LANDAU–ZENER EFFECT
IN SUPERFLUID NUCLEAR SYSTEMS

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The Landau–Zener effect is generalized for many-body systems with pairing residual interactions. The microscopic equations of motion are obtained and the $^{14}$C decay of $^{223}$Ra spectroscopic factors are deduced. An asymmetric nuclear shape parametrization given by two intersected spheres is used. The single particle level scheme is determined in the frame of the superasymmetric two-center shell. The deformation energy is computed in the microscopic-macroscopic approximation. The penetrabilities are obtained within the WKB approximation. The fine structure of the cluster decay analyzed in the frame of this formalism gives a very good agreement with the experimental ratio of partial half-lives obtained in special conditions.

1. Introduction

Recently, a new method was proposed to approach the fine structure phenomenon of cluster- and alpha-decays in odd-nuclei. These decays modes are considered as superasymmetric fission processes. Assuming the existence of few collective variables, associated to some generalized coordinates describing the nuclear shape parametrization, this approach allows to handle approximately the behavior of many other intrinsic variables. At any values of the shape generalized coordinates, the single particle level scheme and the potential barrier are determined. During the whole process and after the disintegration, the single particle occupation probabilities of the orbitals of interest are computed using the Landau-Zener promotion mechanism alone. This method was successfully used to describe the $^{14}$C- and alpha-decays $^{1,2,3}$. Despite the overlook of residual interactions, the agreement obtained with data gives a strong experimental support to the formalism. Unfortunately, the approach based on the Landau-Zener effect alone is not able to describe the fine structure in the case of the disintegration of even-even nuclei. That motivated us to develop the model in order to actually obtain the final occupation probabilities of the nucleons in different orbitals using the equation of motion derived for pairing residual interactions including the Landau-Zener effect.

The decaying system provides a time dependent single-particle potential in which the nucleons move independently. This description is consistent within the essence of

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the Hartree-Fock approximation where the many body wave function is constrained to be Slater determinants and with the Hartree-Fock-Bogoliubov (HFB) problem where the function is constrained to be of BCS form all time. In the HFB model, the level slippage is treated automatically due to the simultaneous presence of many Slater determinants in the wave function. Such calculations are very cumbersome. A way to bypass this problem is to use the superfluid model constructed over a single particle potential and introducing the Landau-Zener effect to replace the effect given by a part of the residual interactions.

In the next section, the equations describing the microscopic dynamics of a many nucleon system with pairing residual interaction, including the Landau-Zener effect applied to the unpaired nucleon, are deduced. This effect allows the transition of an unpaired nucleon between levels with the same quantum numbers associated to some symmetries of the system. Solving the equations, the response of the system is estimated when the nuclear shape parametrization is changed. In the following, a nuclear shape parametrization given only by two intersected spheres, the radius of the sphere associated to the light fragment kept as constant, will be used. The single degree of freedom remains the elongation characterized by $R$, the distance between the centers of the nascent fragments. Sometimes a normalized elongation $(R_r - R_i)/(R_f - R_i)$ will be used for convenience. The parameter $R_1$ being the radius of the daughter determined from the condition of volume conservation and $R_2 = 41 A_2^{1/3}$ being the radius of the light fragment kept as constant, the next values are determined $R_i = R_1 - R_2$ and $R_f = R_1 + R_2$ for the configuration of the initial nucleus considered as a sphere and for that of two tangent spheres, respectively.

2. Formalism

In order to obtain the equations of motion, we shall start from the variational principle taking the Lagrangian as

$$\mathcal{L} = \langle \phi | H - i\hbar \frac{\partial}{\partial t} + H' - \lambda N | \phi \rangle$$

(1)

and assuming the many-body state formally expanded as a superposition of time dependent BCS seniority one diabatic wave functions

$$| \phi > = \sum_i c_i(t)a_i^+ \prod_{j \neq i} (u_j(t) + v_j(t)a_j^+ a_j^-) | 0 >$$

(2)

Each of these seniority one diabatic wave functions being associated to a state with an unpaired nucleon in the orbital $i$, the orbitals $j \neq i$ being pairwise filled. Furthermore

$$H(t) = \sum_{k > 0} \epsilon_k(t)(a_k^+ a_k + a_k^+ a_k) - G \sum_{k,l > 0} a_k^+ a_k^+ a_l a_l$$

(3)

is the time dependent many-body Hamiltonian with pairing residual interactions,

$$H' = \frac{i}{2} \sum_{i,j} h_{ij}(t) [(u_i a_i - v_i a_i^+)(u_j a_j + v_j a_j^+) + (u_j a_j - v_j a_j^+)(u_i a_i + v_i a_i)]$$

(4)
is the residual interaction between diabatic levels characterized by the same quantum numbers associated to some symmetries of the system. This interaction is responsible for the existence of avoiding level crossing levels in the case of adiabatic states. The presence of this interaction term $H'$ allows the level slippage of an unpaired nucleon and allows the possibility that an unpaired nucleon (or hole) located on the diabatic level $i$ jumps on the level $j$ due to the interaction $h_{ij}(t)$, in the same manner as in the classical two level Landau–Zener effect \(^{4,5}\). The element $h_{ij}(t)$ is different from zero only in the regions of the avoided crossings. The term $-\lambda N$ represents the well known constraint on the total number of particles. In this picture, $a_i^+$ and $a_i$ denote operators for creating and destroying, respectively, a particle in the diabatic state $i$ (not, as usual, in the adiabatic state $i$) and the label $i$ denotes the time-reversed orbital conjugate to, and degenerate with, $i$. Furthermore, $\epsilon_k$ are the single particle energies obtained within the shell model. The blocking correlations (the variation in $\Delta$ and in the $u_j$'s and $v_j$'s due to the change of the blocking level\(^6\)) are neglected.

By performing the variation of the Lagrangian in a way similar as in Ref \(^7\), the next system of coupled differential equations are obtained.

\[
\begin{align*}
\hbar \dot{p}_m & = \sum_m \{ \kappa_i \Delta^*_m - \kappa^*_i \Delta_m \} \\
\hbar \dot{\kappa}_i & = \sum_m \{ (2\rho_i - 1) \Delta_m - 2\kappa_i (\epsilon_i(t) - \lambda(t)) \} \\
\hbar \dot{\rho}_i & = \frac{1}{2} \sum_j h_{mj}(t) (S_{mj} - S_{jm}) \\
\hbar \dot{S}_{jm} & = S_{jm} \left\{ -\frac{1}{2} (| \Delta_m |^2 - | \Delta_j |^2) + (\epsilon_m(t) - \epsilon_j(t)) - \frac{1}{2} \left[ -\frac{\rho_m}{\kappa_m} + 2\kappa^*_m + \frac{\rho_j}{\kappa_j} - 2\kappa^*_j \right] \sum_l p_l \Delta_l - \frac{1}{2} \sum_{k\neq m,j} (h_{mk}(t) S_{jk} - h_{jk}(t) S_{km}) + \frac{1}{2} h_{jm}(t)(p_j - p_m) \right\}
\end{align*}
\]

where

\[
\begin{align*}
p_m & = c_m \rho_m \\
S_{jm} & = c^*_m c_j \\
\Delta_m & = G \sum_{k\neq m} \kappa_k \\
\Delta^*_m & = G \sum_{k\neq m} \kappa^*_k \\
\kappa_k & = u^*_k v_k \\
\rho_k & = | v_k |^2
\end{align*}
\]

In this description, $p_m$ represents the single particle occupation probability of the orbital $m$, and $\rho_k$ is the pairwise occupation probability of the orbital $k$. The previous system is a generalization of the equations of motions obtained in Refs. \(^7,8,9\) and are applied in the case of fine structure of the $^{14}$C emission from $^{223}$Ra. This is a rare phenomenon where the theory \(^{10,11}\) has superseded the experiments \(^{12,13}\). However, the predictions were not able to assess that transitions to the first excited state are favored. The system is solved by taking as initial values those corresponding to the ground state $1i_{11/2}$ of the parent $^{223}$Ra. The starting
3. Results Concerning the $^{14}$C Decay of $^{223}$Ra

The neutron and proton level schemes are obtained with the superasymmetric two-center shell model $^{17}$ improved in Ref. $^1$. The single particle neutron levels are displayed in Fig. 1. The $^{223}$Ra has the spin $\frac{3}{2}$ emerging from $1i_{11/2}$. Adiabatically, this unpaired neutron reaches the $2g_9/2$ level of the daughter $^{209}$Pb. In the frame of this formalism, the fine structure of the $^{14}$C radioactivity can be understood by an enhanced transition probability of the unpaired neutron from the adiabatic level with $\Omega = 3/2$ emerging from $1i_{11/2}$, to the adiabatic level with the same spin projection emerging from $1j_{15/2}$. The level scheme presented in Fig. 1 shows that the $1i_{11/2}$ level reaches adiabatically the $2g_9/2$ state, the $1j_{15/2}$ reaches the $1i_{11/2}$ state, and the $3d_{5/2}$ level reaches $1j_{15/2}$ state of the daughter $^{209}$Pb. In a system with cylindrical symmetries, Landau-Zener transitions can be realized only between levels with the same spin projection $\Omega$. Four avoided level crossings can

$$\frac{(R-R_i)}{(R_f-R_i)}$$

Fig. 1. Neutron level scheme for $^{14}$C spontaneous emission from $^{223}$Ra with respect the normalized elongation. The levels (with $\Omega=3/2$) emerging from $1i_{11/2}$, $1j_{15/2}$ and $3d_{5/2}$ are represented with thick lines.

be observed in Fig. 1 produced at $(R-R_i)/(R_f-R_i) \approx 0.75$, 1, 1.1 and 1.2. Without these interactions, diabatically, the levels $1i_{11/2}$, $1j_{15/2}$ and $3d_{5/2}$ attain $^1$
the $1i_{11/2}$, $1j_{15/2}$ and $2g_{9/2}$ daughter orbitals, respectively.

In Ref. 8, a particular form of the equations (5-8) (without Landau-Zener included) were used to study the dumping (or the dissipation) process. The dissipation was defined as the flow of energy between collective and intrinsic modes. This energy flow must be irreversible. Unfortunately, accordingly with the discussion of Ref. 8, neglecting the Landau-Zener effect, the calculation is microscopically reversible in the sense that if all the coordinates are time-reversed the system retraces its path. An unknown fraction of the energy that is identified as dissipated energy is in fact collective kinetic energy. Through the presence of the Landau-Zener term in the equations of motion the requirement of irreversibility is satisfied. Therefore, The Landau-Zener probabilities $p_m$ describe a true dissipation process, while the variations of $\rho_k$ and $\kappa_k$ values take partially into account the collective kinetic energy. In these circumstances, it can be assumed that, after the disintegration, the spectroscopic factor can be approximated with only the leading term:

$$S_m = |\langle \Phi_0 | \Phi_m \rangle|^2 \propto p_m$$

where $\Phi$ is the BCS wave function given by the equations of motion after the scission, $\Phi_0 = \alpha^+_m \prod_{j \neq m} (u_{0j} + v_{0j} a^+_j a^+_j) | 0 >$, with $m=2g_{9/2}, 1i_{11/2}, 1j_{15/2}$, are the orthogonal set of seniority one wave functions constructed on the asymptotic overlapped energy levels of the daughter and the emitted nucleus with the ground state $u_{0j}$ and $v_{0j}$ values.

The partial half-life for the state $i$ is obtained with the equation in the WKB approximation:

$$T^{m}_{1/2} = \frac{h \ln 2}{2E_v p_m} \exp \left( \frac{2}{\hbar} \int_{R_s}^{R_m} \sqrt{\frac{2A_1 A_2}{A_0} E^m_d} dR \right) = \frac{h \ln 2}{2E_v p_m} \exp(K_m)$$

where $p_m$ is the final occupation probability and has the meaning of a spectroscopic factor. $E^m_d$ is the deformation energy computed in the framework of the Yukawa-plus-exponential model 14 extended for binary systems with different charge densities 15. This deformation energy is corrected within shell effect computed within Strutinsky’s method as in Ref. 16. The deformation energies for the excited states are obtained by adding the difference between the single particle energy of the excited state and the single particle energy of the nucleon in the ground state. The values of $K_i$ are $2.10 \times 10^{27}$, $1.27 \times 10^{29}$ and $1.87 \times 10^{31}$ for transitions to the daughter ground state, first excited state and second excited state, respectively. These deformation energies are plotted with thick line in Fig. 2. $R_s$ is the ground state elongation used as starting point of the decay, $R$ is the channel exit point from the barrier and $E_v$ is the zero point vibration energy. The effort is mainly focused to reproduce the experimental ratio $R$ of the partial half-lives for transitions to the first excited state and to ground state,

$$R = \frac{T_{i11/2}}{T_{g9/2}} = \frac{p_{g9/2} \exp(K_{i11/2})}{p_{i11/2} \exp(K_{g9/2})}$$

which has approximately the value 0.218. A critical parameter in the model is the quantity which characterizes the variations in time of the generalized coordinates.
Fig. 2. The potential barrier $E_d$ measured from the ground state of the parent as function of the normalized elongation coordinate $(R - R_i)/(R_f - R_i)$. The barriers of the first and second excited states obtained by adding the single particle excitation energy are also plotted.
In the present work, it is matter of the velocity \( v \) of the inter-nuclear distance (or the elongation). Some models are able to determine this quantity \(^{18}\), but in the present work the velocity of the elongation is considered as a fitting parameter and its optimal value will be briefly discussed in comparison with previous results. In Fig. 3, the final single-particle occupation probabilities as function of the internuclear distance velocity are plotted. As expected, for large velocities the system becomes mainly diabatic and the occupation probabilities of the first daughter excited state tends to 1 while that of the ground state reaches 0. For lower velocities, the g.s. transitions are favored. In the bottom of Fig. 3, the ratio \( R \) is plotted. When \( v \approx 9 \times 10^4 - 3 \times 10^5 \), \( R \) reaches the vicinity of the experimental value. The value of \( v \) determined in this way is approximately one order of magnitude lower than that determined from the tunneling times of Ref. \(^{18}\) using macroscopic models, that means, approximating the effective mass with a value close to the reduced mass. However, in the region of avoided crossing levels, the more realistic cranking approximation predicts that the inertia increase with orders of magnitude. If the effective mass increases, the deformation velocity must decrease in order to conserve the available energy. Therefore, the lower values of \( v \) are qualitatively consistent with the use of the cranking model for the inertia.

The previous treatment provides a different way to attack the fine structure of superasymmetric fission for odd and even nuclei. To our knowledge, it is the unique model which can explain the favored transitions to the excited state in the case of cluster-decay. Also, the inclusion of the Landau-Zener effect in the equations of motion offers a more complete description of dumping phenomena during the disintegration.

Acknowledgments

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References

13. E. Hourany, G. Berrer-Ronsin, A. Elayi, P. Hoffmann-Rothe, A.C. Mueller, L. Rosier,
Fig. 3. The single particle occupation probabilities $p$ of the daughter $^{209}\text{Pb}$. Bottom, the ratio $R$ between the partial half-lives for transitions to the ground state and first excited state. A full line indicate the experimental $R$-value.


