Figure 2: Schematic representation of the experimental setup for quantum phase retrieval process.

If the quantum phase of an electromagnetic field is unknown, it can be retrieved by measuring the electric field components at different points in space. The retrieved phase provides information about the spatial distribution and temporal evolution of the field. This technique has applications in areas such as optical metrology, communication systems, and quantum computing.

beyond the scope of current technology. Very recently, an all-optical approach to the ultrafast manipulation of electron spin has been demonstrated by making use of the optical ac Stark effect that produces ultra-short effective magnetic field pulses.\cite{[2]}

In what follows, we first elaborate the principle of geometrically manipulating the electron spin and using FR to detect the geometric phase, then discuss the possible experimental implementation by an all-optical approach.

For the sake of generality, assume a mixed initial state of the carrier spins prepared by the pump laser in the conduction band (the hole spins in the valence band can be ignored because of their much shorter relaxation time)

\begin{equation}
\rho_0 = w_0 |0\rangle \langle 0 | + w_1 |1\rangle \langle 1 |.
\end{equation}

Here, $|0\rangle$ and $|1\rangle$ denote, respectively, the spin-down and spin-up states of the excited electron, with respect to the pump laser direction along the $z$-axis (see Fig. 1). Correspondingly, we define the $x$-axis in the plane determined by the pump and probe laser propagating directions, and the $y$-axis perpendicular to this plane. The time evolution of density operator is governed by the Liouville equation, that results in a formal solution

\begin{equation}
\rho(t) = U(t) \rho U(t)\dagger,
\end{equation}

where the operator $U(t)$ describes the quantum evolution starting from the initial (mixed) state $\rho_0$.

To evolve the electron spin state, assume a magnetic field applied in the $x$-$z$ plane with, correspondingly, two projective components $B_x$ and $B_z$. For clarity, we first consider the non-adiabatic geometric evolution of the eigenstates of $\sigma_y$, $\sigma_y |\pm\rangle = \pm |\pm\rangle$, where $\sigma_y = \sigma \cdot e_y$ is the Pauli spin operator directed along the $e_y$ direction ($y$-axis). Specifically, the quantum cyclic evolution is accomplished by the following magnetic field pulses:

\begin{enumerate}[i]
\item Switching on a $\pi$-pulse of magnetic field with components $B_x$ and $B_z$, the state $|+\rangle$ rotates around the magnetic field, from $|+\rangle$ in the $e_y$ direction to $|\rangle$ in the $-e_y$ direction along curve $ABC$ on the Bloch sphere, see Fig. 2. (ii) Suddenly changing the magnetic field to another direction with components $B_x$ and $-B_z$, subjecting to another $\pi$-pulse action the state $|\rangle$ rotates back to $|+\rangle$ along curve $BCDA$ on the Bloch sphere. From the AA phase theory,\cite{[4]} after the above cyclic evolution, the state $|+\rangle$ will acquire a geometric AA phase $e^{\gamma}$, with $\gamma = 4\arctan(B_x/B_z)$. Note that during the above operation, the state vector is always being perpendicular to the magnetic field, thus no dynamic phase is accumulated in the evolution. This idea was first promoted by Suter \emph{et al} in their seminal experiment of demonstrating the AA phase.\cite{[9]} Similarly, the state $|\rangle$ will acquire an AA phase $e^{-\gamma}$ at the same time.

In analogy with the Mach-Zender interferometer where the photon wavefunction is splitted at the first beamsplitter, we consider the consequence of the above rotation on the eigenstates of $\sigma_z$: $|0\rangle = \frac{1}{\sqrt{2}}(|+\rangle - |\rangle)$, and $|1\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |\rangle)$. Straightforwardly, the quantum state evolution is described by

\begin{equation}
\begin{align*}
|0\rangle & \rightarrow \cos \gamma |0\rangle + \sin \gamma |1\rangle, \\
|1\rangle & \rightarrow \cos \gamma |1\rangle - \sin \gamma |0\rangle.
\end{align*}
\end{equation}

We see here that the geometric AA phase plays a role of rotating $|0\rangle$ and $|1\rangle$ into their superposition states. As a consequence, geometric rotation of the electron spin is realized. As emphasized in the geometric quantum computation,\cite{[12]} the major advantage of geometric manipulation is its ability to be fault-tolerant to certain types of errors. Note also that the possible value of $\gamma$ ranges from 0 to 2\pi by controlling the magnetic field components $B_x$ and $B_z$, implying an arbitrary geometric rotation of the spin state.

Below we show that the geometric AA phase can be detected by FR. In the representation of basis states $\{ |1\rangle, |0\rangle \}$, the evolution described by Eq. (3) can be equivalently reexpressed in terms of the evolution operator (matrix) as

\begin{equation}
U(T) = \begin{pmatrix}
\cos \gamma & \sin \gamma \\
-\sin \gamma & \cos \gamma
\end{pmatrix},
\end{equation}

where $T$ is the entire geometric rotation time. Substituting Eq. (4) into (2), the final state of the conduction electron spins after the above geometric operation reads

\begin{equation}
\rho_f = U(T) \rho_0 U(T)\dagger = \begin{pmatrix}
w_1 \cos^2 \gamma + w_0 \sin^2 \gamma & (w_0 - w_1) \cos \gamma \sin \gamma \\
(w_0 - w_1) \cos \gamma \sin \gamma & w_1 \sin^2 \gamma + w_0 \cos^2 \gamma
\end{pmatrix}.
\end{equation}

With this result, any physical quantities can be carried out by statistically averaging the corresponding variables over the system density matrix. In the context of FR experiment, the relevant quantity is the projection of sample magnetization in the propagating direction ($e_k$) of

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig2.png}
\caption{Schematic diagram for geometric rotation of state vector around the magnetic field. Since the state is always perpendicular to the field, there is no dynamical phase accumulation during the evolution.}
\end{figure}
the probe light. The corresponding variable operator is 
\[ \sigma_k = \sigma \times e_k = \cos \alpha \sigma_x - \sin \alpha \sigma_y, \]
where \( \alpha \) is the angle between the pump- and probe-laser propagating directions. Straightforwardly, the component along \( e_k \) of the sample magnetization is calculated as
\[ M_k = \text{Tr}(\rho_k \rho) = (w_1 - w_2) \cos(\alpha - 2\gamma). \] (6)

Two valuable observations on Eq. (6) are in order: (i) The geometric phase \( \gamma \) has clear observable effect in \( M_k \), thus in FR experiment. The dependence of FR angle \( \theta_F \propto M_k \) on the geometric phase \( \gamma \) is in terms of the typical behavior of quantum interferences. In the limit of \( \gamma = 0 \), Eq. (6) reduces to the result of FR experiment that probes the initially excited carrier spins in the semiconductor sample. (ii) The output intensity is an incoherently weighted average of the pure state interference profiles. It is easy to check that for pure state [1] and [0], the interference profiles are, respectively, \( \pm \cos(\alpha - 2\gamma) \). The interference patterns in the initial state \( \rho \). This structure is consistent with the recent work on geometric phases for mixed states in interferometry. [15]

The interferometry described above is also applicable to detect the adiabatic Berry phase. [3] provided the magnetic field can adiabatically complete a closed path in parameter space within the spin relaxation time. This condition can be satisfied in some spin systems, such as the liquid NMR or doped spins in solid-states. Particularly, if the initial state is an eigenstate of the system Hamiltonian, the adiabaticity will ensure that the state in the subsequent time will remain in the same eigenstate of the instantaneous Hamiltonian \( H(B(t)) \) (\( B \) is the time-dependent magnetic field). As \( B \) traces a closed loop in the parameter space, a geometric phase is acquired. For the geometric-phase-based interferometry study, the initial state [0] (or [1]) is a linear superposition of the eigenstates \( \{ \pm \} \) of \( H(B) \), if the magnetic field is initially along the \( y \)-direction. As a result of adiabatically dragging the Hamiltonian along a closed loop, a phase difference between \( \{ + \} \) and \( \{ - \} \) is caused. Noticeably, the phases acquired in this way both have the geometric and dynamic contributions. To detect merely the geometric Berry phase via an interferometry, it is necessary to eliminate the dynamic phase. An applicable approach is to use the refocusing technique known as spin-echo recently developed in NMR experiment. [16] which was also employed recently to eliminate the dynamic phases in geometric quantum computation. [15] The basic idea is to apply the cyclic evolution twice, with the second cyclic evolution retracing the first one but following the reversed path, and with the second application surrounded by a pair of fast \( \pi \)-transformations that swap the states \( \{ + \} \) and \( \{ - \} \).

Now let us return to the possible experimental implementation of the ultrafast geometric manipulation of electron spin and the detection of the non-adiabatic AA phase, which is the major concern of the present study. As we have mentioned earlier, a necessary condition for realizing the geometric manipulation is the cyclic evolution being much faster than the spin relaxation. Viewing that the conduction-electron spin relaxation time is about tens of nanoseconds, we thus need ultra-short magnetic field pulses. We show below how this goal can be achieved by virtue of the recent experiment on the ultrafast manipulation of electron spin by a novel all-optical approach. [2]

The basic idea is to make use of an off-resonance ultrafast laser pulse to induce ac Stark shifts, which are in turn equivalent to the result of an effective magnetic field. To illustrate this, consider the energy level diagram of a semiconductor quantum well shown in Fig. 3(a).

In the absence of magnetic field, the lowest conduction-band (CB) level is two-fold degenerate, denoted by spin states \( \{ \pm 1/2 \} \); and the valence-band (VB) states are denoted, respectively, by \( \{ \pm 3/2 \} \) and \( \{ \pm 1/2 \} \). Switching on a below-bandgap laser pulse (tipping pulse) with, for example, \( \sigma^+ \)-polarization, the laser will virtually couple the state pairs between VB state \( \{ - 3/2 \} \) and CB state \( \{ - 1/2 \} \), as well as CB state \( \{ + 1/2 \} \) and VB state \( \{ - 1/2 \} \). Other possible couplings between laser and electron-hole pairs are forbidden from the standpoint of angular momentum conservation. It is well known that this kind of off-resonance coupling will cause the ac Stark shifts of the relevant energy levels. In particular, the energy shifts can be estimated from the second-order perturbation theory as follows: \( \Delta E_{\pm 1/2}^\pm = |V_{-3/2,-1/2}|^2/\Delta_1 \) for CB state \( \{ - 1/2 \} \), \( \Delta E_{\pm 3/2}^\pm = |V_{-3/2,-3/2}|^2/\Delta_1 \) for VB state \( \{ - 3/2 \} \), \( \Delta E_{\pm 1/2}^\pm = |V_{-1/2,1/2}|^2/\Delta_2 \) for CB state \( \{ + 1/2 \} \), and \( \Delta E_{\pm 3/2}^\pm = |V_{-1/2,3/2}|^2/(-\Delta_2) \) for VB state \( \{ - 1/2 \} \). Here, \( \Delta_1 \) and \( \Delta_2 \) are the detunings of the photon energy with the two pairs states: \( V_{-3/2,-1/2} \) and \( V_{-1/2,1/2} \) are coupling matrix elements of laser with the state pairs. Pivoted in Fig. 3(b) are the shifted levels, with respect to the original unperturbed ones in Fig. 3(a). We notice

| + ⟩ ⟼ | + ⟩
| - ⟩ ⟼ | - ⟩

\( \sigma_+ \)

FIG. 3: ac Stark shift induced by an off-resonance below-bandgap laser pulse. Depicted are, respectively, the original unperturbed energy levels (a) and the shifted ones (b).
here that the level repulsive effect is obvious.

As experimental studies have indicated that the hole spins are either pinned along the quantum well growth direction or dephase rapidly [1, 2] the FR dominantly measures the net effect of the CB electron spins. As a consequence, the ac Stark shifts of CB states $| - 1/2 \rangle$ and $| + 1/2 \rangle$ can be equivalently described as the effect of an effective magnetic field $B_{\text{eff}}$ along the tipping-pulse direction. Denoting the CB level splitting by $\delta_{\text{cb}}$ the effective magnetic field $B_{\text{eff}} = \delta_{\text{cb}}/|g_{c}|\mu_{B}$, where $g_{c}$ is the Lande-$g$ factor, and $\mu_{B}$ is the Bohr magneton. As a rough estimate, corresponding to a CB level splitting of $\delta_{\text{cb}} \approx 1 \text{ meV}$, the effective magnetic field $B_{\text{eff}}$ can be as high as $\approx 20 \text{ T}$. In addition, the persistence time of this effective magnetic field is identical to the laser-pulse duration time, that can be as short as femtoseconds. Therefore, the resulting effective magnetic field can suffice the geometric operation (rotation) of the electron spin under study. To obtain the AA phase resulting interference pattern, the FR experiment can be arranged as follows: (i) After optical excitation using an ultrafast circularly polarized laser pulse, the initial spin state is prepared along the $z$-direction shown in Fig. 1. In the absence of magnetic field, the FR angle of the linearly polarized probe laser keeps unchanged. (ii) Switch on two tipping pulses in succession such that the effective magnetic field induced by the ac Stark effect rotates the electron spin in the manner as shown in Fig. 2. Note that the direction of the effective magnetic field is given by the tipping pulse propagating direction, and its magnitude depends on the detunings and the coupling strengths between the tipping laser and electron-hole pairs. After the action of the tipping pulses, the FR angle will gain a change as show by Eq. (6). (iii) Repeat procedures (i) and (ii) by making changes of the tipping pulse directions. A series of values of the AA phase $\gamma$ can be measured, and accordingly the interference pattern resulting from the geometric AA phase can be obtained. 

In summary, we have elaborated an experimentally accessible scheme to geometrically manipulate electron spin, and to detect the non-adiabatic geometric phase via FR spectroscopy. This study might be of fundamental interest and relevant to the notion of spintronics. Viewing the particular significance of ultrafast manipulation of electron spin, the proposed study can further confirm the novel approach developed in ref. 2. Moreover, the proposed geometric scheme may further stabilize the quantum operation, i.e., being of fault-tolerance to some operational errors. Finally, the proposed interference scheme based on the FR setup can also be applied to detect geometric phases in other spin systems, including the adiabatic Berry phase as long as the spin relaxation time is longer than the experimentally accessible magnetic field duration time.

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