The stability of strange quark matter is studied within the Nambu Jona-Lasinio model with three different parameter sets. The model Lagrangian contains 4-fermion (with and without vector interaction) and 6-fermion terms; the minimum energy per baryon number as a function of the strangeness fraction of the system is compared to the masses of hyperons having the same strangeness fraction, and coherently calculated in the same version of the model, and for the same parameter set. The results show that in none of the different parameter sets strangelets are stable, and in some cases a minimum in the energy per baryon does not even exist.

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Strangelet detection in heavy ion collisions has been proposed long ago [1–5] as a signature of Quark Gluon Plasma (QGP) formation: it has been suggested that rather cold droplets of stable or metastable strange-quark matter may be distilled in heavy-ion collisions during the phase transition from a baryon-rich QGP to hadron matter: the prompt anti-kaon and pion emission from the surface of the fireball could rapidly cool the QGP, thus favouring the condensation into metastable or stable droplets of strange quark matter [4, 5].

The investigation of strangelet stability is therefore rather crucial: it is fundamental to find out whether this system is more or less stable than hyperons, in order to understand which state is more likely to be produced in heavy ion collisions, either hyperons and strange mesons or strangelets. The properties and stability of strangelets have been discussed within the MIT bag model, a confined gas stabilized by the vacuum pressure $B$ [4,6–9]; more recent calculations, employing different quark models [10–12], pointed out that strangelet stability is strongly model dependent.

In this paper we want to discuss the properties of strange quark matter within the NJL model: we consider homogeneous quark matter made up of $u$, $d$ and $s$ quarks; no $\beta$-equilibrium is required: in the system there is a strangeness fraction $R_s = \rho_s/\rho$, $\rho$ being the total baryon density of quarks and $\rho_s$ the baryonic density of strange quarks. Furthermore, electromagnetic interaction has been neglected, so that the minimum of the energy corresponds to an equal number of $u$ and $d$ quarks. We therefore consider the curves corresponding to the minimum energy per baryon number as a function of $R_s$ and compare these curves to the hyperon masses evaluated in the same version of the NJL model and for the same parameter values, in order to understand whether strange matter is more stable than hyperons or viceversa.

Many versions of the NJL model have been used in the past; in our calculation we use a three flavour Lagrangian of the form [13–16]:

$$\mathcal{L}_{NJL} = \mathcal{L}_0 + \mathcal{L}_m + \mathcal{L}_{(4)} + \mathcal{L}_{(6)}$$

where:

$$\mathcal{L}_0 = i \bar{\psi} \gamma^\mu \partial_\mu \psi$$,                                    \hfill (2)

$$\mathcal{L}_m = - \bar{\psi} \hat{\mathbf{m}} \psi$$,                                    \hfill (3)

$$\mathcal{L}_{(4)} = \frac{G}{2} \sum_{k=0}^{8} \left[ (\bar{\psi}\lambda_k \psi)^2 + (i\bar{\psi}\gamma_5\lambda_k \psi)^2 \right] +$$

$$\frac{G_V}{2} \sum_{k=0}^{8} \left[ (i\bar{\psi}\gamma_\mu(\lambda_k \psi)^2 + (i\bar{\psi}\gamma_\mu\gamma_5(\lambda_k \psi)^2 \right],$$

$$\mathcal{L}_{(6)} = -K \left[ \det_{i,j} (\bar{\psi}_i(1 + \gamma_5)\psi_j) + \det_{i,j} (\bar{\psi}_i(1 - \gamma_5)\psi_j) \right].$$

2
In the above $\psi \equiv \begin{pmatrix} u \\ d \\ s \end{pmatrix}$ is the quark field, $\hat{m} \equiv \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$ is the mass matrix, $\lambda_1 \ldots \lambda_8$ are the Gell–Mann flavour matrices, and $\lambda_0 \equiv \sqrt{\frac{2}{3}} I$.

$L(4)$ generates four–leg interaction vertices, while $L(6)$ gives rise to six–leg, flavour-mixing, interaction vertexes; $G$ and $G_V$ are parameters of the model with the dimensions of $[L^2]$ and $K$ is a parameter with the dimensions of $[L^5]$. In the following we will consider both zero and nonzero values for $G_V$, in order to see the effects of a repulsive vector interaction on the stability of strangelets. In the limit $m_i = 0$, $\forall i$, the symmetries of the model Lagrangian are the following ones:

$$U_V(1) \times SU_V(3) \times SU_A(3) \times SU_c(3)$$

where, of course, $SU_c(3)$ is global; $U_A(1)$ is broken by the existence of the axial anomaly.

In the mean field approximation it is possible to evaluate the dynamical quark masses, generated by their interaction with the vacuum, which are given by the following gap equation:

$$m_i^* = m_i - 2G \langle \bar{\psi}_i \psi_i \rangle + 2K \langle \bar{\psi}_j \psi_j \rangle \langle \bar{\psi}_k \psi_k \rangle \quad (i \neq j \neq k)$$

where

$$\langle \bar{\psi}_i \psi_i \rangle = -\frac{3}{\pi^2} \int_0^\Lambda \frac{dp}{p} \frac{m_i^* p^2}{\sqrt{p^2 + (m_i^*)^2}}$$

is the chiral condensate for the $i$-flavour. Since the NJL model is not renormalizable, a three–dimensional regularization with an ultraviolet cut–off $\Lambda$ is introduced.

The energy density of the system in mean field approximation turns out to be:

$$\varepsilon = - \sum_{i=u,d,s} \frac{3}{\pi^2} \int_0^\Lambda \frac{dp}{p} \sqrt{p^2 + (m_i^*)^2} + \left[ \sum_{i=u,d,s} G \langle \bar{\psi}_i \psi_i \rangle^2 \right] +$$

$$\sum_{i=u,d,s} G_V \rho_{V_i}^2 - 4K \langle \bar{uu} \rangle \langle \bar{dd} \rangle \langle \bar{ss} \rangle + \varepsilon_0.$$ 

In the above formula, the constant $\varepsilon_0$ is introduced in order to set the vacuum energy density equal to zero, and $\rho_{V_i}$ is the time component of the vector current.

The dependence of the above formula on $R_s$ and $\rho$ can be easily found by recalling that:

$$\rho_s = R_s \rho$$

$$\rho_{V_u} = \rho_{V_d} = \frac{3(1 - R_s)}{2} \rho$$

$$\rho_{V_s} = 3R_s \rho$$
and:

\[ k_{F_s} = \left( \frac{3\pi^2 \rho_s}{2} \right)^{1/3} \]
\[ k_{F_{u,d}} = \left( \frac{3\pi^2}{2} \rho \left( 1 - R_s \right) \right)^{1/3} \]

\( \rho \) being the total baryon number density in the system \((\rho = N/V)\). In the above the colour degeneracy and baryon number 1/3 of the quarks have been taken into account\(^1\).

From the above formulas, the energy per baryon number turns out to be:

\[ \frac{E_{tot}}{N} = \frac{\epsilon_{tot}}{\rho} \]

In order to investigate the influence of the parameter values on strangelet stability, in this work three different parameter sets will be used:

<table>
<thead>
<tr>
<th>set 1</th>
<th>set 2</th>
<th>set 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_\equiv m_u = m_d = 5.5 \text{ MeV} )</td>
<td>( m_\equiv m_u = m_d = 5.5 \text{ MeV} )</td>
<td>( m_\equiv m_u = m_d = 3.6 \text{ MeV} )</td>
</tr>
<tr>
<td>( m_s = 140.7 \text{ MeV} )</td>
<td>( m_s = 135.7 \text{ MeV} )</td>
<td>( m_s = 87 \text{ MeV} )</td>
</tr>
<tr>
<td>( \Lambda = 602.3 \text{ MeV} )</td>
<td>( \Lambda = 631.4 \text{ MeV} )</td>
<td>( \Lambda = 750 \text{ MeV} )</td>
</tr>
<tr>
<td>( GA^2 = 3.67 )</td>
<td>( GA^2 = 3.67 )</td>
<td>( GA^2 = 3.67 )</td>
</tr>
<tr>
<td>( K\Lambda^5 = 12.36 )</td>
<td>( K\Lambda^5 = 9.29 )</td>
<td>( K\Lambda^5 = 8.54 )</td>
</tr>
</tbody>
</table>

These parameters have been employed in Refs. [16] (set 1), [15] (set 2) and [17] (set 3); in the first two cases, the current masses for the \( u \) and \( d \) quarks are fixed on the basis of isospin symmetry and of limits on the average \( \bar{m} = (m_u + m_d)/2 \) at 1 GeV scale, while the remaining four parameters are fitted to reproduce the masses of \( \pi \), \( K \) and \( \eta' \) mesons, together with the pion–decay constant \( f_\pi \); the third parameter set also reproduces fairly well these experimental values.

We choose for \( G_V \) the two following values, in order to discuss the relevance of the repulsive vector interaction in strangelet formation:

\[ G_V = 0 \quad \quad G_V = 0.5G \]

the second value being motivated, for example, in Ref. [13].

With the above parameter values, the effective quark masses in the vacuum and the chiral condensates turn out to be:

\(^1\)Colour degeneracy is taken into account, even if no explicit gluon field is present in the model Lagrangian.
In Ref. [11] a comparison is made between the minimum energy per baryon with respect to $\rho$ at fixed $R_s$, and the hyperon masses coherently calculated in the same model and for the same parameter values, in order to understand which state is more stable. In the same spirit of this work, we compare our curves to the theoretical hyperon masses evaluated in the NJL model and for the same parameter values: for the first two parameter sets and $G_V = 0$ we used the hyperon masses evaluated in Ref. [18]; following the same techniques presented in Ref. [18], we also calculated the hyperon masses for parameter set 3 and for $G_V = 0.5G$: they are all shown in Table 1. In Fig. 1 we present our results for the three different parameter sets: the continuous lines correspond to $G_V = 0.5G$, the dashed lines to $G_V = 0$; concerning the hyperon masses, empty triangles correspond to $G_V = 0$ and empty squares to $G_V = 0.5G$; full circles are instead the experimental hyperon masses. As it is evident from this figure, for all three parameter sets, with or without vector interactions, the curves corresponding to strangelets turn out to be well above the hyperon masses having the same strangeness fraction: independently of the parameter set used, and of the presence of the vector interaction, strangelets are, therefore, not stable in the NJL model. From the left panel we can see that with the parameter set one and a nonzero value for $G_V$, a minimum in the energy per baryon is present only up to $R_s = 0.7$, as already pointed out in Ref. [10]. In the central panel the results corresponding to parameter set 2 are shown, from which it is evident that the vector interaction further reduces the range of values of $R_s$ corresponding to a minimum in the energy per baryon: in this case in fact a minimum is present only for $0.16 \leq R_s \leq 0.56$. Within parameter set 3 this range is even more reduced, as it appears from the third panel: in the case of $G_V \neq 0$ a minimum is present only for $0.21 \leq R_s \leq 0.45$. With this parameter set, even for $G_V = 0$, the range of $R_s$ which is compatible with the existence of a minimum is limited to: $R_s \leq 0.79$. In order to get a feeling of the occurrence of these minima, we show the curves corresponding to the energy per baryon as a function of $\rho$ for different values of $R_s$ in Fig 2, for the three parameter sets and with $G_V = 0.5G$: full circles indicate local minima.

Our analysis shows that the existence of stable or metastable strangelets is not supported by the NJL model: the curves corresponding to the minimum energy per baryon as a function of the strangeness fraction are always higher than the corresponding hyperon masses coherently calculated in the model for the same parameter values, and in some cases the minimum does not exist; these results seem therefore to indicate that hyperons are more likely to be produced in heavy ion collisions, since they are more stable. This fact confirms the model dependence of the strangelet
stability, which could set serious challenges to the search for these objects in heavy ion collisions.

References

Figure 1: Minimum energy per baryon number as a function of $R_s$ for the three different parameter sets, for $G_V = 0$ (dashed lines) and $G_V = 0.5G$ (continuous lines). Full circles are the experimental hyperon masses, the other dots are the theoretical hyperon masses corresponding to $G_V = 0$ (empty triangles) and $G_V = 0.5G$ (empty squares) respectively.

<table>
<thead>
<tr>
<th>Baryon</th>
<th>$N$</th>
<th>$\Lambda$</th>
<th>$\Xi^0$</th>
<th>$\Omega^-$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{exp}$ (MeV)</td>
<td>938.27</td>
<td>1115.68</td>
<td>1314.9</td>
<td>1672.45</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m_{set 2}$ (MeV)</td>
<td>970.86</td>
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<td>1274.51</td>
<td>1493.10</td>
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<td>$m_{set 3}$ (MeV)</td>
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<td>1067.12</td>
<td>1261.68</td>
<td>1486.26</td>
</tr>
<tr>
<td>$m_{set 1}$ (MeV)</td>
<td>$G_V = 0.5G$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m_{set 2}$ (MeV)</td>
<td>996.77</td>
<td>1101.34</td>
<td>1271.78</td>
<td>1471.33</td>
</tr>
<tr>
<td>$m_{set 3}$ (MeV)</td>
<td>965.78</td>
<td>1128.53</td>
<td>1314.8</td>
<td>1501.91</td>
</tr>
</tbody>
</table>

Table 1: Experimental masses and theoretical masses of hyperons calculated in the NJL model.
Figure 2: Energy per baryon number as a function of $\rho$ for different values of $R_s$: the three panels correspond to the three different parameter sets and $G_V = 0.5G$. Full circles indicate local minima. The dashed lines in the second and third panel indicate the first and last values of $R_s$ corresponding to a minimum: $0.16 \leq R_s \leq 0.56$, and $0.21 \leq R_s \leq 0.45$, respectively.