\[ \Omega_1(x) e^{-i(\omega_{at} + k \cdot z)}, \] where the \( y \)-dependence of the amplitude \( \Omega_1(x) \equiv \Omega_1 \) will be kept implicit. The same applies to other parameters, such as the mean atomic number-density \( n \equiv n(z) \).

The probe beam is described by the electric field operator:

\[ \mathcal{E}(r, t) = \hat{e}(\hbar c k/2\varepsilon_0)^{1/2} \mathcal{E}^1(r, t) e^{-i(\omega_{at} + k \cdot z)} + h.c. \]

where \( \mathcal{E}(r, t) \equiv \mathcal{E} \) is the slowly-varying field, \( \omega = c k \) is the central frequency of probe photons, \( \mathbf{k} = \hat{z} k \) is the wave-vector and \( \hat{e} \perp \hat{z} \) is the unit polarization vector. The dimension of \( \mathcal{E} \) is such that the operator \( \mathcal{E}^1 \) represents the number density of probe photons.

The atoms (initially in the ground level \( g \)) are moving along the \( x \)-axes with an average speed \( \mathbf{v}_a = \hbar \mathbf{k} \mathbf{a}/m \) and the kinetic energy \( \omega_{at} = \hbar^2 \mathbf{k}^2/2m \). The atomic velocities are assumed to be spread over a narrow range around \( \mathbf{v}_a \), so that we can introduce slowly-varying atomic amplitudes as:

\[ \Phi_g = \Psi_g e^{i(\omega_{at} + k \cdot z)}, \]

\[ \Phi_i = \Psi_i e^{i(A \omega_{at} + k \cdot z)} \]

and \( \Phi_i \equiv \Psi_i e^{i(A \omega_{at} + k \cdot z)} \).

Consider first storing of the probe by the first control laser. The following equations hold for the slowly-varying electromagnetic and matter field-operators:

\[
\begin{align*}
\left( \frac{\partial}{\partial t} + c \frac{\partial}{\partial z} \right) \mathcal{E} & = i g \Phi_1 \Phi_e, \quad (1) \\
\left( \frac{\partial}{\partial t} + \mathbf{v}_a \frac{\partial}{\partial x} \right) \Phi_g & = i g \mathcal{E} \Phi_e, \quad (2) \\
\left( \frac{\partial}{\partial t} + i \Delta + \mathbf{v}_a \frac{\partial}{\partial x} + \mathbf{v}_a \frac{\partial}{\partial z} \right) \Phi_e & = \Delta \mathcal{E} \Phi_g, \quad (3) \\
\left( \frac{\partial}{\partial t} + i \Delta + v_a \frac{\partial}{\partial x} + \Delta \Phi_g & = \Delta \mathcal{E} \Phi_e, \quad (4)
\end{align*}
\]

where \( g = \mu \sqrt{c k/2\varepsilon_0} \hbar \) characterizes the strength of the radiation-matter coupling, \( \Delta = \omega_g - \omega_e - \omega_d + \Delta \omega_r \) and \( \Delta = \omega_g - \omega_e - \omega_d + \omega_r \) are the two- and single-photon detunings, \( v_a = h \mathbf{k} \mathbf{a}/m \) is the atomic recoil velocity due to absorption of a probe photon, \( \omega_{at} = h \mathbf{k} \mathbf{a}/m \) is the recoil frequency and \( \Delta \omega_r = h \mathbf{k} \mathbf{a}/2m \). The probe and control beams are assumed to be co-propagating. In this case the overall recoil velocity \( \Delta \mathbf{v}_a = h \mathbf{k} \mathbf{a}/m \) is small and can be neglected in eq.(4). Dissipation of the excited state \( \psi_e \) can be included into the equation (3) replacing \( \Delta \) by \( \Delta - i \gamma \) and adding the appropriate noise operator.

Suppose the probe field \( \mathcal{E} \) is sufficiently weak so that one can disregard the depletion of the ground level \( \psi_g \). Neglecting the last term in eq.(2), one has

\[ \frac{\partial}{\partial t} \Phi_g(r, t) = -i \mathbf{v_a} \frac{\partial}{\partial x} \Phi_g(r, t). \]

It is convenient to introduce the field operators describing annihilation of atomic and spin excitations (excitons):

\[ \psi_u \equiv \psi_u(r, t) = n^{-1/2} \Phi^\dagger_g \Phi_u \]

with \( u = e, q \), where \( n \equiv n(z) \equiv \langle \Phi^\dagger_g \Phi_g \rangle \) describes the spatial profile of the number-density of ground-state atoms. Exploiting relation (5), the equations of motion for the new operators have the form of eqs.(1), (3) and (4) subject to the replacement \( \Phi \rightarrow \psi_u, \Phi_q \rightarrow \psi_e \) and \( \Phi_q \rightarrow n \). Note that the exciton operators \( \psi_e \) and \( \psi_q \) obey approximately Bose-commutation relations even through the constituent atoms may be fermions.

Let us consider the case of exact two-photon resonance \( \gamma \equiv 0 \). This is well justified as the two-photon Doppler shift due to longitudinal and transversal velocity spread of the atoms is strongly diminished due to the chosen geometry. Neglecting terms containing \( \psi_e \) and \( \psi_q \) in eq.(3), one arrives at the adiabatic approximation relating \( \psi_q \) to the electric field amplitude \( \mathcal{E}(r, t) \) as

\[ \psi_q(r, t) = -g n^{1/2} \mathcal{E}(r, t) \]

(7)

with \( \mathcal{E}(r, t) = \mathcal{E}(r, t)/\Omega_1(x) \equiv -\psi_q(r, t)/g n^{1/2} \) being an auxiliary field. Assuming a stationary flow of atoms in the \( x \)-direction, the atomic density \( n \equiv n(z) \) does not depend on \( x \), and thus eqs. (4) and (7) yield

\[ \psi_e(r, t) = \frac{g n^{1/2}}{\Omega_1(x)} \left( \frac{\partial}{\partial t} + \frac{\partial}{\partial x} \right) \mathcal{E}(r, t). \]

(8)

On the other hand, the Rabi frequency \( \Omega_1(x) \equiv \Omega_1 \) is \( z \)-independent, so eqs. (1) and (8) lead to the following equation for \( \mathcal{E}(r, t) \):

\[ \left( \frac{\partial}{\partial t} + \frac{c}{1 + n g} \frac{\partial}{\partial z} + \frac{v_a n g}{1 + n g} \frac{\partial}{\partial x} \right) \mathcal{E}(r, t) = 0, \]

(9)

with \( v_a(x, z) = g n(x, z) \Omega_1^2(z) \) being the group index. Assuming slow propagation, the group index is much larger than unity, so that eq. (9) simplifies to

\[ \left( \frac{\partial}{\partial t} + v_a(z) \frac{\partial}{\partial x} \right) \mathcal{E}(r, t) = 0. \]

(10)

where \( v_a(z) \equiv v_a(x, z) \equiv c/n g(x, z) \) is the group velocity (\( v_a \ll c \)). The dimensionless quantity \( a(x) \equiv a_1(x) = [\Omega_1(x)/\Omega_1(x_1)]^2 \) characterizes the spatial shape of the first control laser centered at \( x = x_1 = 0 \). The \( z \)-dependence of \( v_a(z) \equiv v_a(x, z) \) emerges through the atomic density \( n \equiv n(z) \). It is noteworthy that \( \mathcal{E}(r, t) = -\psi_q(r, t)/\sqrt{v_a/c} \), so the ratio between the number-density of photons and the spin excitations is given by the relative group velocity \( v_a/c \). In other words, the slowly propagating probe beam is made of the EIT (dark state) polaritons [1, 5, 6] comprising predominantly the spin excitations.

Suppose the spatial width of the first control beam \( \Delta x_{1} \) is much larger than that of the incoming probe beam \( \Delta x_{p} \) centered at \( x = x_1 = 0 \). Under this condition, the solution of eq.(10) can be expressed in terms of the incoming electric field at an entry point \( z = z_1 \) as:

\[ \mathcal{E}(r, t) = \frac{\mathcal{E}[\xi(x, z, \Delta x_{1}, \Delta t_{p}(r, x, z)]}{\Omega_1(x_1)} \]

(11)
where \( \xi(x, z) = \int_{-\infty}^{x} a(x') \, dx' - v_0 \int_{z_1}^{z} \left[ \nu_g (z') \right]^{-1} \, dz' \), and 
\( \tau(t, x, z) = t + \xi(x, z) / v_0 - x / v_0 \) obey the proper boundary conditions: \( \xi(x, z = \pm \infty) = x \) and \( \tau(t, x, z = \pm \infty) = t \) for the incoming probe field.

Using the definition of \( \mathcal{E} \) and eq. (11), one finds the temporal and spatial behavior of the operators for electric field and spin excitations:

\[
\mathcal{E}(x, t) = \sqrt{|a(x)|} \left[ \mathcal{E}[\xi(x, z), z_1, \tau(t, x, z)] \right], \quad \nu_g(z) a(x), \quad (12)
\]

\[
\psi_0(z) = \frac{\nu_g^{1/2}}{\Omega_1 (x)} \mathcal{E}[\xi(x, z), z_1, \tau(t, x, z)], \quad (13)
\]

Equations (12) and (13) define the electric and spin components of the EIT polariton. A set of trajectories for such a polariton in the \( x-z \) plane is given by \( \xi(x, z) = \xi_0 \), with \( \xi_0 = 0 \) corresponding to the central trajectory. Due to the motion of the atoms parallel to the \( x \) axis, the polariton is dragged into that direction. Following the spatial profile \( a(x) \) of the coupling beam, its velocity component in the \( z \)-direction, \( \nu_g(z) a(x) \), is further reduced. As soon as \( v_g(z) a(x) \) becomes less than \( v_0 \) the flow of excitation is predominantly determined by the velocity of the atoms \( v_0 \) in the \( x \) direction. If the atomic beam is optically thick in the \( z \)-direction (\( \xi(\infty, z_{max}) > 0 \)), the propagation direction of the polariton may be completely converted from \( z \) to \( x \). The probe field is then stored in the form of a spin excitation that moves with the atoms parallel to the \( x \)-axis and is centered at \( z = z_\infty \), which is a solution of \( \xi(x = \infty, z_{\infty}) = 0 \). This is illustrated in figs. 2 and 3 where we have shown the amplitude of the electric field at different instances of time. Also shown is the effect of the second regeneration laser, which will be discussed later.

As one can see from the figures the dragging of the polariton along with the moving atoms leads to a deformation of the excitation. For large values of \( x \), such that \( a(x) \to 0 \) one has in the vicinity of \( z = z_\infty \)

\[
\xi(x \to \infty, z) \approx - (z - z_\infty) \frac{v_0}{\nu_g (z_\infty)} \quad (14)
\]

\[
\tau(t, x \to \infty, z) \approx - \frac{z - z_\infty}{\nu_g (z_\infty)} - \frac{x - x_\infty}{v_0} \quad (15)
\]

where \( x_\infty \equiv x_\infty (t) = v_0 t \) is the "center of mass" of a stored polariton moving in the \( x \)-direction. Thus if \( \Delta x_p \) and \( 2\Delta \tau_p \) denote the half-width (in \( x \)) of the input probe pulse, the dimensions of the spin-wave are in the limit \( x \to \infty \) :

\[
\Delta x \approx v_0 \Delta \tau_p, \quad \Delta z \approx \frac{\nu_g (z_\infty)}{v_0}, \quad (16)
\]

where we have assumed that \( v_0 \ll \Delta x_p / \Delta \tau_p \). Since the spatial profile of the polariton in the \( y \) direction is not changed, the transfer from an electromagnetic to a matter-wave pulse is associated with a spatial compression of the excitation volume by a factor \( \nu_g (z_\infty) / \nu_0 \). From eq. (16) one can also easily obtain a necessary condition for a complete transfer of the electromagnetic excitation to the atomic beam: Since \( \Delta z \approx \Delta z_\text{atom} \) one finds a minimum ratio of \( v_0 \) to the group velocity at the center

\[
\frac{v_0}{v_g (z_\infty)} \gg \frac{\Delta x_p}{\Delta z_\text{atom}} \quad (17)
\]

If this condition is fulfilled a pulse is completely stored within the atomic beam and does not exit on the back side.

Consider now the regeneration of the probe beam by a spatially separated, i.e., non-overlapping, second control laser centered at \( x = x_2 \) and characterized by a Rabi frequency \( \Omega_2 (x) \). In this case the previous equations (10) - (14) describe the propagation of a probe beam within the entire system, subject to the following replacement: \( a(x) \rightarrow a(x) = a_1 (x) \pm a_2 (x) \) where \( a_2 (x) = \left[ \Omega_2 (x) / \Omega_1 (x) \right]^2 \) characterizes the shape of the second control laser. The upper (lower) sign corresponds to the case where the second control laser propagates in the same (opposite) direction, as compared to the first one [19]. This is because the radiative group velocity of the regenerated probe beam changes sign in the latter case. It is noteworthy that the reversed probe beam experiences a slight shift out of the EIT resonance [5]. Yet such a shift can be neglected in the case where the initial

\[
\text{FIG. 2: Propagation of a probe pulse in a moving EIT medium at times } t = 0, 5, 10, 15, 20, 25, 30, 35, 40, 45, 50 \text{ illuminated by a pair of control lasers propagating in the } +z \text{ direction at } x_1 = 0 \text{ and } x_2 = 5. \text{ The control lasers have equal amplitudes and Gaussian profiles with unity width. The flow and group velocities are } v_0 = 0.1, \text{ and } \nu_g (z) = (1 - 0.95 \exp \{-(z - 2)^2 \})\]

control and probe beams are co-propagating. Note also that the previously considered regeneration of a probe beam is due to the temporal switching of the control laser [1, 2, 3, 4, 5, 6, 7], whereas now the regeneration is induced by a second spatially separated control laser that can be stationary.

Finally let us analyze the output field at a spatial point where $z = z_2$. The maximum electric and spin fields are then concentrated at $x = x_2$, for which $\xi(x_2, z_2) = 0$. In the vicinity of this point, one has $\xi(x, z) \approx \pm a_2(x_2)(x - x_2)$ i.e. the output field has a width $\Delta x_2 = \Delta x_2/ a_2(x_2)$.

In summary, we have investigated a novel scheme of storing and releasing a beam of probe light in a moving atomic medium illuminated by two spatially separated control lasers depicted in fig.1. Beyond the area illuminated by the first control laser, the probe beam transforms into a beam of pure spin excitations moving along the $x$ axis, as one can see from figs.2,3. The regeneration of the probe beam is accomplished by applying the second continuous control laser. Depending on the direction of the latter, the restored probe beam moves either parallel (fig.2), or antiparallel (fig.3) to the initial probe beam.

In contrast to the previous schemes [1, 2, 3, 4, 5, 6, 7], storing and releasing of a probe beam can be now accomplished without switching ‘off’ and ‘on’ of a control laser. This is advantageous if one doesn’t know the exact time of arrival of the probe photons, e.g. if the latter are created via spontaneous processes.

[19] The second control laser should be properly polarized in order to drive the same transition $q \rightarrow e$. 

FIG. 3: Propagation of a probe pulse in a moving EIT medium for conditions of fig.2 but with a counterpropagating (i.e., in $z$ direction) second control field.