Neutrino-nucleon scattering rate in the relativistic random phase approximation

L. Mornas\textsuperscript{a}\textsuperscript{*}

\textsuperscript{a}Departamento de Física, Universidad de Oviedo, E33007 Oviedo, Spain

The in-medium modification to the neutrino-nucleon scattering rate is calculated in the relativistic random phase approximation in the framework of a hadronic meson exchange model, in view of applications to neutrino transport in supernovae and protoneutron stars.

1. Introduction

Neutrino transport is an important ingredient of the numerical simulation of supernova collapse and of the cooling of the resulting protoneutron star. In the conditions of high density and temperature prevailing in this situation, the neutrino opacities will be appreciably modified by medium effects. This has been confirmed by the calculations of several groups (see \textit{e.g.} [1–3] and references therein).

We perform here a calculation of the effect of correlations on this parameter in the random phase approximation. This contribution is an extension of the results presented in [3] to the more commonly used parametrizations of the nuclear interactions NL3 and TM1 [4,5]. The reader is referred to [3] for detailed analytical expressions as well as for a more complete list of bibliographical references.

In the dense and hot nuclear matter present in the protoneutron star, the mean free path of the neutrino is determined mostly by its interactions with the baryons. Here we will consider only the nucleons. The scattering process occurs \textit{via} the neutral current, while the emission and absorption processes involve the charged current. In both cases, the differential cross section is given by

\[
\frac{d\sigma}{dE_\nu d\Omega} = \frac{G_F^2}{64\pi^3} \frac{E_\nu'}{E_\nu} Tm(S_{\mu\nu} L_{\mu\nu})
\]

It is proportional to the imaginary part of the lepton current $L_{\mu\nu}$ contracted with the structure function of the nucleons $S_{\mu\nu}$. When the nucleons are free, the structure function is proportional, with a thermodynamical factor, to the mean field value of the polarization

\[
\Pi_{W W}^{\mu\nu, MF}(q) = \int d^4p \ Tr [\Gamma_{W}^{\mu'} G(p) \Gamma_{W}^{\nu'} G(p + q)]
\]

where $G$ is the propagator of the nucleon and $\Gamma_{W}$ is the vertex for the weak interaction.

\textsuperscript{*}e-mail: lysiane@pinon.ccu.uniovi.es

Work supported in part by the Spanish Grant n° MCT-00-BFM-0357
The RPA correlations are introduced by replacing the mean field polarization by the solution of the Dyson equation

\[
\Pi_{WW}^{\mu\nu\text{RPA}} = \Pi_{WW}^{\mu\nu\text{MF}} + \Pi_{WS}^{\mu\nu\text{MF}} D_{SS\alpha\beta}^{(0)} \Pi_{SW}^{\beta\nu\text{RPA}}
\]

with \( D_{SS}^{(0)} \) representing the nuclear interaction in free space\(^2\). As can be easily seen on the diagramtical representation, this corresponds to an infinite series of loop diagrams, which may also be resummed by defining a screened nuclear interaction \( D_{SS}^{\text{RPA}} \)

\[
\Pi_{WW}^{\mu\nu\text{RPA}} = \Pi_{WW}^{\mu\nu\text{MF}} + \Pi_{WS}^{\mu\nu\text{MF}} D_{SS\alpha\beta}^{\text{RPA}} \Pi_{SW}^{\beta\nu\text{MF}}
\]

2. Polarization insertions

We will describe the nuclear interaction by a relativistic field model with meson exchange. Non-linear meson-meson interactions are introduced in order to obtain a better description of the properties of nuclear matter at saturation. We consider here two widely used parametrizations of the nuclear interaction, NL3 and TM1 [4,5], which are fitted to the properties of nuclei and bulk nuclear matter. The interaction piece of the corresponding Lagrangian reads

\[
\mathcal{L}_{\text{int}} = \bar{\psi} \left( -g_\sigma \sigma + g_\omega \gamma^\mu \omega_\mu + g_\rho \gamma^\mu \rho_\mu \tau \right) \psi - \frac{1}{3} b m_N \sigma^3 - \frac{1}{4} c \sigma^4 + \frac{1}{4} d (\omega_\alpha \omega^\alpha) \tag{5}
\]

The values of the parameters are collected in the following table.

<table>
<thead>
<tr>
<th></th>
<th>( m_\sigma \text{ [MeV]} )</th>
<th>( g_\sigma )</th>
<th>( g_\omega )</th>
<th>( g_\rho )</th>
<th>( b/g_\omega^3 )</th>
<th>( c/g_\omega^4 )</th>
<th>( d/g_\omega^5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>NL3</td>
<td>508.19</td>
<td>10.217</td>
<td>12.867</td>
<td>4.4744</td>
<td>2.0551 \times 10^{-3}</td>
<td>-2.6511 \times 10^{-3}</td>
<td>0.0</td>
</tr>
<tr>
<td>TM1</td>
<td>511.20</td>
<td>10.029</td>
<td>12.614</td>
<td>4.6322</td>
<td>-1.5082 \times 10^{-3}</td>
<td>6.1123 \times 10^{-5}</td>
<td>2.8166 \times 10^{-3}</td>
</tr>
</tbody>
</table>

These interactions do not include the \( \delta \) meson considered in [3] nor the tensor coupling of the \( \rho \) meson. The Lagrangian also does not include a pion term; however it was shown in [3] that the neutrino-nucleon scattering rate does not depend on the pion and only very weakly on the details of the residual contact interaction\(^3\). These parametrizations have also been shown to yield good agreement with the available data on the longitudinal response for quasielastic electron scattering on nuclei in the RPA approximation. (cf. e.g. [6]).

We present in this contribution results for the neutrino-nucleon scattering rate. In this case, we need to replace the piece \( D^{\text{RPA}} \) describing the screened nuclear interaction by the matrix of propagators for the neutral mesons \( \sigma, \omega \) and \( \rho \) dressed at RPA level in asymmetric nuclear matter. The detailed expression for these propagators with meson mixing in all channels was derived in [7].

One should be aware that, in models with non linear meson couplings, the meson propagators are not only dressed by particle-hole loops, but also include a contribution of the non linearities. When performing the derivation of the propagators, one finds that

---

\(^2\)The indices \( W \) or \( S \) stand for a vertex with a weak or strong coupling respectively.

\(^3\)This statement does not hold for the emission and absorption processes which involve charged \( \rho \) and \( \pi \) exchange, corrected at short distance by contact terms.
this amounts to replacing the masses of the mesons appearing in the expressions of the propagators given in Eqs. (32-47) of [7] as follows:\footnote{The index \( H \) indicates that the values of the meson fields are taken in the Hartree approximation, and \( \eta^2 = \eta^\mu \eta_\mu \) with \( \eta^\mu \) defined below Eq. (6)}:

\[
\begin{align*}
m^2_\sigma &\rightarrow M^2_\sigma = m^2_\sigma + 2 b m_\sigma \sigma_H + 3 c \sigma_H^2 \\
m^2_\omega &\rightarrow M^2_{\omega T} = m^2_\omega + d \omega^2_H \quad \text{(trans.) or} \\
M^2_{\omega L} &= \frac{(m^2_\omega + d \omega^2_H)(m^2_\omega + 3 d \omega^2_H)}{m^2_\omega + d \omega^2_H (-2 \eta^2 + 3)} \quad \text{(long.)}
\end{align*}
\]

3. Numerical results

We calculated the differential and total cross sections in asymmetric matter with a proton fraction \( Y_p \). The proton fraction is determined by the condition that \( \beta \) equilibrium is realized: \( \hat{\mu} = \mu_n - \mu_p = \mu_e - \mu_\nu \). In cold neutron stars, the neutrinos can leave the star unhindered. On the other hand, in the conditions we are considering here they are still trapped inside the star, and the chemical potential of the neutrino therefore takes a finite value \( \mu_\nu = (6\pi^2 \rho Y_\nu)^{1/3} \). The lepton fraction \( Y_L = Y_e + Y_\nu \) is determined by neutrino transport (diffusion equation or Boltzman), to which the cross section serves as input. A typical value is \( Y_L \simeq 0.4 \).

It is convenient to decompose the polarizations and propagators onto orthogonal projectors formed with the vectors and tensor available in the problem, \textit{i.e.} the metric \( g^{\mu\nu} = \text{diag}(1,-1,-1,-1) \), the hydrodynamic velocity \( u^\mu \) and the transferred momentum \( q^\mu \).

\[
\Pi^\mu_\nu \text{RPA} = \Pi_T T^{\mu\nu} + \Pi_L \Lambda^{\mu\nu} + \Pi_Q Q^{\mu\nu} + i \Pi_E E^{\mu\nu} \quad (6)
\]

\[
\begin{align*}
\Lambda^{\mu\nu} &= \frac{\eta^\mu \eta^\nu}{\eta^2} \quad ; \quad \eta^\mu = u^\mu - \frac{q \cdot u}{q^2} q^\mu \quad \text{(longitudinal)} \\
T^{\mu\nu} &= g^{\mu\nu} - \frac{\eta^\mu \eta^\nu}{\eta^2} - \frac{q^\mu q^\nu}{q^2} \\
E^{\mu\nu} &= \epsilon^{\mu\nu\rho\lambda} \eta_\rho q_\lambda \quad \text{(axial)} \quad ; \quad Q^{\mu\nu} = \frac{q^\mu q^\nu}{q^2} \quad \text{(does not contribute)}
\end{align*}
\]

The contraction of the lepton current with the polarization can be expressed by means of three structure functions \( R_1, R_2 \) and \( R_5 \) related to the previous polarizations by

\[
\begin{align*}
R_1 &= -\frac{2}{1 - e^{-z}} \Im \left[ -\frac{q^2}{\eta^2} \Pi_L + \frac{w^2}{\eta^2} \Pi_T \right] \\
R_2 &= \frac{2}{1 - e^{-z}} \Im \left[ \Pi_T \right] \\
R_5 &= \frac{2}{1 - e^{-z}} \Im \left[ \Pi_E \right] \\
\text{with} \quad z = \frac{\omega - \Delta \mu}{k_B T} \quad \text{and} \quad q^\mu = (\omega, q)
\end{align*}
\]
of the total result. The corrections will therefore arise mostly from the subdominant longitudinal and axial-vector polarizations $\Pi_L$ and $\Pi_E$. As an example we show here the transverse structure function for the TM1 and NL3 parametrizations.

![Graph](image)

Figure 1. Transverse response function $R_2$ at twice saturation density, temperature $T=30$ MeV and a fixed momentum transfer $k=100$ MeV and a fixed momentum transfer $k=100$ MeV, and the NL3 and TM1 parametrizations, compared to the reduction factor obtained in [3].

An estimate\(^5\) of the mean free path is given by the inverse of the total cross section obtained by integrating the differential one over the allowed energy and momentum transfer, with a Pauli blocking factor for the outgoing lepton. For the parameter sets NL3 and TM1, the total neutrino-neutron scattering cross section is found to be reduced by RPA correlations at high density by a factor 20% to 25%. At low density and moderate temperature, on the other hand, RPA correlations would yield an enhancement; however the validity of the model becomes questionable in this range. The reduction factor obtained here is somewhat stronger than the one quoted in [3]. Actually we observe that parameter sets adjusted to give a lower effective mass at saturation yield stronger reduction factors\(^6\).

REFERENCES

7. L. Mornas, nucl-th/0107002

---

\(^5\)This is only an estimate since other processes contribute.

\(^6\)\(m^*=0.63\) for TM1 and 0.6 for NL3 while it was adjusted to 0.8 in [3]