Stabilizing dilaton and baryogenesis

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Abstract

Entropy production by the dilaton decay is studied in the model where the dilaton acquires potential via gaugino condensation in the hidden gauge group. Its effect on the Affleck-Dine baryogenesis is investigated with and without non-renormalizable terms in the potential. It is shown that the baryon asymmetry produced by this mechanism with the higher-dimensional terms is diluted by the dilaton decay and can be regulated to the observed value.

I. INTRODUCTION

Many gauge-singlet scalar fields arise in the effective four-dimensional supergravity which could be derived from string theories. Among them the dilaton $S$ has a flat potential in all orders in perturbation theory [1]. Therefore some non-perturbative effects are expected to generate the potential whose minimum corresponds to the vacuum expectation value (VEV). The most promising mechanism of the dilaton stabilization and the supersymmetry breaking is the gaugino condensation in the hidden gauge sector [2–5].

In cosmological consideration, even if the dilaton acquires potential through such non-perturbative effects, there are some difficulties to relax the dilaton to the correct minimum, as pointed out by Brustein and Steinhardt [6]. Since the potential generated by multiple
gaugino condensations is very steep in the small field-value region, the dilaton would have large kinetic energy and overshoot the potential maximum to run away to infinity.

As a possible way to overcome this problem, Barreiro et al. [7] pointed out that the dilaton slowly rolls down to its minimum with a little kinetic energy due to large Hubble friction in the case that the background fluid dominates the cosmic energy density. As a result, the dilaton could be trapped and would oscillate around the minimum.

When the dilaton decays, however, the remaining energy density is transformed to radiation. One may worry about a huge entropy production by the decay, because it could dilute initial baryon asymmetry [8]. Therefore in order to obtain the observed baryon asymmetry, as required by e.g. nucleosynthesis, it is necessary to produce a larger asymmetry than the observed one at the outset.

The attractive mechanism to produce large baryon asymmetry in supersymmetric models was proposed by Affleck and Dine [9]. However, the baryon-to-entropy ratio produced by this mechanism, $n_b/s$, is usually too large. So if we take into account the entropy production after the baryogenesis, we can expect that the additional entropy may dilute the excessive baryon asymmetry to the observed value, as pointed out in e.g. [10,11].

In this paper, we investigate whether the dilution by the dilaton decay can regulate the large baryon asymmetry produced by the Affleck-Dine mechanism to the observed value. The paper is organized as follows. In §II and §III, we describe the potential and the dynamics of the dilaton. Then, in §IV we estimate the baryon asymmetry generated by the Affleck-Dine mechanism taking into account dilution by the dilaton decay. Section V is devoted to the conclusion. We take units with $8\pi G = 1$.

II. DILATON POTENTIAL

We consider the potential of the dilaton non-perturbatively induced by multiple gaugino condensates. In string models, the tree level Kähler potential is given by

$$K = -\ln(S+S^*) - 3\ln(T+T^* - |\Phi|^2),$$

where $S$ is the dilaton, $T$ is the modulus and $\Phi$ represents some chiral matter fields [12].

The effective superpotential of the dilaton [2] generated by the gaugino condensation is given by

$$W = \sum_a \Lambda_a e^{-\alpha_a S},$$

where $\Lambda_a$ and $\alpha_a$ are constants which depend on the hidden gauge groups, for example, $\alpha_a = 8\pi^2/N_a$ for the pure SU($N_a$) Yang-Mills theories. Since at least two condensates are required to form the potential minimum, we consider a model with two condensates. Then the indices of the gauge group are $a = 1, 2$, and we take $10 \lesssim \alpha_1 \lesssim \alpha_2$. We also assume that $\Lambda_1$ and $\Lambda_2$ have opposite sign and satisfies $\Lambda_1/\Lambda_2 < -1$ [4].

The potential $V$ for scalar components in supergravity is given by

$$V = e^K \left[ (K^{-1})^i_j D_i W (D_j W)^* - 3|W|^2 \right],$$

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\[ D_i W = \frac{\partial W}{\partial \Phi^i} + \frac{\partial K}{\partial \Phi^i} W, \]  

where

\[ K_{ij} = \partial^2 K / \partial \Phi^i \partial \Phi^j \]  

and the inverse \((K^{-1})_{ij} \) is defined by \((K^{-1})_{ij} K_{jk} = \delta_i^k \). In the region, \( \alpha \text{Re} S \gg 1 \), the potential Eq. (3) can be rewritten as

\[ V(S) \simeq e^K (K^{-1})^S |\partial_S W|^2 \]

\[ = (S + S^*) |\alpha_1 \Lambda_1|^2 e^{-\alpha_1 (S + S^*)} \left| 1 + \frac{\alpha_2 \Lambda_2}{\alpha_1 \Lambda_1} e^{-(\alpha_2 - \alpha_1) S} \right|^2. \]  

(5)

In this potential the imaginary part of \( S \) has a minimum at \( \text{Im} S = 0 \). So we assume \( \text{Im} S = 0 \) and concentrate on the behavior of \( \text{Re} S \) hereafter. Then we find that the potential minimum, \( S_{\text{min}} \), is given by

\[ S_{\text{min}} = \frac{1}{\alpha_2 - \alpha_1} \ln \left( \frac{\alpha_2 - \Lambda_2}{\alpha_1 \Lambda_1} \right). \]  

(6)

We assume \( S_{\text{min}} \simeq 2 \) to reproduce a phenomenologically viable value of the gauge coupling constant of grand unified theory \( [5] \). On the other hand, the position of the local maximum of potential, \( S_{\text{max}} \), is given by

\[ S_{\text{max}} = S_{\text{min}} + \frac{1}{\alpha_2 - \alpha_1} \ln \left( \frac{\alpha_2}{\alpha_1} \right). \]  

(7)

For \( S \ll S_{\text{min}} \), the potential (5) can be approximated as

\[ V(S) \simeq (S + S^*) V_0 e^{-\alpha_2 (S + S^*)}, \]  

(8)

where \( V_0 \simeq |\alpha_2 \Lambda_2|^2 \). Thus the dilaton potential depends only on \( \text{Re} S \). The potential (5) has the minimum at \( S_{\text{min}} \) and the local maximum at \( S_{\text{max}} (> S_{\text{min}}) \). For \( S \gg S_{\text{cr}} \equiv S_{\text{min}} - 1/(\alpha_2 - \alpha_1) \), the approximate expression for the potential (8) breaks down. Around the potential minimum \( S_{\text{min}} \), the potential (5) becomes

\[ V(S) \simeq |\alpha_1 \Lambda_1|^2 e^{-2\alpha_1 S_{\text{min}} (\alpha_2 - \alpha_1)^2 (S - S_{\text{min}})^2}. \]  

(9)

However, one can see from the Kähler potential (1) that the variable \( \text{Re} S \) does not have the canonical kinetic term. Therefore we introduce the canonically normalized variable, \( \phi \), as

\[ \phi \equiv \frac{1}{\sqrt{2}} \ln \text{Re} S. \]  

(10)

### III. STABILIZATION MECHANISM FOR THE DILATON

Here, after reviewing the mechanism for dilaton stabilization proposed by Barreiro et al. \([7]\), we estimate the relic energy density of the dilaton and the amount of the entropy density produced by its decay. We will consider the situation that the universe after inflation
contains the dilaton, $\phi$, and a fluid with the equation of state, $p = (\gamma - 1)\rho$, where $\gamma$ is a constant. For example, $\gamma = 4/3$ for radiation or $\gamma = 1$ for non-relativistic matter. The latter includes oscillating inflaton field or/and the Affleck-Dine (AD) condensate $\phi_{AD}$.

In the spatially flat Robertson-Walker space-time,

$$ds^2 = -dt^2 + a(t)^2 dx^2,$$  

(11)

with the scale factor $a(t)$, the Friedmann equations and the field equation for $\phi$ read

$$\dot{H} = -\frac{1}{2}(\rho + p + \dot{\phi}^2),$$

(12)

$$\ddot{\phi} = -3H\dot{\phi} - \frac{dV(\phi)}{d\phi},$$

(13)

$$H^2 = \frac{1}{3} \left[ \rho + \frac{1}{2} \dot{\phi}^2 + V(\phi) \right],$$

(14)

where $H = \dot{a}/a$ is the Hubble parameter and a dot denotes time differentiation. We define the new variables,

$$x \equiv \frac{\dot{\phi}}{\sqrt{6}H}, \quad y \equiv \frac{\sqrt{V(\phi)}}{\sqrt{3}H},$$

(15)

and the number of e-folds $N \equiv \ln(a)$.

Then, the equations of motion can be rewritten as

$$x' = -3x - \sqrt{\frac{3}{2}} \frac{\partial V}{V} y^2 + \frac{3}{2} x \left[ 2x^2 + \gamma(1 - x^2 - y^2) \right],$$

(16)

$$y' = \sqrt{\frac{3}{2}} \frac{\partial V}{V} xy + \frac{3}{2} y \left[ 2x^2 + \gamma(1 - x^2 - y^2) \right],$$

(17)

$$H' = -\frac{3}{2} H \left[ 2x^2 + \gamma(1 - x^2 - y^2) \right],$$

(18)

where the prime denotes a derivative with respect to $N$. In terms of these variables, the Friedman equation becomes $x^2 + y^2 + \rho/(3H^2) = 1$. We see that $x^2$ and $y^2$ are respectively the ratios of the kinetic and potential energy densities of the dilaton to the total energy density. We consider the case of the universe dominated by the background fluid, so the inequalities, $x^2, y^2 \ll 1$, hold. Then Eq. (18) can easily be solved and the solution is

$$H = H_0 e^{-3\gamma N/2}.$$  

(19)

Next we introduce another new variable,

$$x_s \equiv \frac{d\Re S}{d\phi} x.$$  

(20)

Then Eqs. (16) and (17) can be respectively rewritten as

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\[ x_s' = -3x_s + \frac{3}{2} \gamma x_s + 2\alpha_2 \sqrt{\frac{3}{2}} \left( \frac{d\text{Re}S}{d\phi} \right)^2 y^2, \]  
\[ y' = -2\alpha_2 \sqrt{\frac{3}{2}} x_s y + \frac{3}{2} \gamma y, \]  
where we have used the relation \(-2\alpha_1 V = \partial V / \partial \text{Re}S\). Now we examine the stationary points in the above equations. From \(y' = 0\), we find

\[ x_s = \sqrt{\frac{3}{2} \frac{\gamma}{2\alpha_2}}. \]  
(23)

On the other hand, from \(x_s' = 0\) we see

\[ y^2 = \frac{3(2 - \gamma)\gamma}{2(2\alpha_2)^2} \left( \frac{d\text{Re}S}{d\phi} \right)^{-2}. \]  
(24)

Except for the factor \((d\text{Re}S/d\phi)^{-2}\), Eqs. (23) and (24) represent the scaling solution for the scalar field with exponential potential [13]. In spite of the presence of the factor \((d\text{Re}S/d\phi)^{-2}\), we can verify that the deviation from the scaling solution is small enough, as already shown in [7]. Thus we can write the solution as

\[ \text{Re}S = \frac{3\gamma}{2\alpha_2} N + \frac{1}{\alpha_2} \ln \left[ \frac{4V_0}{(2 - \gamma)\gamma} \left( \frac{2\alpha_2}{3H_0} \right)^2 \right] + \epsilon(N), \]  
(25)

where

\[ \epsilon(N) = \frac{2}{\alpha_2} \ln \left\{ \frac{3\gamma}{2\alpha_2} N + \frac{1}{\alpha_2} \ln \left[ \frac{4V_0}{(2 - \gamma)\gamma} \left( \frac{2\alpha_2}{3H_0} \right)^2 \right] \right\}. \]  
(26)

denotes the deviation from the scaling solution; we find \(x_s'' = \epsilon(N)'' \ll 1\).

Figure 1 depicts time evolution of \(\text{Re}S\) as a function of \(N\) for various initial values of the dilaton field amplitude. As is seen there if the dilaton relaxes to the scaling solution before reaching \(S_{\text{min}}\), its energy is small enough to prevent overshooting. Attractor behavior in a different situation has been studied in [14]. The Hubble parameter at \(S = S_{\text{min}}\) is estimated as

\[ H_{\text{min}} \simeq \frac{2\alpha_2}{3} \sqrt{\frac{2V_0}{(2 - \gamma)\gamma}} e^{-\alpha_2 S_{\text{min}}}, \]  
(27)

from Eqs. (19) and (25). On the other hand, as is seen from Eq. (5), the mass of the dilaton in vacuum is given by

\[ m_\phi \simeq 2\alpha_1 \sqrt{V_0} e^{-\alpha_1 S_{\text{min}}}. \]  
(28)

We therefore find \(m_\phi \simeq H_{\text{min}}\), so the dilaton begins to oscillate immediately when it approaches the potential minimum. Since the mass of the gravitino is given by \(m_{3/2} \simeq \Lambda_2 e^{-\alpha_2 S_{\text{min}}},\) the mass of the dilaton is
\[ m_\phi \simeq \alpha_1^2 m_{3/2} \]
\[ \simeq 10^2 \text{TeV} \left( \frac{m_{3/2}}{1 \text{TeV}} \right) \left( \frac{\alpha_1}{10} \right)^2. \]  

When the dilaton \( S \) approaches the critical point \( S_{\alpha} \), the single exponential approximation (8) breaks down and the scaling behavior terminates. Then the energy density of the dilaton is estimated as
\[ \rho_\phi^{in} = \frac{1}{2} \dot{\phi}^2 + V \bigg|_{H=H_{\text{min}}} = 3H^2 (x^2 + y^2) \bigg|_{H=H_{\text{min}}} \]
\[ = \frac{3}{2} \left( \frac{\gamma}{2\alpha_1} \right)^2 \rho \left( \frac{\Re S}{d\phi} \right)^{-2} \left( 1 + \frac{2 - \gamma}{2\gamma} \right) \bigg|_{H=H_{\text{min}}} \]
\[ \simeq 10^{-3} \rho \left( \frac{10}{\alpha_1} \right)^2 \left( \frac{2}{\Re S} \right)^2 \text{ for } \gamma = 4/3. \] 

Indeed, the numerical calculation gives a close value \( \rho_\phi \simeq 10^{-4} \rho \) at the beginning of the oscillation. After that, the dilaton begins to oscillate and the energy density decreases as \( a(t)^{-3} \) until it decays.

In Fig. 2, we present the energy density of the dilaton in the universe filled by the \( \gamma = 4/3 \) background fluid at the beginning of the oscillation regime for various model parameters, \( \alpha_1 \) and \( \alpha_2 \). The values along the contour lines represent the energy density \( \rho_\phi \) in the unit of \( 10^{-4} \rho \). The case with \( \gamma = 1 \) is depicted in Fig. 3, where we find smaller energy density of the dilaton by a factor of \( \sim 3 \). These figures are drawn in the two-parameter space, though the potential contains four parameters as seen from Eq. (5). The other two have been fixed by setting \( m_{3/2} = 1 \) TeV and \( S_{\text{min}} = 2 \) [5].

The decay of the dilaton produces huge entropy. If we assume that the background fluid is radiation with \( \gamma = 4/3 \), at the time \( t = H_{\text{min}}^{-1} \) its temperature is \( T \sim 10^{11} \) GeV and the entropy density is
\[ s = \frac{4\pi^2}{90} g_* T^3, \]
where \( g_* \sim 10^2 \) is the effective number of relativistic degrees of freedom. By using Eq. (30), we find that the entropy density increases by the factor
\[ \Delta = \frac{T}{T_D} \left( \frac{\rho_\phi}{\rho} \right) \bigg|_{H_{\text{min}}^{-1}} \simeq 10^9 \left( \frac{T}{10^{11} \text{GeV}} \right) \left( \frac{10^{-2} \text{GeV}}{T_D} \right) \left( \frac{\rho_\phi / \rho |_{H_{\text{min}}^{-1}}}{10^{-4}} \right), \] 

when the dilaton decays at
\[ H = \Gamma_D \simeq m_\phi^3 \simeq 10^{-22} \text{GeV} \left( \frac{m_\phi}{10^2 \text{TeV}} \right)^3, \]
\[ T_D \simeq m_{\phi}^{3/2} \simeq 10^{-2} \text{GeV} \left( \frac{m_\phi}{10^2 \text{TeV}} \right)^{3/2}, \]
is the reheating temperature after decay of the dilaton.
IV. AFFLECK-DINE BARYOGENESIS

The Affleck-Dine mechanism is an efficient mechanism of baryogenesis in supersymmetric models [9]. In fact, it is too efficient and the produced baryon asymmetry, \(n_b/s\), is in general too large. However, additional entropy release by the dilaton decay may significantly dilute the baryon asymmetry [10,11] and we examine this possibility here.

A. Original Affleck-Dine mechanism

First, we investigate the originally proposed Affleck-Dine mechanism with a flat potential up to \(\phi_{AD} \sim 1\) [9]. We consider the situation that there are the dilaton and the Affleck-Dine condensate in radiation dominated universe. As mentioned above, at the moment \(H = m_\phi\), the dilaton begins to oscillate with the initial energy density \(\rho_\phi \simeq 10^{-4}\rho_\gamma\), where \(\rho_\gamma\) is the energy density of the background radiation. Then, on the other hand, the AD condensate is expected to take a large expectation value, \(\phi_{AD} \sim 1\), above which its potential blows up exponentially.

The amplitude of the AD field and its energy density remains constant while the Hubble parameter is larger than \(m_{AD}\), where \(m_{AD}\) is the mass of the AD condensate. One should keep in mind, however, that though the energy density, \(\rho_{AD}\), remains constant the baryon number density decreases as \(1/a^3\) if baryonic charge is conserved. So if baryon charge is accumulated in “kinetic” motion of the phase of \(\phi_{AD}\) it would decrease as \(1/a^3\). If, on the other hand, the AD-field is frozen at the slope of not spherically symmetric potential then baryonic charge of the AD-field is not conserved and after \(H < m_{AD}\) both radial and angular degrees of freedom would be “defrosted” and baryonic charge may be large.

As we noted above, when \(H \simeq m_{3/2}(\simeq m_{AD})\), the AD field begins to oscillate. Its energy density at that moment becomes comparable to that of the radiation, while the energy density of the dilaton is estimated as

\[
\rho_\phi|_{H=m_{3/2}} = \left(\frac{m_\phi}{m_{3/2}}\right)^{1/2}\frac{\rho_\phi}{\rho_\gamma}|_{H=m_\phi} \rho_\gamma|_{H=m_{3/2}} \simeq 10^{-3}\rho_\gamma|_{H=m_{3/2}},
\]

for the initial energy density \(\rho_\phi = 10^{-4}\rho_\gamma\) and \(m_\phi \simeq 10^2 m_{AD}\). After that the universe becomes dominated by the oscillating AD condensate and enters into approximately matter dominated regime (see below).

The energy density of the condensate and its baryon number density are given respectively by the expressions:

\[
\rho_{AD} = m_{AD}^2 \phi_{AD}^2, \quad n_b = \kappa m_{AD}\phi_{AD}^2,
\]

where \(\kappa = n_b/n_{AD} < 1\) is a numerical coefficient and \(n_{AD}\) is the number density of the AD field.

The rate of evaporation of the condensate, given by the decay width of the AD field into fermions, \(\Gamma_{AD} = C m_{AD}\) with \(C = 0.1 - 0.01\), is quite large. When the Hubble parameter becomes smaller than \(\Gamma_{AD}\), thermal equilibrium would be established rather soon. However, the condensate would evaporate very slowly and disappear much later [15]. The low evaporation rate is related to a large baryonic charge and relatively small energy density of the
condensate. Below we will find the temperature and the moment of the condensate evaporation repeating the arguments of ref. [15]. Let us assume that the condensate evaporated immediately when \( H = \Gamma_{AD} \) producing plasma of relativistic particles with temperature \( T_{AD} \) and chemical potential \( \mu_{AD} \). The temperature can be estimated as \( T_{AD} \approx \rho_{AD}^{1/4} \) and since \( T_{AD} \gg m_{AD} \) the chemical potential is given by

\[
\mu_{AD} \approx \frac{n_b}{T_{AD}^2} = \kappa \phi_{AD} \gg m_{AD}, \tag{37}
\]

if \( \kappa \) is not very small. On the other hand, chemical potential of bosons cannot exceed their mass. It means that instantaneous evaporation of the condensate is impossible. The process of evaporation proceeds rather slowly with an almost constant temperature of the created relativistic plasma. During the process of evaporation the energy density of the latter was small in comparison with the energy density of the condensate, except for the final stage when the condensate disappeared.

The cosmological baryon number density and energy densities are given by the equilibrium expressions:

\[
\frac{n_{b,\text{tot}}}{T^3} = \frac{2N_f N_c B_q}{6\pi^2} (\xi_q^3 + \pi^2 \xi_q) + \frac{1}{2\pi^2} \int_0^\infty dy y^2 \left[ \frac{1}{\exp(\epsilon - \xi) - 1} - \frac{1}{\exp(\epsilon + \xi) - 1} \right] + B_c \tag{38}
\]

\[
\frac{\rho_{\text{tot}}}{T^4} = \frac{\pi^2 g_\ast}{30} + \frac{72 N_f (N_c + 1) \pi^2}{15} \left[ 1 + \frac{30}{7} \left( \frac{\xi_q}{\pi} \right)^2 + \frac{15}{7} \left( \frac{\xi_q}{\pi} \right)^4 \right] + \frac{1}{2\pi^2} \int_0^\infty dy y^2 \epsilon \left[ \frac{1}{\exp(\epsilon - \xi) - 1} + \frac{1}{\exp(\epsilon + \xi) - 1} \right] + \rho_c \tag{39}
\]

where \( y \equiv p/T \) is dimensionless momenta, \( \epsilon \equiv \sqrt{y^2 + m_{AD}^2}/T \), \( \xi \equiv \mu_{AD}/T \) and \( \xi_q \equiv \mu_q/T \) are dimensionless chemical potential of the AD field and quarks, respectively. \( N_f = 6 \) and \( N_c = 3 \) are the numbers of flavors and colors and factor 2 came from counting spin states, \( B_q = 1/3 \) is the baryonic charge of quarks while the baryonic charge of the AD field is assumed to be 1, and \( B_c \) and \( \rho_c \) are baryon number density and energy density of the condensate normalized to \( T^3 \) and \( T^4 \), respectively. The first term in \( \rho_{\text{tot}} \) includes energy density of light particles with zero charge asymmetry and \( g_\ast \) is the number of their species. The second term includes the contribution from leptons with the same chemical potential as quarks - it is given by \( (N_c + 1) \).

For definiteness, let us assume that the AD field decays into the channel \( \phi_{AD} \to 3q + l \) and taking into account that the sum of baryonic and leptonic charges is conserved\(^1\), so that \( B - L = 0 \), we find

\[
\mu_q = \mu_l = \frac{\mu_{AD}}{4}. \tag{40}
\]

\(^1\)One may wonder if this assumption is inappropriate because the baryon asymmetry created in this channel would be washed out by anomalous electroweak processes [16]. As will be seen later, however, we can avoid this difficulty because in most cases of our interest the AD condensate evaporates at a lower temperature when these anomalous processes are no longer effective.
Before complete evaporation of the condensate, the chemical potential of AD-field remains constant and equal to its maximum allowed value $m_{AD}$. Thus the only unknowns in these expressions are the temperature and the amplitude of the field in the condensate. According to Eq.(36), from Eqs. (38) and (39) we obtain

$$\frac{B_c}{\rho_c} = \kappa \frac{T}{m_{AD}}. \quad (41)$$

The same relation was true for the initial values of $n_{b,\text{tot}}/T^3$ and $\rho_{\text{tot}}/T^4$. Assuming that the ratio $n_{b,\text{tot}}/\rho_{\text{tot}}$ remains the same during almost all process of evaporation, though it is not exactly so, we can exclude $B_c$, $n_{b,\text{tot}}$, $\rho_c$ and $\rho_{\text{tot}}$ from expressions (38) and (39) and find one equation that permits to calculate the plasma temperature in presence of evaporating condensate as a function of the baryonic charge fraction in the initial condensate, $\kappa$. We find

$$m_{AD}/T \simeq 20, \text{ for } \kappa = 1, \quad (42)$$
$$m_{AD}/T \simeq 2, \text{ for } \kappa = 0.1. \quad (43)$$

Exact solution of the problem demands much more complicated study of the evolution of energy density according to the equation $\dot{\rho} = -3H(\rho + P)$, while the evolution of baryonic charge density is determined by the conservation of baryonic charge which is assumed to be true at the stage under consideration and thus $n_{b,\text{tot}} \propto a^{-3}$. The temperature of plasma found in this way would not be much different from the approximate expressions presented above.

Using the above-calculated plasma temperature (43) we find that at the moment of condensate evaporation (when $\mu_{AD} = m_{AD}$) the cosmological energy and baryon number densities of the created relativistic plasma are given by

$$\rho_p \simeq 1000T^4, \quad (44)$$
$$n_b \simeq 50T^3, \quad (45)$$

for $\kappa = 1$, and

$$\rho_p \simeq 70T^4, \quad (46)$$
$$n_b \simeq 1.75T^3, \quad (47)$$

for $\kappa = 0.1$.

We see that a large baryon asymmetry prevents from fast condensate evaporation, though the interaction rate could be much larger than the expansion rate. From Eq. (45) or (47) and the baryon number conservation

$$n_b = \kappa m_{AD} \left( \frac{a_{\text{AD}}}{a(t)} \right)^3 (\phi_{\text{AD}}|_{H=m_{AD}})^2, \quad (48)$$

we find
\[ (\frac{a_{ev}}{a_{AD}})^3 = \kappa \frac{m_{AD}}{n_b} (\phi_{AD}|_{H=m_{AD}})^2 \]  
\[ \simeq 160 \left( \frac{\phi_{AD}|_{H=m_{AD}}}{m_{AD}} \right)^2 \simeq 10^{33}, \text{ for } \kappa = 1, \]  
\[ \simeq 0.46 \left( \frac{\phi_{AD}|_{H=m_{AD}}}{m_{AD}} \right)^2 \simeq 3 \times 10^{30}, \text{ for } \kappa = 0.1, \]

where \(a_{AD}\) and \(a_{ev}\) are the value of the scale factor at the moment \(H = m_{AD}\) and that at the evaporation of the AD field, respectively. Then at the evaporation the Hubble parameter and the baryon-to-entropy ratio are respectively given by

\[ H_{ev} = (\rho_p/3)^{1/2} \simeq \begin{cases} 
2 \times 10^{-14} \text{ GeV for } \kappa = 1, \\
5 \times 10^{-13} \text{ GeV for } \kappa = 0.1 
\end{cases} \]  

\[ \frac{n_b}{s}|_{ev} \simeq 1, \text{ for } \kappa = 1, \]  
\[ \frac{n_b}{s}|_{ev} \simeq 0.04, \text{ for } \kappa = 0.1, \]

as follows from Eqs. (44), (45), (46) and (47).

Now we have to calculate the ratio of the baryon asymmetry to the entropy of the plasma after thermalization of the products of dilaton decay. Initially, at the moment of evaporation of AD-condensate the energy density of the dilaton is roughly \(10^{-3}\) with respect to the energy density of plasma. The latter is dominated by chemical potential \(\mu = m_{AD} > T\). When the universe expanded by the factor \(\rho_{AD}/\rho_{\phi}|_{ev} \equiv a_{eq}/a_{ev} \simeq 10^3\) the dilaton starts to dominate and the relativistic expansion regime turns into matter dominated one at \(a = a_{eq}\). To the moment of the dilaton decay the energy density of the dilaton becomes larger than the energy density of the plasma formed by the evaporation of the AD-condensate by the factor \(a_d/a_{eq} = (H_{eq}/H_d)^{2/3}\), where \(a_d\) and \(H_d\) are the scale factor and the Hubble parameter at the time of the dilaton decay. Keeping in mind that \(H_{eq} = (a_{eq}/a_{ev})^2H_{ev} = 10^{-6}H_{ev}\), we obtain the dilution factor by the dilaton decay

\[ \Delta = \left( \frac{\rho_\phi}{\rho_{AD}} \right)^{3/4}|_{d} = \left( \frac{H_{ev}a_{eq}^2}{H_{d}a_{eq}^2} \right)^{1/2} = \frac{\rho_\phi}{\rho_{AD}}|_{ev} \left( \frac{H_{ev}}{H_{d}} \right)^{1/2} \]  

Thus for \(\kappa = 1\) the dilution factor is only 14, while for \(\kappa = 0.1\) it is 70.

Finally we find the baryon asymmetry after the dilaton decay is given by

\[ \frac{n_b}{s} = \frac{n_b}{s}|_{ev} \frac{1}{\Delta} \simeq 0.1 - 0.001. \]

Thus in this model the dilution of originally produced asymmetry from the decay of AD field is not sufficient.
B. Affleck-Dine mechanism with non-renormalizable potential

As we have seen, additional entropy production due to the dilaton decay is too small to dilute the baryon asymmetry generated in the original Affleck-Dine scenario. Therefore it is necessary to suppress the generated baryon asymmetry. The presence of the non-renormalizable terms can reduce the expectation value of the AD field during inflation. As a result, the magnitude of the baryon asymmetry can be suppressed. Hence we introduce the following non-renormalizable term in the superpotential to lift the Affleck-Dine flat direction as a cure to regulate the baryon asymmetry [17].

\[ W = \frac{\lambda}{n M^{n-3}} \phi^n_{AD}, \]  \hspace{1cm} (57)

where \( M \) is some large mass scale.

The potential for the AD field in the inflaton-dominated stage reads

\[ V(\phi_{AD}) = -c_1 H^2 |\phi_{AD}|^2 + \left( \frac{c_2 \lambda H \phi^n_{AD}}{n M^{n-3}} + \text{H.C.} \right) + |\lambda|^2 \frac{|\phi_{AD}|^{2n-2}}{M^{2n-6}}, \]  \hspace{1cm} (58)

where \( c_1 \) and \( c_2 \) are constants of order unity. The first and the second terms are soft terms which arise from the supersymmetry breaking effect due to the vacuum energy of the inflaton. The minimum of the potential is estimated as

\[ |\phi_{AD}| \simeq \left( \frac{HM^{n-3}}{\lambda} \right)^{1/(n-2)}. \]  \hspace{1cm} (59)

After the end of inflation and till the time when \( H \simeq m_{3/2} \), the AD field traces the instantaneous minimum, (59). Here we consider the inflation models whose reheating temperature is low enough to avoid the overproduction of the gravitinos without additional dilution [18]. Therefore the decay rate of the inflaton is smaller than the gravitino mass, \( i.e. \Gamma_I \lesssim m_{3/2} \).

Note, however, that this constraint may be unnecessary because of additional dilution created by the dilaton decay.

When \( H \sim m_{3/2} \), the low energy supersymmetry breaking terms appear. Then the potential for the AD field becomes

\[ V(\phi_{AD}) = m^2_{AD} |\phi_{AD}|^2 + \left( \frac{Am_{3/2} \phi^n_{AD}}{n M^{n-3}} + \text{H.C.} \right) + |\lambda|^2 \frac{|\phi_{AD}|^{2n-2}}{M^{2n-6}}, \]  \hspace{1cm} (60)

where \( A \) is a constant of order unity, and the ratio of the energy density of the AD field \( \rho_{AD} \) to that of the inflaton \( \rho_I \) is given by

\[ \frac{\rho_{AD}}{\rho_I} \simeq \left( \frac{m_{3/2} M^{n-3}}{\lambda} \right)^{2/(n-2)}, \]  \hspace{1cm} (61)

up to numerical coefficients depending on \( c_1, c_2 \) and \( A \). The typical value is \( \rho_{AD}/\rho_I \simeq 10^{-16}(M/\lambda) \) for \( n = 4 \) and \( \rho_{AD}/\rho_I \simeq 10^{-8}(M^3/\lambda)^{1/2} \) for \( n = 6 \).

As a beginning, let us assume first that the inflaton decayed exactly at \( H = m_{AD} \) and after that the universe was dominated by relativistic matter. Note that the corresponding
reheating temperature is \( T_R \simeq \sqrt{T_I} \simeq \sqrt{m_{AD}} \approx 10^{10} \) GeV. The evaporation of the AD condensate into relativistic plasma would be different from the evaporation into cold plasma considered in the previous subsection. Due to interaction with plasma the products of the evaporation acquire much larger temperature than in the case of the evaporation into vacuum. Since the energy density of the condensate is negligible in comparison with the total energy density of the plasma, the temperature of the latter drops in the usual way, \( T \propto 1/a \), in contrast to the previously considered case when \( T = \text{const} \). Since the temperature of the plasma is high, \( T \gg m_{AD} \), the baryon number density is 

\[
 n_b = B_c T^3 + C_B T^2 \mu 
\]

where \( C_B \sim 1 \) is a constant coefficient, \( \mu \leq m_{AD} \) is the value of the chemical potential, and we have neglected terms of the order of \( \mu^3 \). Since \( n_b \propto a^{-3} \), the ratio of \( a_{ev} \) to \( a_{AD} \) is

\[
 \frac{a_{ev}}{a_{AD}} = \frac{n_b|_{H=m_{AD}}}{m_{AD} T_R^2} \simeq \frac{\rho_{AD} \sqrt{\rho_I}}{\rho_I} \left( \frac{m_{AD}}{H=m_{AD}} \right) ; \tag{62}
\]

(compare to Eq. (49)). Here we took for the initial value of the baryonic charge density \( n_b|_{H=m_{AD}} = \kappa m_{AD} \phi_{AD}^2 \) with \( \kappa \sim 1 \). Hence for \( n = 4 \) the condensate would evaporate instantly, while for \( n = 6 \) evaporation would take place at \( a_{ev}/a_{AD} \simeq 10^8 (M^3/\lambda)^{1/2} \).

To be more precise, however, we must take into account that the interaction rate of the condensate is \( \Gamma_{AD} = (0.1 - 0.01) m_{AD} \) and the evaporation cannot start before \( H = \Gamma_{AD} \). At that moment the plasma temperature would be smaller by the factor \( (m_{AD}/\Gamma_{AD})^2 = 10^2 - 10^4 \) and the baryon number density of the condensate would be smaller by \( (m_{AD}/\Gamma_{AD})^6 \). Correspondingly the red-shift of the end of evaporation should be shifted by a factor \( (m_{AD}/\Gamma_{AD})^2 \) with respect to the beginning of evaporation and it means that it would remain the same with respect to the initial moment \( H = m_{AD} \).

As we have already noted, for \( n = 4 \) the condensate decays quickly and the baryon number density produced in the decay is diluted by the plasma created by the inflaton decay as

\[
 \frac{n_b}{s} \simeq \frac{T_R}{m_{AD} \rho_I} \left( \frac{m_{AD}}{10^{10} \text{GeV}} \right) \left( \frac{1 \text{TeV}}{m_{AD}} \right) \left( \frac{M}{\lambda} \right) . \tag{63}
\]

where \( T_R \) is the reheating temperature of the inflaton and we used the estimate of Eq. (61). The result does not depend upon the moment of the decay of AD-condensate as long as its energy density remains sub-dominant. If an additional dilution by the dilaton is taken into account the baryon asymmetry may be even smaller than the observed one.

The situation is different for \( n = 6 \) when the condensate decays late. At some stage the energy density of non-relativistic condensate would be larger than the energy density of relativistic matter from the decay of the inflaton and the evaporation would proceed in the way described in the previous section with a constant temperature of the products of evaporation, \( T \simeq m_{AD}/20 \) for \( \kappa = 1 \) and \( T \simeq m_{AD}/2 \) for \( \kappa = 0.1 \). An additional contribution of “sterile” energy density of dilaton which changes only the total expansion rate is not essential if the reaction rate \( \Gamma_{AD} \) is larger than the expansion rate, which is typically the case.

If we formally calculate the ratio of the energy density of the radiation which came from the inflaton decay to the energy density of the condensate at the moment of the evaporation of the AD field using (61) and (62), we obtain
and find that the energy density of AD field could be larger than that of the radiation from
the inflaton if \( T_R > \lambda \sqrt{m_{AD}} M^{-3} \). Then the energy density of the decay products of the
inflaton become smaller than that of the AD field before the decay of the condensate and
the character of evaporation would change from evaporation into hot plasma to evaporation
into vacuum. In this case the energy density of plasma at the moment of evaporation is
given by Eq. (44) or Eq. (46) and does not depend upon the previous thermal history.

In this case, the ratio of baryon number density to the photon number density after
condensate decay is given by Eq. (45) or Eq. (47). It remains constant till the decay of
the dilaton. The dilution factor after its decay can be calculated in the same way as in
the previous subsection with the difference that the relative energy of the condensate is
considerably smaller. In this case, the dilution factor (55) becomes

\[
\Delta \simeq 10^8 \left( \frac{T_{AD}}{10^2 \text{GeV}} \right) \left( \frac{10^{-2} \text{GeV}}{T_D} \right) \left( \frac{\rho_\phi/\rho_{AD}}{10^4} \right),
\]

where we used that \( \rho_\phi = 10^{-4} \rho_I \) and \( \rho_{AD} = 10^{-8} \rho_I \) at the reheating time. Repeating similar
calculations, we obtain

\[
\frac{n_b}{s} = \frac{n_b}{s_{\text{ev}}} \Delta \simeq 10^{-11} \left( \frac{1 \text{TeV}}{T_{AD}} \right) \left( \frac{T_D}{10^{-2} \text{GeV}} \right),
\]

where we assumed \( \kappa = 0.1 \) and took \( n_b/s_{\text{ev}} \sim 10^{-2} \). This result is quite close to the observed
value of the asymmetry.

Finally, we study the case with low reheating temperature, \( T_R < \lambda \sqrt{m_{AD}} M^{-3} \), corre-
sponding to much later reheating time, \( t_{\text{rh}} \gg M_{3/2} \). In this case, the energy density of the
AD field never becomes larger than that of the radiation from the inflaton decay. After
inflaton have decayed, the AD field decay. Then the baryon-to-entropy ratio is given by

\[
\frac{n_b}{s} \bigg|_{\text{ev}} \simeq \frac{n_b}{s_{\text{ev}}} \frac{T_R}{n_{AD} \rho_{AD} \rho_I} \bigg|_{t_{\text{rh}}},
\]

After that, the dilaton decays with releasing the huge entropy. The dilution factor due to
the decay is estimated as

\[
\Delta \simeq 10^6 \left( \frac{T_R}{10^8 \text{GeV}} \right) \left( \frac{10^{-2} \text{GeV}}{T_D} \right) \left( \frac{\rho_\phi/\rho_I}{H_{\text{min}}^{-1}} \right). \tag{68}
\]

Therefore, for the \( n = 6 \) Affleck-Dine condensate, we obtain the final baryon asymmetry as

\[
\frac{n_b}{s} = \frac{n_b}{s_{\text{ev}}} \Delta \simeq 10^{-10} \left( \frac{\kappa}{0.1} \right) \left( \frac{M^3}{\lambda} \right)^{1/2} \left( \frac{1 \text{TeV}}{m_{AD}} \right) \left( \frac{T_D}{10^{-2} \text{GeV}} \right), \tag{69}
\]

This expression implies that the final baryon asymmetry is independent of the reheating
temperature of inflation.

In the present model, the supersymmetry breaking is caused by the F-term of the dilaton.
Therefore, when the dilaton decays, it can decay into gravitinos through their mass term
and this process could lead to overproduction of the gravitinos. The constraint derived in
[19] to avoid the overproduction is \( m_\phi \gtrsim 100 \text{ TeV} \). The mass of the dilaton, (29), in the
model considered here is in the allowed region.
V. CONCLUSION

In this paper, we have studied the Affleck-Dine baryogenesis in the framework of the string cosmology. In string models, the dilaton is ubiquitous and does not have any potential perturbatively. We adopted the non-perturbatively induced potential of the dilaton via the gaugino condensation in the hidden gauge sector. Then we set phenomenologically desired values for the gravitino mass and the VEV of the dilaton.

The attractive mechanism to stabilize the dilaton at the desired minimum was proposed by Barreiro et al. [7]. They did not estimate the energy density of the oscillating dilaton. It is estimated in the presented paper where we have found $\rho_\phi \simeq 10^{-4} \rho$ at $H = m_\phi$. This energy transforms into the radiation after the decay of the dilaton before nucleosynthesis because the mass $m_\phi \simeq 10^2$ TeV is sufficiently high.

We have discussed cosmological baryogenesis in this model. In the above-mentioned cosmological history with the entropy production, the Affleck-Dine baryogenesis might be the only workable mechanism for baryogenesis. We have investigated the Affleck-Dine baryogenesis with and without non-renormalizable terms. We have shown that while the original Affleck-Dine scenario produces too much baryon asymmetry even if there is the dilution by the dilaton decay, the model with $n = 6$ non-renormalizable terms can lead to the appropriate baryon asymmetry.

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FIG. 1. Evolution of $\text{Re} \ S$ as a function of $N$ for various initial conditions with $S_{\text{min}} = 2$ in case of $\gamma = 4/3$. We set the initial values of the velocity and the Hubble expansion rate as $\frac{d\text{Re}S}{dt}|_0 = 0$, and $H_0 = 1$, respectively.
FIG. 2. Energy density of the dilaton at the beginning of the oscillations for various model parameters. The number associated with each contour line represents the value of \( \rho_\phi \) normalized by \( 10^{-4} \rho \). Here we adopt \( \gamma = 4/3 \).

FIG. 3. Same as Fig. 2 except for \( \gamma = 1 \).