.1 M\(_{\odot}\) for both Oph B2 and Oph C (see Fig. 1). The total MCS masses in both Ophiuchus B and C are equal to the fraction \( \epsilon = 0.13 \) times the cloud core masses.

The collision rate for MCS \( i \) is now obtained by integrating over the IMF,
Oph B2: $M_{\text{cloud}} = 50 \, M_\odot$, $R_{\text{cloud}} = 0.055 \, \text{pc}$, $\varepsilon = 0.13$, $C = 50$, $M_{\text{min}} = 0.1 \, M_\odot$

Oph C: $M_{\text{cloud}} = 34 \, M_\odot$, $R_{\text{cloud}} = 0.1 \, \text{pc}$, $\varepsilon = 0.13$, $C = 187$, $M_{\text{min}} = 0.1 \, M_\odot$

Figure 2. The ratio of collision to collapse rates is shown versus the MCS mass for an idealized case representative of the cores in Ophiuchus. The MCS masses are distributed according to the IMF and they have smooth density profiles. The lower four curves are for the flat parts of the MCS only, while the upper four curves include the envelopes also. $M_t$ is the turnover mass (in $M_\odot$) in the IMF.
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\[ \omega_{\text{collision}}(M_i) = \frac{\pi^{21/2} v}{4\pi R_{\text{cloud}}^3/3} \int_{M_{\text{min}}}^{\infty} n(M_j) (R_i + R_j)^2 \left[ 1 + \frac{2G(M_i + M_j)}{2v^2 (R_i + R_j)} \right] dM_j \] (13)

The results of this model are shown in Figure 2. The ratios of the collision rates to the average collapse rates for MCS are plotted as a function of MCS mass. Solid lines are for an idealized model of Oph B and dashed lines are for Oph C. Two different values of the turn-over mass in the IMF, \( M_t \), are assumed. The lower set of curves is for the MCS cores only, which come from the flat part of the Whitworth & Ward-Thompson profile only. That is, it uses \( \rho_{\text{flat}} \) for the collapse rate, and \( R_{\text{flat}}(M) \) for the collision cross section.

Figure 2 reproduces the ratios of the collision rates to the collapse rates found in Figure 1 when only the flat parts of the density profiles are considered. These ratios are small, so the flat parts should not collide with each other before they collapse into stars. However, if the MCS have envelopes where the density drops down to the average value in the cloud core, which corresponds to an extra factor of \( a \sim 3 \) in radius, then the envelopes will interact significantly before they get involved with the collapse. Thus MCS in dense cluster environments should not have significant envelopes.

3 GLOBULAR CLUSTERS

Other models were considered for more massive clusters. When the cluster mass is large, the largest MCS mass is also large, forcing us to modify the IMF so that, for example, 1000 \( M_\odot \) stars do not form. We follow the discussion in Elmegreen (2000) and use for this case an IMF of the form:

\[ n(M)dM = n_0 M^{-2.3} \left[ 1 - \exp \left( -M^2/M_t^2 \right) \right] \exp \left( -M/100 M_\odot \right) dM, \] (14)

which has a cutoff at high mass. The IMFs in equations (12) and (14) are shown in Figure 3 (multiplied by \( M \) to convert to a log \( M \) coordinate). The dotted line is from equation (12) for \( M_t = 0.1 \); it blends with the solid line at low mass, which is from equation (14) using the same \( M_t \). The dashed line is from equation (14) using \( M_t = 1 M_\odot \) and the same total cluster mass as the solid line. The two straight lines show slopes of \(-1.3\) and \(-1.6\) for comparison; these bracket the IMF observations in the indicated mass range. There are no observations of an IMF for masses much larger than 100 \( M_\odot \), so the fall off is purely conjectural at this time.

The result for massive clusters are shown in Figures 4 and 5. Figure 4 shows the ratio of rates versus MCS mass for two clouds with \( M_{\text{cloud}} = 6000 \) and \( 6 \times 10^5 \ M_\odot \), and with
Figure 3. Models for the initial stellar mass function used in the figures here. The solid line has a turnover mass $M_t = 0.1 \, M_\odot$ and the dashed line has $M_t = 1 \, M_\odot$. Both have upper mass turnovers too, while the dotted line is a power law to arbitrarily high mass.
equal star formation efficiencies ($\epsilon = 0.3$) and densities ($\rho_{\text{cloud}} = 7.8 \times 10^{-20} \, \text{gm cm}^{-3}$), corresponding to radii of $R_{\text{cloud}} = 1.08 \, \text{pc}$ and $5 \, \text{pc}$, respectively. The density contrast, $C = 160$, was also taken to be the same in each case, and comparable to that in Ophiuchus. Two values of the turnover mass, $M_t$ are considered, while $M_{\text{min}} = 0.1 \, M_\odot$ for all cases.

Figure 4 shows several things. First, the maximum mass increases with the total stellar mass, $\epsilon M_{\text{cloud}}$, as expected for a statistical ensemble (i.e., the solid lines go to larger proto-

Figure 4. The ratio of rates for two clusters more massive than Ophiuchus, showing an increase in the ratios to unacceptably high values for $\sim 10^5 \, M_\odot$ clusters, even for the cores (i.e., the flat parts of the density profiles), when the density contract $C$ is as low as observed in Ophiuchus.
stellar mass than the dashed lines). Second, the ratio of collision to collapse rates increases with MCS mass because of the increasing collision cross section. Third, the ratio is much lower for the MCS cores than for envelopes, as in Figure 2; it is also lower for higher turnover mass, $M_t$. The reason for the $M_t$ dependence is that higher $M_t$ corresponds to relatively fewer low mass stars, and this decreases the number density for collisions in the field.

Figure 4 suggests a problem with star formation in massive clusters. If the densities of the MCS-equivalent objects are the same as in the Ophiuchus cores, then the envelopes of the massive stars will strongly interact with most of the lower mass MCS, probably accreting them. Even the cores of the MCS in the highest mass clouds (solid lines in the figure) will interact (by tidal forces) strongly with other MCS and with the protostars already formed. This implies that massive clusters cannot have MCS with the same densities as those in Ophiuchus (i.e., $10^7 - 10^8 \text{ m(H}_2\text{)} \text{ cm}^{-3}$).

Figure 5 shows the same clouds and clusters as in Figure 4 but now with compression factors ($C = 59, 3460$) that scale with the first (dashed lines) and second (solid lines) powers of the Mach number from virialized motions, assuming a sound speed of $0.3 \text{ km s}^{-1}$. The assumption here is that MCS might be produced in self-gravitating shocks that have average compression factors proportional to some power of Mach number. This power is likely to be close to 1 when the shock moves across magnetic field lines, and 2 when the shock moves parallel to the field lines. The shocks presumably arise from supersonic turbulence in the cloud cores where MCS form, so the turbulent speed is taken to be the cloud virial speed, which is the same as $v$ in equation (13).

Figure 5 also shows cases with very large turnover masses, $M_t = 10 \text{ M}_\odot$. When $M_t \leq 1 \text{ M}_\odot$, only the Mach-squared scaling is strong enough to make the MCS so small that their cores avoid severe collisions. In all cases, the envelopes collide. McKee & Tan (2002) discuss the last stages of massive star formation and note that it requires very high initial densities and pressures. This is consistent with the high value of $C$ found here from close-packing constraints in the massive clusters where massive stars tend to form. One implication of this result is that the formation of stars from MCS in massive clusters should be faster than in low mass clusters.
Figure 5. The ratio of rates for a $2 \times 10^5 \, M_\odot$ cluster, like a globular cluster, with two cases for the density contrast factor. Only when the density contrast factor is very large, scaling with the square of the cloud Mach number in this model, can MCS and protostars co-exist in a cluster environment without destructive interactions.
Figures 4 and 5 suggest that the MCS mass where collisions are important \( (\omega_{\text{collision}}/\omega_{\text{collapse}} \sim 1) \) becomes insensitive to the IMF turnover mass, \( M_t \), when the ratio between the maximum stellar mass and \( M_t \) is large. Once this ratio exceeds several hundred, most collisions are between MCS with masses above \( M_t \) and then the value of \( M_t \) does not matter.

Higher \( M_t \) means fewer low-mass collision partners and more freedom for the massive MCS to collapse without interference from neighbors. For a given modest MCS density contrast, \( C \), denser and more massive stellar clusters should have higher IMF turnover masses because of crowding limitations. However, sufficiently large \( C \) gives all cores a low ratio of collision to collapse rates, and then the IMF does not depend on crowding.

An increase in thermal temperature in a cloud core should decrease the density contrast in shocks and decrease \( C \). This increases the relative importance of collisions and can increase \( M_t \) by removing the low mass MCS. The dependence of \( M_t \) on \( C \) for a constant ratio of rates is approximately \( M_t \propto C^{-2} \) from Figure 5. This comes from the mutual dependencies of \( \omega_{\text{collision}}/\omega_{\text{collapse}} \) on \( C^{-3.5/3} \) (see Sect. 5) and \( M_t^{-0.6} \) for the “core only” case with \( M_t \) in the range from 1 to 10. From these mutual dependencies, \( \omega_{\text{collision}}/\omega_{\text{collapse}} \) is constant if \( M_t^{-0.6} \propto C^{3.5/3} \), which gives \( M_t \propto C^{-1.94} \).

Such a \( C \) dependence is comparable to the dependence of the thermal Jeans mass \( (M_J) \) on compression ratio, which would predict \( M_J \propto C^{-2} \) for \( C \) proportional to the square of the Mach number. This comes from the dependence of the Jeans mass on \( T^2 \) for constant pressure, and the dependence of the Mach number on \( T^{-1/2} \) for constant virial speed in the cloud.

The similar dependencies of \( M_t \) and \( M_J \) on \( C \) imply that the thermal Jeans mass and the crowding limitations both determine the turnover mass in the IMF. If two regions have different temperatures with all else the same, then \( M_t \) should be larger in the warmer region in proportion to the thermal Jeans mass and the crowding conditions will stay about the same. This means that warm cloud cores can produce a top-heavy IMF because of a combination of increased thermal Jeans mass and increased crowding.

No clear dependencies between cluster density and IMF have been found (Massey & Hunter 1998), although some IMF observations suggest they could be present. For example, the clustered embedded stars in Orion have a shallower IMF than the unclustered embedded stars (Ali & DePoy 1995), and the LMC clusters in regions of high stellar density tend to
have shallower slopes than those in regions of low stellar density (J.K. Hill et al. 1994; R.S. Hill et al. 1995). Similarly, the extreme field stars in the LMC have a steeper IMF slope (Massey et al. 1995) than the dense and massive cluster 30 Dor, which has a shallow slope like the Salpeter value (Massey & Hunter 1998). Shallow slopes correspond to proportionally more massive stars, and this can be the result of a previous stage in the evolution of the cluster where the low mass MCS coalesced into higher mass MCS. The problem with these interpretations is that other processes, such as mass segregation, can change the slope of the IMF in the same way as MCS coalescence.

Variations in $M_t$ have also been difficult to establish observationally. The old globular clusters in the Milky Way halo tend to have normal $M_t$, namely about half a solar mass (Paresce & De Marchi, 2000), whereas a young globular cluster in M82 has $M_t$ higher than normal by a factor of $\sim 5$ (Smith & Gallagher 2001). The cluster masses and densities are about the same.

This lack of a clear correlation between the IMF and the density or mass of a cluster (over and above possible changes in the thermal Jean mass) implies that crowding effects are not particularly important in either the protostellar phase or in an earlier phase, such as that producing the MCS in Ophiuchus. This can only be the case if the MCS that eventually form stars build up from smaller objects at nearly constant density, and if this density scales with at least the square of the Mach number in the cloud core.

Crowding and coalescence in the MCS phase will be very important in the cores of globular clusters if the temperature is large or the compression factor scales directly with the Mach number, as for a strongly magnetic gas. In this case, supermassive stars can form by the coalescence of MCS. Such stars may either form massive black holes directly, or they may continue to coalesce as stellar remnants until several together make a massive black hole. This is the scenario for black hole formation in dense clusters proposed by Ebisuzaki et al. (2001), but modified slightly here to include the MCS phase, when collisions are more important than in the main-sequence or post-main sequence phases.

5 SCALING OF THE RESULTS

The curves in Figures 4 and 5 have a similar dependence on MCS mass that comes mostly from the mass dependence for the square of the MCS radius, which is approximately as $M^{2/3}$. Lower $M_t$ clusters have slightly higher ratios of rates because of the correspondingly
larger MCS spatial densities. Lower $M_t$ clusters also produce slightly steeper slopes in the figures. The steeper slope at low $M_t$ arises because the terms $(R_i + R_j)$ and $(M_i + M_j)$ are dominated by the radius and mass of the test object, $R_i$ and $M_i$, when most of the field MCS are at much smaller radii and masses. This occurs at low $M_t$, considering that the integral in equation (13) is dominated by the field MCS mass at the peak of the IMF, which is close to $M_t$. When $M_t$ is high, most of the field interactors are large and massive, and then the dependencies of $(R_i + R_j)$ and $(M_i + M_j)$ on $R_i$ and $M_i$ alone diminish. The test mass, $M_i$, is the quantity plotted on the abscissa in the figures.

The vertical shifts between curves may be understood in terms of the dependencies of the ratio of rates on the compression factor, $C$, and the cloud mass, $M_{\text{cloud}}$. In Figure 4, the cloud mass in two cases differs by a factor of 100 while the cloud density and the MCS densities and radii are the same. The cloud mass enters into the velocity dispersion $v$, the cloud radius, $R_{\text{cloud}}$, and the total number of MCS, which appear in equation (13). For the assumed constant cloud density, the only remaining dependence is on $v$, which scales as $M_{\text{cloud}}^{1/3}$. Thus the dashed and solid lines for both core and envelope models are displaced vertically in the figure by $100^{1/3} = 4.64$, which is 0.66 in the log, as observed.

In Figure 5, the compression factor in two cases differs by a factor of 58.6 for the same cloud mass and radius. For the envelopes, the MCS radii scale with $C^{1/4}$ times the core radii, and the core radii scale with $C^{-1/3}$ for a given MCS mass. Thus the MCS envelope radii scale with $C^{-1/12}$ at constant mass when $C$ is large. The ratio of collision to collapse rates scales with radius as $R^{3.5}$, which is as $C^{-3.5/12}$. Considering the ratio 58.6 for $C$, this means that the dashed and solid curves for the envelope case in Figure 5 should be vertically displaced by $58.6^{-3.5/12} = 0.30$, which is −0.5 in the log, as observed approximately. For the cores, the MCS radii scale with $C^{-1/3}$ for a given MCS mass and so the ratio of rates scales with $C^{-3.5/3}$. This gives a vertical displacement between the dashed and solid lines for the cores that is equal to a factor of $58.6^{-3.5/3} = 0.0085$, which is −2.1 in the log, as observed.

The displacement between the envelope and core cases results mostly from the $C^{1/4}$ ratio between envelope and core radii, given the $R^{3.5}$ dependence for the ratio of rates at constant MCS mass. Thus in Figure 5, the vertical displacement for the $C = 59$ case is $59^{3.5/4} = 35.4$, which is 1.55 in the log, and the vertical displacement for the $C = 3460$ case is $3460^{-3.5/4} = 1249$, which is −3.1 in the log. Both are approximately correct compared to the displacements in the Figure.
6 CONCLUSIONS

The mm-wave continuum sources in Ophiuchus are not interacting strongly at the present time and should collapse to stars independently, preserving their masses (Motte et al. 1998). Previous versions of these MCS should have interacted more strongly if their densities were lower at fixed masses, and MCS in more massive clusters should interact more strongly too. If such interactions are destructive, or if they modify the mass function, then such a previous stage could not have occurred. This might imply that the MCS formed elsewhere and fell into the cloud core after they had their high densities. However, such a model would not explain the hierarchical positions of these MCS, as seen in both Ophiuchus and Serpens (Testi, et al. 2000); MCS ballistic motions over one or more core dynamical times would mix them up.

A high ratio of rates might also imply that collisions were important in the past, particularly among the most massive MCS. However, pure collisional agglomeration leads to a shallow mass function, \( M^{-1.5}dM \) (Field & Saslaw 1965), not the observed \( M^{-2.3}dM \) function at high MCS mass (Motte et al. 1998; Testi & Sargent 1998; Johnstone et al. 2001). Accretion of ambient gas by moving protostars may give a better mass function (Bonnell et al. 1998), but such models are not collision-dominated like the high \( \omega_{\text{collision}}/\omega_{\text{collapse}} \) cases here.

The most likely scenario for the formation of the MCS in Ophiuchus and elsewhere is that they grow inside high-density turbulent shocks and become unstable when they reach a sufficiently large mass. Such a process would not have a previous stage where low density MCS were strongly interacting. It would also account for the hierarchical positioning of MCS in cloud cores (because turbulence makes hierarchical structure), the inferred Mach number dependence of the MCS density with increasing cloud core mass, the lack of smoothly varying envelopes, and the required rapid formation time of the MCS, considering their high densities. Models of turbulence-induced star formation are in Elmegreen (1993), Padoan (1995), Heitsch, Mac Low, & Klessen (2001), Klessen, Heitsch, & Mac Low (2000), Klessen (2001), Ossenkopf, Klessen, & Heitsch (2001), Padoan, et al. (2001), and Wuchterl & Klessen (2001). Other formation scenarios might be preferred outside of dense clusters.

For turbulence-induced star formation, the number of stars that finally forms in a cluster should be larger than the number of MCS that are present at any one time because the MCS phase is shorter than the lifetime of the whole core. The ratio of the number of MCS to the
number of final star counts should be the ratio of corresponding evolution times, which is the inverse square root of the ratio of densities, $C^{-1/2}$. This ratio equals $\sim 0.1$ for the Ophiuchus cores and could be as small as 0.01 for super star clusters ($M \sim 10^5 \, M_\odot$). The MCS-equivalent phase in massive clusters should also be associated with higher densities, possibly in proportional to the square of the Mach number. Thus objects with densities of $10^7 \, \text{cm}^{-3}$ in Ophiuchus would appear in a similar phase of evolution having densities of $\sim 10^9 - 10^{10} \, \text{cm}^{-3}$ in super star clusters. Of course, observations with a sensitivity to a particular density will always see the observable value, so comparisons between low and high mass clusters should look for the same pre-stellar phases in terms of structure and kinematics when comparing densities.

The IMF should be affected by crowding in the sense that warmer, more crowded environments should have a higher low-mass limit to the power law part and proportionally more massive stars to conserve total mass. The temperature dependencies of the thermal Jeans mass and the low mass limit from crowding are about the same.

REFERENCES