Recent conjectures that there are mesoscopically “large” extra dimensions, through which gravity propagates have interesting implications for much of physics. The scenario implies gross departures from Newton’s law of gravity at small length scales. Testing departures from Coulomb’s law on sub-millimetre scales is hard.

It is now possible to routinely create Bose-Einstein condensates with de Broglie wavelengths of order a μm and total size of order 10μm. BEC condensates move coherently under gravitational acceleration, and I propose that the transverse fringe shift due to the acceleration of pair of interfering BECs passing a dense linear mass may be measurable, and provide direct evidence for anomalous gravitational acceleration. Ideally such experiments are best carried out in free fall to maximise the time spent by a BEC in the non-Newtonian regime.

I.

Recent conjectures have postulated that the hierarchy problem in physics may be resolved if two (or more) of the extra dimensions postulated by extensions of the standard model of particle theory, are compactified on mesoscopic scales - with effective radii much larger than the Planck scale [3,13]. A particularly interesting possibility is in the case of $n = 2$ mesoscopic compactification, in which case the implied scale, $R_c$, for the extra dimensions is of the order of 0.1 mm. In the simplest theory, the standard model gauge fields are restricted to (or near) the 3-dimensional brane on which we live, and only gravity propagates into the bulk of the large extra dimensions (LEDs). The resulting theory has a number of interesting physical implications including possible resolution of the “hierarchy problem” and anomalous cross-sections for particle productions at moderately high energies ($\gtrsim 10^7$ eV) [2,4,9]. There may also be astrophysical implications [16,5].

An immediate implication of LEDs is that Newton’s law fails on small scales, and is replaced by an effective potential gradient

$$\nabla \Phi(r) = -(n + 1) \frac{m_1 m_2}{M_{pl}^{n+2}} \frac{1}{r^{n+2}} r \ll R_c$$

(1.1)

where $n$ is the number of large extra dimensions, and $M_{pl-n}$ is the higher dimensional Planck mass, implying an effective 4-D Planck mass $M_{pl} \sim M_{pl-n}^{1+n/2} R_c^{n/2}$ [3].

The laboratory experimental constraints on deviations from Newton’s law on scales less than 1 cm [10] are very weak, so the conjecture is not directly excluded by direct experiments, although experiments in progress will either detect the predicted deviation, or constrain $R_c$ (or equivalently $M_{pl-n}$). If correct, LEDs have many implications for physics on different scales, some of which will be tested in the near future. Experiments to directly measure deviations from Newtonian gravity on sub-millimetre scales are underway (see [17]).

It is now possible to routinely generate, in the laboratory, Bose-Einstein condensates with de Broglie wavelengths of order a micron and total sizes of order 10μm, and to manipulate and transport coherent ensembles of $\gtrsim 10^6$ ultra-cold atoms [6,1,14,8]. Atom interferometry can be used to obtain both high precision measurements of absolute gravitational acceleration and gravitational gradients [12,15]. Also, clearly, the dynamics of a Bose-Einstein condensate are affected by gravitational forces [7].

A linear cylindrical mass, with length $l(\gg R_c)$, produces a transverse gravitational acceleration $g_T = 2G\mu/r$ for $r > R_c$, where $\mu = \pi \rho a^2$ is the mass per unit length of a cylinder density $\rho$ and radius $a$. As a simple example, consider a thin walled hollow glass cylinder, with a segment length $l$ filled with a high density substance (such as mercury, lead or gold), and another comparable length segment empty. If we conjecture that $R_c < 0.1$ mm, but $R_c > 0.01$ mm, then for $a = 10$ μm the gravitational field close to the cylinder is non-Newtonian with $g_T \approx 2G\mu/r^3$. So the transverse acceleration close to the filled cylinder may be two orders of magnitude larger than in the Newtonian case. Using a cylinder with a filled core and a contiguous empty core segment would allow “blind tests” of anomalous transverse acceleration by sliding the cylinder so that either the empty or the dense core were in the path of the BEC pair. The homogeneous structure would hopefully lead to consistent systematic offsets due to surface effects and other non-gravitational perturbations.

A Bose-Einstein condensate falling past such a cylinder will experience a transverse impulse. Falling under gravity, the BEC will have vertical speeds of order cm/sec, and traverse the conjectured non-Newtonian regime in a few milliseconds. The resultant transverse impulse $\Delta v_T = \int -\nabla \Phi dt \sim 4 \times 10^{-10}$ cm/sec. The resultant Doppler shift would be of the order of $\mu$Hz for reasonable atomic transitions, which is not practical for direct detection. If a pair of BEC condensates is sent past the cylinder on either side the resultant transverse shift will be a order 10 femtometres. Launching the BEC pair verti-
cally upwards will roughly double the shift and eliminate some sources of systematic error. With a de Broglie wavelength of a micrometer, measuring the phase shift due to the presence of cylinder with dense core, as opposed to an empty cylinder would require measuring the phase accuracy to $O(1/n)$, where $n$ is the number of atoms in the BEC, which can be done in principle [11].

Additional precision may be obtained by doing multiple vertical traverses of the system. However, the limiting factor above is the high vertical speed of the BEC as it falls under gravity. If the speed of the BEC crossing the putative non-Newtonian regime is smaller, the transverse impulse is proportionately larger and the transverse phase shift is also proportionately large as the BEC can undergo transverse coasting due to the anomalous transverse impulse for longer (up to the intrinsic lifetime set by the ballistic expansion of the BEC). Conducting such an experiment in orbit, for example on the International Space Station, might produce phase shifts four orders of magnitude larger, and correspondingly easier to measure. The resultant transverse velocity would also be correspondingly larger, and might conceivably be detectable, particularly if the BEC is allowed to traverse past an array of multiple dense cylinders.

Interferometry of coherent mesoscopic atomic ensembles may be used to detect anomalous acceleration due to non-Newtonian gravity on sub-millimetre scales. A simple experimental setup of a thin cylinder with dense core, and a contiguous control segment with empty core should produce anomalous transverse velocity shifts in pairs of Bose-Einstein condensates leading to fringe shifts upon the subsequent interference by the pairs, that are measurable in principle. The experiment is best done in free fall, since the gravitational acceleration on Earth’s surface limits the coasting time in the setup postulated here.