We investigate the dynamics of relativistic spinning test particles in the spacetime of a rotating black hole using the Papapetrou equations. We use the method of Lyapunov exponents to determine whether the orbits exhibit sensitive dependence on initial conditions, a signature of chaos. In the case of maximally spinning equal-mass binaries (a limiting case that violates the test-particle approximation) we find unambiguous positive Lyapunov exponents that come in pairs $\pm \lambda$, a characteristic of Hamiltonian dynamical systems. We find no evidence for nonvanishing Lyapunov exponents for physically realistic spin parameters, which suggests that chaos may not manifest itself in the gravitational radiation of extreme mass-ratio binary black-hole inspirals (as detectable, for example, by LISA, the Laser Interferometer Space Antenna).