Dependence of Nuclear Level Density on Vibrational State Damping

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The response function approach is proposed to include vibrational state in calculation of level density. The calculations show rather strong dependence of level density on the relaxation times of collective state damping.

1 Introduction

Level density $\rho$ is one of the main quantities to define characteristics of nuclear decay. The collective states can strongly effect on level density, specifically, at low excitation energies [1]-[6]. Calculation of the variation factor $K$ of level densities is the simplest method to estimate effect of the vibrational states on level densities. The factor $K$ is the ratio of level densities with and without allowing for vibrational states. The level densities can be calculated within framework of statistical approach by standard saddle-point method[1] or modified one[7]. It was found[8] that factor $K$ is practically independent of type of statistical approach. Therefore we will use below standard saddle-point method as a simplest way for calculation of the variation factor of level density.

In this contribution work the effect of the vibration state damping on nuclear level density is studied on the base of response function approach in wide range of excitation energies. The dependence of variation factor $K$ on damping of vibrational state is investigated.
2 Response function method for calculation of the level density variation

Variation factor of level density is defined as

\[ K = \frac{\rho}{\rho_0}, \]  

where \( \rho \) and \( \rho_0 \) are level densities with and without allowing for vibrational states.

The level density has the following form within framework of statistical approach in standard saddle-point method [1]

\[ \rho(U, A) = \left(4\pi^2 D\right)^{-1/2} \exp S(\alpha_0, \beta_0), \]  

where \( S(\alpha_0, \beta_0) = -\alpha_0 A + \beta_0 E + \ln Z(\alpha_0, \beta_0) \) is entropy of nucleus with mass number \( A \) at excitation energy \( U = E - E_{g.s} \) with \( E_{g.s} \) for ground state energy; \( Z(\alpha, \beta) = \text{Tr} \left[ \exp \left( -\beta \hat{H} + \alpha \hat{A} \right) \right] \) - partition function; \( \hat{H} \) - nuclear Hamiltonian; \( \hat{A} \) - operator of particle number; \( D \) - determinant of second partial derivatives of logarithm of partition function with respect to parameters \( \alpha \) and \( \beta \). The saddle-points \( \alpha_0, \beta_0 \) define temperature \( (T = 1/\beta_0) \) and chemical potential \( (\mu = \alpha_0/\beta_0) \). They are solutions of the system of thermodynamic state equations \( A = \partial \ln Z/\partial \alpha, \ E = -\partial \ln Z/\partial \beta. \)

We use conception of random phase approximation and suppose that collective vibrational states of the multipolarity \( L \) are formed by the separable residual interaction \( V_{res}^k = k \hat{Q}_L \hat{Q}_L^+ \), where \( \hat{Q}_L \) is one-body operator of multipole momentum with multipolarity \( L \), \( \hat{Q}_L \propto r^L Y_{LM} \), and \( k \) is a coupling constant.

The partition function can be presented in the form [3] \( Z = Z_0 \cdot \Delta Z \), where \( Z_0 \) is partition function of the independent particle model. The quantity \( \Delta Z \) can be expressed by variation of thermodynamic potential \( \Delta \Omega \) in the form \( \Delta Z = \exp(-\beta \Delta \Omega) \) with the following equation for \( \Delta \Omega \)

\[ \Delta \Omega = \int_0^k d\varepsilon' \left\langle \hat{Q}_L \hat{Q}_L^+ \right\rangle /2 = \frac{2L+1}{2\pi} \int_0^k d\varepsilon' \int_{-\infty}^{+\infty} \frac{-1}{e^{\beta \varepsilon' - 1}} \text{Im} X^k_L d\varepsilon'. \]  

Here, \( X^k_L \) is nuclear response function [9] on external field proportional to \( \hat{Q}_L \) with frequency \( \omega = \varepsilon/\hbar \); symbol \( \left\langle ... \right\rangle \) denotes averaging over total Hamiltonian with residual interaction \( V_{res}^{k'} \). The final expression in right-hand side of the eq.(3) is obtained after transformation of relationship for
ΔΩ with the use of Green function method[10]. Finally variation of the partition function is presented in the following form

\[ \Delta Z(\beta) \equiv \Delta Z_{vib} = \prod_j \left| \frac{1 - \exp \left( -\left( E_j^0 + i\Gamma_j^0 \right) \beta \right)}{1 - \exp \left( -\left( E_j + i\Gamma_j \right) \beta \right)} \right|^{2L+1} \]

(4)

with the use of transformation similar to that one from Ref.[11] Here, \( E_j, \Gamma_j \) are real and imaginary parts of solutions of the equation

\[ k - 1/X_0^L(\varepsilon) = 0, \]

(5)

and \( E_j^0, \Gamma_j^0 \) are real and imaginary parts of roots of the equation \( 1/X_0^L(\varepsilon) = 0 \) with \( X_0^L \) for response function of independent particle model \( (k = 0) \).

3 Calculations and conclusion

Semiclassical Vlasov-Landau approach for response function[9] was used in the calculations. The nuclear mean field and relaxation processes can be easy taken into account in this approach. Collision integral was considered in relaxation time approach. Three different parametrizations of relaxation time \( \tau_c \) were adopted with taken into account retardation effects[12, 13].

1) Expression within framework of transport theory[12, 13]

\[ \bar{\hbar}/\tau_c(\varepsilon, T) = \left( (\varepsilon/2\pi)^2 + T^2 \right)/4.07 \text{ MeV}. \]

(6)

2) Relaxation time due to model of doorway state

\[ \bar{\hbar}/\tau_c(\varepsilon, T) = 2\pi \left( \frac{K_m}{A^3} \right) \frac{g^3(\varepsilon + U)^2}{2 \frac{3 + \bar{n}}{3}}, \]

(7)

where \( \bar{n} = 0.843 a T \); \( K_m \) is defined from agreement between relaxation times given by eqs.(6) and (7) at \( T = 0 \) and at GQR energy; \( g = g(\varepsilon_F) = 6a/\pi^2 \) with level density parameter \( a = A/8 \text{ MeV}^{-1} \).

3) Model of doorway state

\[ \bar{\hbar}/\tau_c(\varepsilon, T) = 2\pi \left( \frac{K_B}{A^3} \right) \frac{g^3(\varepsilon + U)}{8}, \]

(8)

where \( K_B \) is defined from agreement with relaxation times given by eqs.(6) and (8) at \( T = 0 \) and at GQR energy.

The dependence of variation factor \( K \) on excitation energy is shown on fig.1 for nucleus \(^{56}\text{Fe} \). Low-lying quadrupole state with energy \( E_{2+}^{\text{exp}} = \)}
0.847 MeV was taken into account; histogram is microscopic calculation and experimental values[4]. Solid line shows calculation with relaxation time given by eq.(6), dash - with relaxation time according to eq.(7), dotted - eq.(8) for \( \tau_c \).

Parametrization of relaxation time given by eq. (8) leads to small value of variation factor in comparison with other parametrization of relaxation times. Level density rather strongly depends on a relaxation times. It can give an additional possibility for investigation of the relaxation time dependence on temperature and collective state energy.

References


Fig. 1: Variation factor of level density for $^{56}$Fe at different relaxation times; lines: solid - relaxation time due to (6), dash - (7), dotted - (8); histogram - microscopic calculation and experimental values [4].