Non-Markovian collision integral in Fermi-systems

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The non-Markovian collision integral is obtained on the base of the Kadanoff-Baym equations for Green functions in a form with allowance for small retardation effects. The collisional relaxation times and damping width of the giant isovector dipole resonances in nuclear matter are calculated. For an infinite Fermi liquid the dependence of the relaxation times on the collective vibration frequency and the temperature corresponds to the Landau’s prescription.

1 Introduction

The dissipative properties of many-body systems, specifically transport coefficients for viscosity as well as the damping of the collective excitations, are strongly governed by the interparticle collisions. In semi-classical approach these collisions are described by collision integral in kinetic equation. It allows us to involve a viscosity into motion equations similar to the phenomenological Navier-Stockes ones [1]-[6]. An advantage of such approach is a conceptual clarity and a possibility to use many results from a general macroscopic physics.

For correct description of the collision relaxation rates in systems with a fast variation of the mean field, it is necessary to incorporate the memory (retardation) effects [7]-[16]. We investigate the form of the non-Markovian linearized collision integral with retardation using the Kadanoff-Baym equations [17, 18] for Green functions. This method enables immediately to take into account self-consistent mean field.

The non-Markovian collision term of the semiclassical linearized Vlasov-Landau equation was obtained with the use of the Kadanoff-Baym equations for the Green functions in Refs. [11, 14]. The kinetic equation was considered in one-component Fermi liquid with the use of the linear approximation on deviation from overall equilibrium with a nonequilibrium distribution function \( \delta f = f(\vec{p}, \vec{r}, t) - \bar{f}(\bar{\epsilon}) \propto \exp(-i\omega t) \), where \( f(\vec{p}, \vec{r}, t) \) is an exact distribution function and \( \bar{f} \) is an equilibrium distribution function at equilibrium single-particle energy \( \bar{\epsilon} \). An expression for linearized collision integral was obtained in the Born approximation on two-body collisions in the following general form:

\[
J(\vec{p}, \vec{r}, t) = J^{(1)}(\vec{p}, \vec{r}, t) + J^{(2)}(\vec{p}, \vec{r}, t),
\]
where the first component corresponds to variation of the distribution function and second one is connected with variation $\delta U = \epsilon(\vec{p}, \vec{r}, t) - \bar{\epsilon}$ of mean field with $\epsilon(\vec{p}, \vec{r}, t)$ for actual single-particle energy.

One can see that this relationship does not agree with general form of the linearized Landau-Vlasov equation in Fermi liquid in the limiting case of the states with week time-dependence of nonequilibrium component of distribution function, when $\partial \delta f / \partial t \rightarrow 0$. Indeed, the linearized Landau-Vlasov equation in quasi-homogeneous systems can be presented in the following form:

$$\frac{\partial \delta f}{\partial t} + \frac{p}{m} (\vec{p} \cdot \vec{r}) \delta \bar{f} = J,$$

where $\delta \bar{f}$ is a linear deviation of the exact distribution function $f(\vec{p}, \vec{r}, t)$ from the distribution function of the equilibrium shape $\bar{f}(\epsilon)$ evaluated with actual single-particle energy $\epsilon = \bar{\epsilon} + \delta U$,

$$\delta \bar{f} = \delta f - \frac{df}{d\bar{\epsilon}} \delta U = \delta f - \bar{f}(\epsilon).$$

The derivative $\partial \delta f / \partial t$ can be omitted in Eq. (2) for slightly time-dependent states and the collision integral should be a functional, $\Psi$, of the $\delta \bar{f}$,

$$J = \Psi(\delta \bar{f}) \equiv \frac{p}{m} (\vec{p} \cdot \vec{r}) \delta \bar{f},$$

in order to the Landau-Vlasov equation would be fulfilled in this case. The relationships (1) and (4) are generally in contradictions and derivation of the form of non-Markovian collision integral from Refs. [11, 14] can be revised.

It was pointed in Refs. [3, 19] that collisional integrals of the type (4), $J \equiv J(\delta \bar{f})$, are general form of the collision integrals between quasiparticles in Fermi liquid without retardation and they lead to a local equilibrium state described by distribution function $f_{l.e.} = \bar{f}(\epsilon = \bar{\epsilon} + \delta U)$. Due to this we use in the following this terminology.

In this contribution an expression for non-Markovian collision integral of the linearized Vlasov-Landau transport equation from Refs. [11, 14] is modified in Sect. 2 for the case of slightly time-dependent states, i.e. in a form which allows for reaching the local equilibrium in system.

In Sect. 3 calculations of the relaxation times and damping width of the collective vibration in nuclear matter with the use collision integral with retardation effect are presented.

2 Non-Markovian linearized collision integral within semiclassical approach

In order to obtain linearized Vlasov-Landau equation with collision integral we use the mixed $\{\vec{p}, \vec{r}\}$ Weyl-Wigner representation for single-particle Green functions (correlation functions)
\[ G^>(1, 1'), G^<(1, 1') : \]
\[
 f(\vec{p}, \vec{r}, t) \equiv g^<(t, t; \vec{p}, \vec{r}) = \int d\vec{r}' \exp\left(-\frac{i}{\hbar} \vec{p} \cdot \vec{r}'\right) g^<(t, t; \vec{r} + \frac{\vec{r}'}{2}, \vec{r} - \frac{\vec{r}'}{2}); \quad (5)
\]
\[
 \vec{r}' = \vec{r}_1 - \vec{r}_1', \vec{r} = \frac{1}{2}(\vec{r}_1 - \vec{r}_1'),
\]
where \( f(\vec{p}, \vec{r}, t) \) is one-body distribution function and Green functions are determined by\cite{17, 18}
\[
 G^>(1, 1') = -i \langle \Psi(1) \Psi^+ (1') \rangle, \quad t_1 > t_1',
\]
\[
 G^<(1, 1') = i \langle \Psi^+ (1') \Psi(1) \rangle, \quad t_1 < t_1'.
\quad (6)
\]
Here, \( \Psi^+ (1) \) and \( \Psi(1) \) are the operators of creation and annihilation of a fermion; the
symbol 1 includes both space and time variables, namely, \( 1 \equiv \{ \vec{r}, t \} \) (we omit isotopic and spin variables); and the expectation values in Eq.(6) are calculated for the ground state or for the ensemble of initial states if a temperature of system is not zero.

The Green functions \( G^>(1, 1') \) and \( G^<(1, 1') \) satisfy the Kadanoff-Baym equation
\[
 \begin{bmatrix}
 i\hbar \frac{\partial}{\partial t_1} + \frac{\hbar^2}{2m} \nabla_1^2 - U(1) \\
 -i\hbar \frac{\partial}{\partial t'_1} + \frac{\hbar^2}{2m} \nabla_1'^2 - U(1')
 \end{bmatrix} G^>(1, 1') = I^>_{1}(1, 1') - I^>_{2}(1, 1'),
\]
\[
 \begin{bmatrix}
 i\hbar \frac{\partial}{\partial t_1} + \frac{\hbar^2}{2m} \nabla_1^2 - U(1) \\
 -i\hbar \frac{\partial}{\partial t'_1} + \frac{\hbar^2}{2m} \nabla_1'^2 - U(1')
 \end{bmatrix} G^<(1, 1') = J^<_{1}(1, 1') - J^<_{2}(1, 1'),
\quad (7)
\]
where the single-particle potentials \( U(1), U(1') \) are determined by the relations
\[
 U(1)G^<(1, 1') = \int d\vec{r}_2 \Sigma_0(\vec{r}_1, \vec{r}_2, t_1)G^<(\vec{r}_2, t_1; 1'),
\]
\[
 U(1')G^>(1, 1') = \int d\vec{r}_2 G^>(1; \vec{r}_2, t_1')\Sigma_0(\vec{r}_2, \vec{r}_1', t_1')
\quad (8)
\]
with
\[
 \Sigma_0(\vec{r}_1, \vec{r}_2, t) = -i\delta(\vec{r}_1 - \vec{r}_2) \int d\vec{r} v(|\vec{r}_1 - \vec{r}|)G^<(\vec{r}, t_1; \vec{r}, t_1) + iv(|\vec{r}_1 - \vec{r}_2|)G^<(\vec{r}_1, t_1; \vec{r}_2, t_1),
\quad (9)
\]
and \( v(|\vec{r}_1 - \vec{r}_2|) \) for two body potential.

The functions \( I^>_{1, 2} \) and \( J^>_{1, 2} \) are correlation integrals of the form
\[
 I^>_{1}(1, 1') = \frac{1}{\hbar} \int_{t_0}^{t_1} dt_2 \Sigma^>(1, 2) - \Sigma^<(1, 2)) G^>(2, 1'),
\]
\[
 I^>_{2}(1, 1') = \frac{1}{\hbar} \int_{t_0}^{t_1} dt_2 \Sigma^>(1, 2) \left[ G^>(2, 1') - G^<(2, 1') \right],
\]
\[
 J^>_{1}(1, 1') = \frac{1}{\hbar} \int_{t_0}^{t_1} dt_2 \left[ G^>(1, 2) - G^<(1, 2) \right] \Sigma^>(2, 1'),
\]
\[
 J^>_{2}(1, 1') = \frac{1}{\hbar} \int_{t_0}^{t_1} dt_2 G^>(1, 2) \left[ \Sigma^>(2, 1') - \Sigma^<(2, 1') \right].
\quad (10)
\]
These correlation integrals have the general form with allowance for retardation effect. It is assumed as usually that interaction between particles start at the time $t_0 = -\infty$.

In order to obtain the Landau-Vlasov equation the following suggestions are adopted [11, 14]: A) Born approximation for two-body scattering; B) the time variation of the nonequilibrium distribution function $\delta f \propto \exp(-i\omega t)$ is taken periodic one with real frequency during all interval of the time changing ($-\infty \leq t' \leq t$); C) linear approximation on deviation of one-body Green functions from their equilibrium values are used; D) Fermi system is considered as quasi-homogeneous one in coordinate space.

The linearized Vlasov-Landau equation has the form (2) and can be presented as [11, 14]

$$\frac{d}{dt} f(\vec{p}, \vec{r}, t) + \{\varepsilon, f\} = J(\vec{p}, \vec{r}, t). \tag{11}$$

Here,

$$\{\varepsilon, f\} \equiv \frac{\partial}{\partial \vec{p}} \varepsilon \cdot \frac{\partial}{\partial \vec{r}} f - \frac{\partial}{\partial \vec{r}} \varepsilon \cdot \frac{\partial}{\partial \vec{p}} f$$

are the Poisson brackets and

$$\epsilon(\vec{p}, \vec{r}, t) = \frac{p^2}{2m} + U(\vec{p}, \vec{r}, t) \tag{12}$$

is the classical energy of a particle in the mean field $U(\vec{p}, \vec{r}, t) = \bar{\epsilon} + \delta U(\vec{p}, \vec{r}, t)$, where expression for $\delta U$ can be expressed in terms of the Landau interaction amplitude $F(\vec{p}, \vec{p}')$ for two-body collision scattering matrix:

$$\delta U = \frac{g}{N_F} \int \frac{d^3p'}{(2\pi \hbar)^3} F(\vec{p}, \vec{p}') \delta f(\vec{p}', \vec{r}, t), \tag{13}$$

where $N_F = 2p_F m^* / (g \pi^2 \hbar^3)$, $p_F$ is the Fermi momentum, $m^*$ is the effective mass of nucleon and $g$ is the spin degeneracy factor.

The linearized collision integral has form (1) with (see, Eqs.(42),(43) and (45),(46) of Ref.[14] for details)

$$J^{(j)}(\vec{p}, \vec{r}, t) = 2 \int \frac{d^3p_2 d^3p_3 d^3p_4}{(2\pi \hbar)^3} W(\vec{p}_1, \vec{p}_2, \vec{p}_3, \vec{p}_4) \delta(\Delta \vec{p}) B^{(j)}(\vec{p}, \vec{r}, t). \tag{14}$$

Here, $W(\{\vec{p}_i\}) = (d\sigma/d\Omega) 4(2\pi \hbar)^3/m^2$ is the probability of two-body collisions with the initial momenta $\vec{p}_1 = \vec{p}, \vec{p}_2$ and final ones $\vec{p}_3, \vec{p}_4$; $d\sigma/d\Omega$ is in-medium differential cross-section (in Born approximation);

$$B^{(1)}(\vec{p}, \vec{r}, t) = \sum_{k=1}^{4} \delta f_k(t) \frac{\partial}{\partial f_k} \left[ \delta_+(\Delta \bar{\epsilon} + \hbar \omega) + \delta_-(\Delta \bar{\epsilon} - \hbar \omega) \right],$$

$$B^{(2)}(\vec{p}, \vec{r}, t) = Q(\{\vec{f}_j\}) \frac{\Delta(\delta U(t))}{\hbar \omega} \left[ \delta_+(\Delta \bar{\epsilon} + \hbar \omega) - \delta_-(\Delta \bar{\epsilon} - \hbar \omega) \right] - \left[ \delta_-(\Delta \bar{\epsilon} + \hbar \omega) - \delta_+(\Delta \bar{\epsilon} - \hbar \omega) \right], \tag{15}$$
where \( \bar{f}_k \equiv \bar{f}(\bar{p}_k, \bar{r}) \); \( \partial Q(\{\bar{f}_j\})/\partial \bar{f}_k \) are the derivatives of the is the Pauli blocking factor \( Q \) with respect to \( \bar{f}_k \),

\[
Q(\{\bar{f}_j\}) = (1 - \bar{f}_1)(1 - \bar{f}_2)\bar{f}_3\bar{f}_4 - \bar{f}_1\bar{f}_2(1 - \bar{f}_3)(1 - \bar{f}_4).
\]

(16)

The \( \bar{\epsilon}_i = \bar{\epsilon}(\bar{p}_i, \bar{r}) \) and \( \Delta U_j \) are the equilibrium single-particle energy and the variation of the mean field for particle with momentum \( \bar{p}_i \) respectively; \( \Delta \bar{\epsilon} = \bar{\epsilon}_1 + \bar{\epsilon}_2 - \bar{\epsilon}_3 - \bar{\epsilon}_4 \), \( \Delta(\Delta U) \equiv \Delta U_1 + \Delta U_2 - \Delta U_3 - \Delta U_4 \), \( \Delta \bar{p} = \bar{p}_1 + \bar{p}_2 - \bar{p}_3 - \bar{p}_4 \). The equilibrium distribution function \( \bar{f}_k \equiv \bar{f}(\bar{p}_k, \bar{r}) \) depends on the equilibrium single-particle energy \( \bar{\epsilon}_k \equiv \bar{\epsilon}(\bar{p}_k, \bar{r}) \); \( \bar{f}_k = \bar{f}(\bar{\epsilon}_k) \). It equals the Fermi function evaluated at the temperature \( T \), \( \bar{f}(\bar{\epsilon}_k) = 1/[1 + \exp((\bar{\epsilon}_k - \mu)/T)] \).

Note that the generalized functions \( \delta_+ \), \( \delta_- \) appearing in Eq. (15) include also integral contribution,

\[
\delta_+(x) = \frac{1}{2\pi} \int_{-\infty}^{0} d\tau \ e^{-ix\tau} = \frac{i}{2\pi} \frac{1}{x + i0} = \frac{1}{2} \delta(x) - \frac{1}{2\pi i} \mathcal{P}\left(\frac{1}{x}\right), \quad \delta_-(x) = \delta_+(x),
\]

(17)

where \( \delta(x) \) is the delta function and the symbol \( \mathcal{P} \) denotes the principal value of integral contribution. The integral terms of the \( \delta_\pm \), corresponding to virtual transitions, are usually rejected in the \( J \) because they assumed to be included by renormalizing the interactions between particles [8]. This corresponds to substitution of the \( \delta(x)/2 \) for \( \delta_\pm \) in Eq. (15), i.e., to taking into account real transitions with conservation of energy. We will consider below only these transitions.

Now we will modify the expression for quantity \( B^{(1)} \). We present the nonequilibrium component \( \delta f \) of the distribution function in the form

\[
\delta f(\bar{p}_j, \bar{r}, t) = -\nu(\bar{p}_j, \bar{r}, t) \frac{\partial \bar{f}(\bar{\epsilon}_j)}{\partial \bar{\epsilon}_j}.
\]

(18)

Then the quantity \( B^{(1)} \) can be written as

\[
B^{(1)}(\bar{p}, \bar{r}, t) = -\frac{1}{2} \sum_{k=1}^{4} \nu_k \frac{\partial Q(\{\bar{f}_j\})}{\partial \bar{\epsilon}_k} \left[ \delta(\Delta \bar{\epsilon} + \hbar \omega) + \delta(\Delta \bar{\epsilon} - \hbar \omega) \right]
\]

\[
= \frac{1}{2} \Delta \nu Q(\{\bar{f}_j\}) \frac{\partial}{\partial \hbar \omega} \left[ \delta(\Delta \bar{\epsilon} + \hbar \omega) + \delta(\Delta \bar{\epsilon} - \hbar \omega) \right] - \delta B^{(1)},
\]

(19)

where \( \Delta \nu \equiv \nu_1 + \nu_2 - \nu_3 - \nu_4 \), \( \nu_k = \nu(\bar{p}_k, \bar{r}, t) \) and

\[
\delta B^{(1)} = \frac{1}{2} \sum_{k=1}^{4} \frac{\partial}{\partial \bar{\epsilon}_k} \left\{ \nu_k Q(\{\bar{f}_j\}) \left[ \delta(\Delta \bar{\epsilon} + \hbar \omega) + \delta(\Delta \bar{\epsilon} - \hbar \omega) \right] \right\}
\]

\[
+ \frac{1}{2} \sum_{k=1}^{4} Q(\{\bar{f}_j\}) \left[ \delta(\Delta \bar{\epsilon} + \hbar \omega) + \delta(\Delta \bar{\epsilon} - \hbar \omega) \right] \frac{\partial \nu_k}{\partial \bar{\epsilon}_k}.
\]

(20)

The first component in the Eq.(20) determines a probability flux of colliding particles which is connected with possibility of variation of the energy \( \bar{\epsilon}_k \) when the values of other
energies ($\tilde{\epsilon}_j \neq \tilde{\epsilon}_k$ and $\hbar\omega$) are fixed. This term should be equal zero because of fixing the total energy in initial or final states and therefore it does not contribute to the total number of the collisions $N$:

$$N(\tilde{p}) \equiv \int_0^\infty \text{d}J(\tilde{p}, \epsilon), \quad \tilde{p} \equiv \tilde{p}/p.$$

A relative dynamical component $\nu_k$ of the distribution function is slowly dependent on energy and it can be considered (at least for low temperatures $T \ll \epsilon_F$) as a function of the momentum direction rather than of the momentum: $\nu_k \equiv \nu(\tilde{p}_k, \vec{r}, t) = \nu(\tilde{p}_k, \epsilon_F, \vec{r}, t)$. Therefore the second component in the Eq.(20) is also negligible and the term $\delta B^{(1)}$ should be omitted from the Eq.(19), $\delta B^{(1)} = 0$. The expressions for $B^{(j)}$ take the form

$$B^{(1)}(\tilde{p}, \vec{r}, t) = \frac{1}{2} \Delta \nu Q\{{\tilde{f}_j}\} \frac{\partial}{\partial \hbar\omega} [\delta(\Delta \epsilon + \hbar\omega) + \delta(\Delta \epsilon - \hbar\omega)],$$

$$B^{(2)}(\tilde{p}, \vec{r}, t) = \frac{1}{2} Q\{{\tilde{f}_j}\} \frac{\Delta(\delta U(t))}{\hbar\omega} \{[\delta(\Delta \epsilon + \hbar\omega) - \delta(\Delta \epsilon)] - \delta(\Delta \epsilon - \hbar\omega) - \delta(\Delta \epsilon)]\}.$$

The shift in energy $\Delta \epsilon$ by $\hbar\omega$ in the arguments of the $\delta$-functions of the expressions (1), (14), (22) for the collision integral agrees with the interpretation of the collisions in the presence of the collective excitations proposed by Landau [20]. According to this interpretation, high-frequency oscillations in Fermi liquid can be considered as phonons, that are absorbed and created at the two-particle collisions.

As it was discussed in the introduction, the form of expressions 1), (14), (22) for collision integral is not correct in general case and an additional modification of these expressions are needed. The incorrectness results from approximations which were made at the kinetic equation derivation.

Because of assumptions both on undamped behaviour of the distribution function throughout all time interval ($-\infty \leq t' \leq t \to \infty$) and on small magnitude of two-body interaction, one can expect overestimation of retardation effects in the foregoing expression for collision integral. It means that from physical point of view this relationship should be fulfilled only in the case of small $\omega$.

Therefore, we replace the derivatives of the form $\partial \delta(\Delta \epsilon + \hbar\omega)/\partial \hbar\omega$ and $\partial \delta(\Delta \epsilon - \hbar\omega)/\partial \hbar\omega$ in $B^{(1)}$ of the Eq.(22) by the finite differences $(\delta(\Delta \epsilon + \hbar\omega) - \delta(\Delta \epsilon))/\hbar\omega$ and $(\delta(\Delta \epsilon) - \delta(\Delta \epsilon - \hbar\omega))/\hbar\omega$, respectively. Then we combine the resulting expression together with contribution $B^{(2)}$ arising from mean-field variation and finally obtain the linearized collision integral for Fermi liquid in the following form

$$J(\tilde{p}, \vec{r}, t) = \int \frac{d\tilde{p}_2d\tilde{p}_3d\tilde{p}_4}{(2\pi\hbar)^6} W\{{\tilde{p}_i}\}\delta(\Delta \tilde{p}) \Delta \chi Q \frac{\delta(\Delta \epsilon + \hbar\omega) - \delta(\Delta \epsilon - \hbar\omega)}{\hbar\omega}.$$

Here, $\Delta \chi \equiv \chi_1 + \chi_2 - \chi_3 - \chi_4; \chi_k = \chi(\tilde{p}_k, \vec{r}, t)$ is a function determining relative deviation of distribution function from local equilibrium state, $\delta \tilde{f}$, (3):

$$\delta \tilde{f} = \delta f - \frac{df}{d\epsilon} \delta U = f(\tilde{p}, \vec{r}, t) - f\text{, l.e.} = -\chi \frac{df}{d\epsilon}, \quad \chi = \nu + \delta U.$$
With the use of the algebraic relation [4]

\[(1 - \bar{f}_1)(1 - \bar{f}_2)\bar{f}_3\bar{f}_4 - \bar{f}_1\bar{f}_2(1 - \bar{f}_3)(1 - \bar{f}_4)\exp\left(\frac{\mp\hbar\omega}{T}\right)\delta(\Delta\epsilon \pm \hbar\omega) = 0, \quad (25)\]

the Eq. (23) can be presented as

\[J(\vec{p}, \vec{r}, t) = \int \frac{d\vec{p}_0d\vec{p}_2d\vec{p}_4}{(2\pi\hbar)^6} W(\{\vec{p}_i\}) \delta(\Delta\vec{p}) \Delta\chi \bar{f}_1\bar{f}_2(1 - \bar{f}_3)(1 - \bar{f}_4) [\Phi(h\omega, T) - \Phi(-h\omega, T)], \quad (26)\]

where \(\Phi(h\omega, T) = \delta(\Delta\epsilon + h\omega)\left[\exp(-h\omega/T) - 1\right]/2\hbar\omega\).

The collision integral of the form (23) or (26) depends on the variation \(\delta\bar{f}, J \equiv J(\delta\bar{f})\). It was mentioned in the introduction this behaviour is in line with general properties of the Vlasov-Landau equation in the Fermi-liquid [4, 3] at \(\partial\delta\bar{f}/\partial t = 0\). It provides driving distribution function towards its local equilibrium value. This behaviour is in line with general properties of the Vlasov-Landau equation in the Fermi-liquid [4, 3] at \(\partial\delta\bar{f}/\partial t = 0\).

The expressions (23), (26) depend only on the occupation probability \(P_{2p2h} \equiv \bar{f}_1\bar{f}_2(1 - \bar{f}_3)(1 - \bar{f}_4)\) of the 2p-2h states in the phase space. This fact leads to interpretation of the collision damping with linearized collision term as the relaxation process due to the coupling of one-particle and one-hole states to more complicated 2p − 2h configurations.

The form of the collision integral (26) in the Markovian limit (\(\omega \to 0\)) coincides with the standard expression for the collision integral in Fermi-liquid without retardation effects [4, 3] because in this case the term in square brackets of Eq.(26) tends to the value \(-\delta(\Delta\epsilon)/T\).

The equation (23) for some special explicit form of the quantity \(\chi_j\) was used at first in Refs. [9, 21, 22]. The derivation of the collision integral (23) is performed in Ref. [15] within framework of the extended time-dependent Hartree-Fock model. The expressions for the distortion functions \(\chi_j\) corresponding to a perturbation approach on collision term and including the amplitudes of the random phase approximation were used in this method.

Note that the expression for the collision integral in two-component Fermi-system is obtained from Eq.(26) in the same manner as done in Ref.[23] under the assumption that chemical potentials and the equilibrium distribution functions are the same for protons and neutrons.

3 Calculations of the relaxation times and nuclear matter viscosity

The collision integral can be used to calculate collisional relaxation times governing the dissipative properties of different physical quantities [2, 4, 9, 10, 11, 12, 24, 25, 26]. Below we calculate relaxation times, \(\tau_{\ell}(\pm)\), of collective vibrations in two-component nuclear
matter consisting of neutron and protons subsystems. These collective relaxation times is determined by interparticle collisions within the distorted layers of the Fermi surface with multipolarity $\ell$:

$$\frac{1}{\tau_{\ell}^{(\pm)}} = \int_{0}^{\infty} d\epsilon_1 \int d\Omega_p J_{\ell}^{(\pm)}(\hat{p}, \epsilon_1)Y_{\ell 0}(\hat{p})/ \int_{0}^{\infty} d\epsilon_1 \int d\Omega_p \delta f^{(\pm)}Y_{\ell 0}(\hat{p}),$$

(27)

where $Y_{\ell m}(\hat{p})$ is the spherical harmonic function and $J^{(\pm)}(\hat{p}, \epsilon)$ are the linear combinations of the collision integrals for protons $J_p$ and neutrons $J_n$ in nuclear matter: $J^{(\pm)} = (J_p \pm J_n)/2$; $\delta f^{(\pm)} = (\delta f_p \pm \delta f_n)/2$. These times are proportional to the relaxation times $\tau^{(\pm)}$ defining the damping widths $\Gamma^{(\pm)}_{\ell}(L)$ of the isoscalar ($+$) and the isovector ($-$) vibrations with frequency $\omega [12, 23, 26, 27]$ in regime of rare collisions with $\omega \tau^{(\pm)} > 1$ in the Fermi liquid. In particular, the collisional damping widths of giant resonances with dipole ($L = 1$) and quadrupole ($L = 2$) multipoles resemble the widths in the relaxation rate approach

$$\Gamma^{(\pm)}_{\ell}(L) = \hbar/\tau^{(\pm)}_{\ell}(L), \quad \tau^{(-)}_{\ell}(L = 1) = \tau^{(-)}_{\ell=1}, \quad \tau^{(+)}_{\ell}(L = 2) = \tau^{(+)}_{\ell=2},$$

(28)

in the case when nuclear fluid dynamical model with relaxation is used [23, 26]. The collisional damping width [12] of zero sound in the Fermi liquid with its relative velocity $S_r \approx 1$ is also given by Eq.(28) but with the use of the $\tau^{(+)}_{\ell=\infty} \propto \tau^{(+)}_{\ell=2}$ for $\tau^{(\pm)}_{\ell}(L)$. The time $\tau^{(+)}_{\ell=2}$ at $\omega = 0$ is the thermal relaxation time determining the viscosity coefficient of the Fermi liquid [25].

The time $\tau^{(+)}_{\ell=2}$ at $\omega = 0$ is the thermal relaxation time determining the viscosity coefficient of the Fermi liquid.

The analytical expressions for collective relaxation times of the damping of the collective vibrations with frequency $\omega$ has the following general form in low-temperature and low-frequency limits ($T, \hbar \omega \ll \epsilon_F$)[12, 23, 28]

$$\frac{\hbar}{\tau_{\ell}^{(\pm)}} = \frac{1}{\alpha_{\ell}^{(\pm)}} \left\{ (\hbar \omega)^2 + (2\pi T)^2 \right\}, \quad \frac{1}{\alpha_{\ell}^{(\pm)}} = \frac{2m}{3\pi \hbar^2} \left[ < \sigma_{av}^{(\pm)} \Phi_{\ell}^{(\pm)} > + < \sigma_{pn}^{(\pm)} \Phi_{\ell}^{(\pm)} > \right],$$

(29)

where $\sigma_{av}^{(\pm)} = (\sigma_{nn}^{(\pm)} + \sigma_{pp}^{(\pm)})/2$; $\sigma_{jj'}^{(\pm)} \equiv d\sigma_{jj'}/d\Omega$ is in-medium differential cross-section for scattering of the nucleons $j$ and $j'$ (here, $j = n$ or $p$, and similarly $j' = p$ or $n$). The symbol $< \ldots >$ in Eq.(29) denotes averaging over angles of the relative momenta of the colliding particles,

$$< \ldots > = \frac{1}{\pi} \int_{0}^{\pi} d\phi \sin(\phi/2) \int_{0}^{\pi} d\theta (\ldots).$$

(30)

The functions $\Phi_{\ell}^{(\pm)}$ define the angular constraint on nucleon scattering within the distorted layers of the Fermi surface with multipolarity $\ell$:

$$\Phi_{\ell}^{(\pm)} = 1 \pm P_\ell(\cos \phi) - P_\ell((\hat{p}_3 \hat{p}_1)) \mp P_\ell((\hat{p}_4 \hat{p}_1)),$$

(31)
where the scalar products $(\hat{p}_3\hat{p}_1)$ and $(\hat{p}_4\hat{p}_1)$ are given as
\[
(\hat{p}_3\hat{p}_1) = \cos^2(\phi/2) + \sin^2(\phi/2) \cos \theta,
(\hat{p}_4\hat{p}_1) = \cos^2(\phi/2) - \sin^2(\phi/2) \cos \theta.
\]  

Due to the momentum conservation and conditions $p_i = p_F$, the angle $\theta$ agrees with the scattering angle in the center-of-mass reference frame of two nucleons. The angle $\phi$ defines the magnitudes of the relative momenta $k_i = (\vec{p}_2 - \vec{p}_1)/2$ and $k_f = (\vec{p}_4 - \vec{p}_3)/2$ before and after collision, respectively. The value of total momentum, $P = \vec{p}_1 + \vec{p}_2$, also depends on a magnitude of the $\phi$. We have
\[
\vec{k}_i \vec{k}_f = k^2 \cos \theta, \quad k^2 = k_i^2 = k_f^2 = p_F^2 \sin^2(\phi/2), \quad \vec{P}^2 = 4p_F^2 \cos^2(\phi/2).
\]  

Therefore the relative kinetic energy $E_{rel}$ of two nucleons as well as the energy of centrum mass motion $E_{cm}$ are dependent on angle $\phi$
\[
E_{rel} = \frac{k^2}{m} = 2\epsilon_F \sin^2(\phi/2), \quad E_{cm} = \frac{P^2}{2m} = 2\epsilon_F \cos^2(\phi/2)
\]  

and the total energy $E_{tot} = E_{rel} + E_{cm}$ holds only fixed, $E_{tot} = 2\epsilon_F$. Therefore the in-medium differential cross-sections $\sigma'_{j,m}$ of two nucleon scattering depend on the relative momenta $k_i$ and $k_f$ at fixed total energy rather than at fixed relative kinetic energy $E_{rel}$, because the magnitude of $E_{rel}$ changes with angle $\phi$ between colliding particles. The transfer momenta $q = k_i - k_f = \vec{p}_3 - \vec{p}_1$ and $q' = - (k_i + k_f) = \vec{p}_1 - \vec{p}_4$ for scattering due to direct and exchange interactions respectively are also functions of the $\phi$ and $\theta$: $q = 2k(\phi) \sin(\theta/2)$ and $q' = 2k(\phi) \cos(\theta/2)$.

Now we estimate the collisional relaxation times in the case of the isotropic scattering with independent of energy the angle-integrated cross sections $\sigma_{jj'}$. Performing angular integration in (29) with the use of Eqs.(30) and (31) we find that $1/\tau_{\ell<\ell_0}^{(\pm)} = 0$ and

\[
\frac{\hbar}{\tau_{\ell}^{(\pm)}} = \frac{1}{\alpha_{\ell}^{(\pm)}} \left[ (h\omega/2\pi)^2 + T^2 \right], \quad \frac{1}{\alpha_{\ell}^{(\pm)}} = \frac{8m}{3\hbar^2} \left[ c_0 \sigma_{av} + d_0^{(\pm)} \sigma_{np} \right],
\]  

\[
c_\ell = 1 - \frac{2 - (-1)^\ell}{2\ell + 1}, \quad d_{\ell}^{(-)} = \frac{1 - (-1)^\ell}{2\ell + 1}, \quad d_{\ell}^{(+)} = d_{\ell=0}^{(-)} = c_{\ell=0} = c_{\ell=1} = 0,
\]  

where $\sigma_{av} = [\sigma_{pp} + \sigma_{nn} + 2\sigma_{np}]/4$ is the in-medium spin-isospin averaged nucleon-nucleon cross section. The magnitude of the in-medium cross section $\sigma_{jj'}$ is taken usually proportional to the value of the free space cross section $\sigma_{jj'}^{(f)}$ with a factor $F = \sigma_{jj'}/\sigma_{jj'}^{(f)}$, so that the parameter $\alpha_{\ell}^{(\pm)}$ can be rewritten in the form
\[
\alpha_{\ell}^{(\pm)} = \tilde{\alpha}_{\ell}^{(\pm)}/F, \quad \tilde{\alpha}_{\ell}^{(\pm)} = 4.18 / \left[ c_\ell + 1.3d_{\ell}^{(\pm)} \right], \quad MeV.
\]
Here, the values $\sigma_{av}^{(f)} = 3.75 \text{ fm}^2$ and $\sigma_{np}^{(f)} = 5 \text{ fm}^2$ are adopted [9, 27]; they correspond to the free space cross sections near Fermi energy. The dependence on $\ell$ for relaxation times $\tau_{\ell}^{(\pm)} \equiv \tau_{\ell,f}^{(\pm)}$ with the use of free space cross sections is shown in Fig. 1; $\hbar\omega = 13.43 \text{MeV}$. The Figs. 2, 3 describe relative relaxation times versus multipolarity $\ell$ for cross section parameterizations using Gogny and Skyrme effective two-body forces with parameters from [15, 30]. The relative relaxation times presented in Fig. 3 were calculated using cross section on Fermi surface. Solid and dashed lines connect the values which correspond to isoscalar and isovector modes of vibrations respectively.

The magnitudes of the relaxation times are different for isoscalar and isovector modes of vibrations and they are dependent on the multipolarity $\ell$. The collisional relaxation times rather slowly vary with collective motion mode at isotropic scattering with energy independent free cross sections. As seen from the Fig. 3, the relaxation times calculated with effective interaction between nucleons are greater than that ones with cross-section in free space. It means that in-medium cross-section between nucleons in nuclear matter is smallest than in free space ($\approx 20\%$ for Skyrme forces and $\approx 60\%$ for Gogny interaction). The relaxation times $\tau_{\ell}^{(\pm)}$ depend on frequency $\omega$ due to the memory effects in the collision integral. The temperature dependence arises from smearing out the equilibrium distribution function near the Fermi momentum in heated nuclei. The collisional rates $1/\tau_{\ell}^{(\pm)}$ are quadratic both in temperature and in frequency with the same relationship between the components much as in the zero sound attenuation factor of heated Fermi liquid within the Landau prescription [20, 26, 9, 15].

According to response function approach the damping width of giant isovector dipole resonance (GDR) at the temperature $T$ is presented in the form [29]

$$
\Gamma(T) = 2 q\gamma \frac{E_r^2 + E_0^2}{(E_r^2 - E_0^2)^2 + (2\gamma E_r)^2},
$$

(37)

where $\gamma$ is determined by relaxation time $\tau_c(h\omega = E_r, T)$,

$$
\gamma = \frac{\hbar}{\tau_c(h\omega = E_r, T)},
$$

(38)

$E_r$ is an energy of giant dipole resonance; $E_0 = 41 A^{-1/3} \text{ MeV}$. Here, we determine quantity $q$ from equality of the GDR width in cold nuclei with corresponding experimental value $\Gamma_{exp}$: $\Gamma(T = 0) = \Gamma_{exp}$. The temperature dependence of the GDR width according to Eq.(38) in atomic nuclei $^{208}Pb$ and $^{120}Sn$ are shown on Figs.6. Experimental data are taken from Refs.[31, 32] and they are indicated by points. The relaxation time $\tau_c$ is taken as equal to $\tau_{\ell=1}^{(-)}$: dash lines in Fig. 4 correspond to calculations with Skyrme forces; dash-dot lines - Gogny interaction; solid lines - calculations with free space cross-section for two-nucleon collisions. The temperature behaviour of the damping width is in rather close agreement with the ones of experimental data.
It is seen from Figs. 4 that in rare collision regime the dependence of the GDR widths on the collective vibration frequency and the temperature has the following form \( \Gamma \propto (\hbar \omega)^2 + 4(\pi T)^2 \), which corresponds to the Landau’s prescription [20, 33, 34].

The relaxation times rather slowly vary with multipolarity of the Fermi surface distortions governed by collective motion and two-body collisions. It gives possibility to use approximately the relaxation time ansatz for collision integral.

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References


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Fig. 1: The relaxation times $\tau_{\ell,f}^{(\pm)}$ versus multipolarity $\ell$ in cold nucleus $^{208}\text{Pb}$ with free space cross section.
Fig. 2: The relative relaxation times $\tau_{\ell}^{(\pm)}/\tau_{\ell,f}^{(\pm)}$ versus multipolarity $\ell$ for different cross section parameterizations.
Fig. 3: The relative relaxation times $\tau^{(\pm)}_\ell / \tau^{(\pm)}_{\ell,f}$ versus multipolarity $\ell$ for different cross section parameterizations calculated on Fermi surface.

Fig. 4: The temperature dependence of the GDR width according to Eq.(38) in nuclei $^{208}$Pb and $^{120}$Sn: dash lines - calculations with Skyrme forces; dash-dot lines - Gogny interaction; solid lines - calculations with free space cross-section for two-nucleon collisions.