1. INTRODUCTION

The quest to find ordered structures in the system of satellites of the Milky Way galaxy dates back to 30 years ago when possible alignments of globular clusters and/or dwarf galaxies along wide streams were first noted (Hodge & Michie 1969; Lynden-Bell 1976; Kunkel & Demers 1977; Kunkel 1979; Lynden-Bell 1982). The mounting consensus for scenarios in which the accretion of satellites has a major role in the formation of the outer halo of the Galaxy (Searle & Zinn 1978; Zinn 1993) prompted a new burst of such kind of studies since the mid ‘90s up to the present day (Majewski 1994; Lynden-Bell & Lynden-Bell 1995; Fusi Pecci et al. 1995; Dinescu, Girard & van Altena 1999; Palma, Majewski & Johnston 2002). Despite the many interesting suggestions, none of the quoted studies was able to provide a conclusive proof of the reality of the alignments, mainly because of the overwhelming difficulty to assess the statistical significance of structures formed by inherently small numbers of objects.

However, recent theoretical and observational achievements may help us to look into the problem from a different and more fruitful perspective:

1. N-body simulations of the process of Galaxy assembly, starting from standard cosmological conditions (Cold Dark Matter - CDM), strongly suggest that the hierarchical merging of satellites is the main driver of galaxy formation (see Moore et al. 1999; Klypin et al. 1999, and references therein).

2. Independent of the assumed cosmology, it has been demonstrated that the accretion/disruption of satellites into the halo of a larger galaxy may leave long-lived relics in the form of streams of stellar (and/or dark matter) remnants that remain aligned to the orbital path of the parent satellite (Johnston, Hernquist & Bolte 1996; Johnston et al. 1999; Ibata et al. 2001a; Ibata & Lewis 1998; Mayer et al. 2001).

3. Convincing observational evidence of the clumpy and "filamentary" nature of the Galactic halo have been provided by many different groups (e.g.)sdss,vivas,dohm,yanny,ivez,carb,ami1,ami2,maj99.

The in vivo example of a satellite accretion/disruption is provided by the Sagittarius dwarf Spheroidal galaxy (Sgr dSph; Ibata et al. 1994, 1997), which is currently merging with the Milky Way, and is carrying its own globular cluster system (i.e., M 54, Ter 8, Arp 2 and Ter 7, previously believed to be normal Galactic globulars). There is now clear observational evidence that the Sgr dSph is losing stars under the strain of the Milky Way tidal field. These tidally-removed stars are found along a huge (and quite coherent) stream extending all over the sky (Sgr Stream, see) and references therein|carb,dohm,david,ssds, tracing the orbit of the parent galaxy.

The contribution of the Sagittarius galaxy to the halo field star population, but it also had a significant role in the building-up of the globular cluster system of the Milky Way.
The orbit has a planar rosette structure, with the pole of the orbit located at \( \ell = 90^{\circ}, b = -13^{\circ} \) (i.e. a nearly polar orbit), and peri- and apo-Galactic distances of 15 kpc and 60 kpc respectively. The derived orbit has been successfully compared with the observed position of the Sgr Stream (Ibata et al. 2001a,b), providing also remarkable indications that the dark halo of the Milky Way is nearly spherical.

In this framework it is a tantalizing application to look for other halo globulars that may be correlated with the orbital path of the Sgr dwarf, and which could be lying in the Sgr Stream. In particular, we look for the phase-space coincidence of outer halo globulars with the computed orbit of the Sgr dSph from 1 Gyr ago up to the present day, searching for the most recent episodes of globular cluster loss, i.e. the ones whose traces are most likely to be still detectable.

2. LOOKING FOR STRUCTURES

For our comparison we selected from the catalogue by Harris (1996) the 35 globular clusters in the range of galactocentric distance 10 kpc \( \leq R_{GC} \leq 40 \) kpc. Among these, 33 have also measured radial velocity \( V_r \). For sake of brevity and clarity we will call this sample the Outer Halo Sample (OHS), in the following. With this selection we avoid the central part of the Galactic halo where it is less likely that ordered structures can survive for a long time, and we leave out of the sample the handful of clusters lying outside of \( R_{GC} \geq 60 \) kpc, a region that lies beyond the Sgr Stream according to the IL98 orbit\(^1\). The adopted OHS does not include the known Sgr globulars, to avoid the detection of the obvious signal of their clustering around the center of the Sgr galaxy.

In Figure 1 we show the OHS clusters (small solid circles) and the Sgr orbit in the planes formed by the rectangular Galactocentric coordinates\(^2\) \((X, Y, Z, \text{ in kpc})\) and in the \( R_{GC} \) [kpc] vs. \( V_r \) [km/s] plane. The large full circles are the known Sgr globulars, which we also show in the plots for completeness. Note that these clusters lie around the end of the orbit corresponding to the present time \((t = 0)\). We highlight (with encircled solid circles) six more clusters that lie remarkably close to the orbit in all the considered planes. These clusters are: Pal 12 (\(?, \) whose association to the Sgr Stream has been already established by )\(]pal12,dav12,NGC 4147,NGC 5634,NGC 5053,Pal 5\) and Ter 3. Is this association real or could it be the mere occurrence of a chance alignment? Though chance alignments in the four-dimensional phase space \((X, Y, Z, V_r)\) are not expected to be very likely, the key point is to quantify the probability that the observed structure could have originated from a statistical fluctuation. To do this we will compare the observed distribution - and its phase space distance to the Sgr orbit - with synthetic samples (having the same dimension as the OHS) extracted from a model representing an unstructured parent halo.

The most conservative comparison that can be made is with a model that closely resembles the observed radial and velocity distribution of the OHS. Figure 2 (upper panel) shows that the cumulative radial distribution of the OHS is well reproduced by a spherical halo model with a density distribution \( \rho \propto R^{-1.6} \). A Kolmogorov-Smirnov (KS) test shows that the probability that the OHS is drawn from the \( \rho \propto R^{-1.0} \) model is \( \approx 90\% \). On the other hand, the probability that the same sample is drawn from the other two models shown for comparison \( (\Phi \propto R^{-1.9}, \Phi \propto R^{-2.5}) \) is \( \leq 15\% \). Doubts may be cast on the appropriateness of a spherical model. It may be conceived that if the true parent halo is flattened, some excess of clustering of the observed points along an orbit with low inclination may artificially emerge in the comparison with a spherical model. This is clearly not the case, however, since the IL98 orbit is nearly polar, i.e. it is almost perpendicular to the Galactic Plane (see Figure 1). In the lower panel of Figure 2 it is shown that the observed distribution of radial velocity of the OHS is well reproduced by a Gaussian distribution with \( <V_r> = -38 \) km s\(^{-1}\) and \( \sigma_V = 175 \) km s\(^{-1}\). According to a KS test the probability that the observed sample is drawn from the model distribution is \( \approx 90\% \).

In the following simulations we extract all the synthetic samples from a spherical and isotropic model with \( \Phi(R_{GC}) \propto \frac{1}{R_{GC}^6} \) and with the Gaussian distribution of radial velocities shown in Figure 2. For each simulated cluster (as well as for all the OHS ones) we computed the spatial distance from the nearest point in the Sgr orbit \( (D_{orb}, \text{ in kpc}) \) and the difference between their radial velocity and the one predicted from the computed orbit at that point \( (\Delta V_r = V_r(orb) - V_r(\text{obs}) \text{ kpc}) \).

In Figure 3, the \( D_{orb} \) values of the selected clusters (large filled circles) are plotted against their \( \Delta V_r \). The encircled points are the six clusters highlighted in Figure 1. A sample of 10000 synthetic clusters (dots) extracted from the adopted model is also shown, for comparison, in the upper panel of Figure 3. The OHS clusters show a remarkable over-density toward the Sgr orbit, that lies in the origin of the axis in the considered plane. The dashed dotted lines enclose the points whose observed radial velocity is within \( \pm 60 \) km s\(^{-1}\) of the velocity predicted by the IL98 orbit. Note that the expected velocity dispersion of the Sgr debris along the Sgr Stream is \( \sigma \approx 60 \) km s\(^{-1}\), according to Ibata et al. (2001b). The continuous vertical segments are placed at \( D_{orb} = 6, 12, \text{ and } 18 \) kpc.

The lower panel of Figure 3 is arranged in the same way, but in this case the dots represent the distribution of the points in the best-fitting Ibata et al. (2001a) simulation that retains a bound core to the present day. The boldface dots are the particles that remain bound to Sgr or were bound less than 3 Gyr ago, while the ordinary dots are particles that were already unbound at that time. Note the remarkable similarity with the distribution of the OHS clusters. In particular, the correlation between the particles that have flown in the Sgr Stream in recent times (less than 3 Gyr ago) and the OHS clusters with

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\(^1\) There is a 28 kpc wide gap in the radial distribution of Galactic globular clusters (see, e.g. Zinn 1985, and references therein). There are only five clusters beyond \( R_{GC} = 40 \) kpc, namely Pal 14, Eridanus, Pal 3, NGC 2419, Pal 4, and AM1. Their respective galactocentric distances are \( R_{GC} = 65, 83, 90, 98, 99, \text{ and } 117 \text{ kpc} \), much beyond the apogalactization of the Sgr orbit. The adopted outer radial threshold \( (R \leq 40 \) kpc, quite similar to the one adopted by Palma, Majewski & Johnston 2002, i.e. \( R_{GC} \leq 30 \) kpc) provides the selection of a homogeneous sample without significant gaps.

\(^2\) This Galactocentric coordinate system is defined such that the origin lies at the Galactic Center; at the Solar position, \((-8,0,0)\), the \( Y \)-axis points in the direction of Galactic rotation; while the \( Z \)-axis towards the North Galactic Pole.
Fig. 1.— The distribution of Galactic globular clusters with $10 \text{kpc} \leq R_{GC} \leq 40 \text{kpc}$ (OHS; small full circles) in the $Y$ vs. $Z$, $X$ vs. $Z$, $X$ vs. $Y$, and $V_r$ vs. $R_{GC}$ planes. The large full circles are the known Sgr globulars M 54, Arp 2, Ter 8 and Ter 7 while the encircled points mark the clusters that are near the Sgr orbital path in all of the planes shown here (NGC 4147, Pal 12, NGC 5634, Pal 5, NGC 5053, and Ter 3). The continuous line is the orbit of the Sgr dSph since the present time (at the position of the Sgr clusters) to 1 Gyr ago, according to Ibata & Lewis (1998).
Fig. 2.— Upper panel: the cumulative radial distribution of the OHS globulars (thick continuous line) is compared to three different model distributions in the considered radial range (10 kpc $\leq R_{GC} \leq 40$ kpc). According to a Kolmogorv-Smirnov test, the probability that the observed distribution is drawn from the model $\Phi \propto R^{-1.6}$ is $P \approx 90\%$, while the other two models shown score $P \leq 15\%$. Lower panel: the observed cumulative distribution of radial velocities for the same clusters (thick continuous line) is compared with a Gauss distribution with $<V_r> = -38$ km s$^{-1}$ and $\sigma_V = 175$ km s$^{-1}$. Also in this case the probability that the observed distribution is drawn from the considered model is $P \approx 90\%$. 
$|\Delta V_r| < 60$ km s$^{-1}$ at any $D_{orb}$ is quite striking.

We define $N_{D_{<6}}$ as the number of synthetic or real clusters having $-60$ km s$^{-1} < \Delta V_r < +60$ km s$^{-1}$ and $D_{orb} < x$ kpc. For example, it can be seen from Figure 3 that there are five OHS clusters with $|\Delta V_r| < 60$ km s$^{-1}$ and $D_{orb} < 6$ kpc, thus $N_{D_{<6}} = 5$. In the same way we find $N_{D_{<12}} = 10$ and $N_{D_{<18}} = 14$ from the observed sample. Now the question is what is the probability that a sample having $N_{D_{<x}}$ greater or equal to the observed one ($N_{D_{<x}}$) is drawn from the assumed unstructured model? To answer this question we randomly extracted 10000 samples of 32 synthetic clusters from the assumed model, and for each of them we measured $N_{D_{<6}}, N_{D_{<12}},$ and $N_{D_{<18}}$.

Figure 4 reports the distributions of $N_{D_{<6}}$ (upper panel), $N_{D_{<12}}$ (middle panel), and $N_{D_{<18}}$ (lower panel) for the 10000 simulated samples. The respective observed values are indicated by a dashed line. In all of the considered cases the mean and modal $N_{D_{<x}}$ values of the distribution are much smaller than the observed ones. $N_{D_{<6}} \geq 5$ occurs for only 200 simulated samples in 10000 (2% of the cases), $N_{D_{<12}} \geq 10$ occurs only for 42 samples in 10000 (0.4%), and $N_{D_{<18}} \geq 14$ occurs only in 2 cases in 10000 (0.02%). It can be concluded that it is highly improbable that the observed clustering of the OHS globulars around the Sgr orbit occur by chance alignment. It is important to remark again that the model from which the simulated samples have been extracted has the same $R_{GC}$ and $V_r$ distribution of the observed sample. Thus the Sgr orbit appears to be a strongly preferred subset of the phase-space for the Galactic globulars in the considered range of $R_{GC}$.

Finally, we note that the simulated samples that have a significant probability of realization (e.g., $P \geq 20\%$) have $N_{D_{<6}} \leq 3$, $N_{D_{<12}} \leq 6$, and $N_{D_{<18}} \leq 8$, significantly lower than the observed values. Hence, the significance of the detected phase-space structure cannot be due to the actual correlation of just a couple of clusters with the Sgr orbit. According to the distributions shown in Fig. 4, the probability that $N_{D_{<18}} \geq 10$ is $P \sim 5\%$. Therefore it can be stated, with 95% confidence, that at least 4 real associations are needed to produce the observed signal of $N_{D_{<18}} = 14$.

The above test demonstrates that the observed phase-space clumping around the orbit of the Sgr dwarf galaxy is highly unlikely to have occurred by a chance coincidence, if the halo globular cluster population is indeed distributed according to the simple spherical model described above. To address this concern, we performed a second test, using “bootstrapped” artificial data sets constructed from the real OHS. The artificial samples were constructed by rotating the position of each real globular cluster by a random azimuthal angle about the Galactic center, and introducing a random flip perpendicular to the Galactic plane. This implicitly assumes, of course, that the globular cluster distribution is symmetric about the Galactic center, as well as above and below the Galactic plane. A slight complication arises from the fact that we do not know the proper motion of many of the clusters, so we cannot deduce the heliocentric radial velocity that the cluster should have at its new position. We therefore have to assume some model for the halo velocity distribution: we take a simple Gaussian model, and draw random realizations of the total space velocity $\vec{V}$ consistent with the observed heliocentric velocity $V_r$ (that is, we calculate the conditional probability of $\vec{V}$ given $V_r$). Having defined thereby the 3-dimensional velocity vector, we rotate the vector to the new position, and project it along the line of sight to the Sun (corrections for the peculiar motion of the Sgr and for Galactic rotation are made). With a Gaussian model that has $\sigma = 175$ km s$^{-1}$ and $V_r = -38$ km s$^{-1}$, we find that out of 10000 artificial data sets, $F(N_{D_{<6}} \geq 5) = 399$, $F(N_{D_{<12}} \geq 10) = 102$, and $F(N_{D_{<18}} \geq 14) = 18$. For comparison, for a model with $\sigma = 110$ km s$^{-1}$ (or $\sigma = 150$ km s$^{-1}$) and $V_r = 0$ km s$^{-1}$, the statistics are as follows: $F(N_{D_{<6}} \geq 5) = 463(425)$, $F(N_{D_{<12}} \geq 10) = 140(121)$, and $F(N_{D_{<18}} \geq 14) = 35(28)$, where the result in brackets refers to the higher velocity dispersion model. According to these tests using “bootstrapped” random data sets, the observed phase-space clumping is highly unlikely to have occurred by chance, in agreement with our previous test. The fact that the observed structure has a slightly lower statistical significance with the “bootstrapped” samples than was deduced from the previous test with its perfectly isotropic and isothermal random samples is not surprising. This is presumably a consequence of occasional chance reappearances of (part of) the structure of the real OHS in the “bootstrapped” samples.

As a final remark, we note that the region of the $\Delta V_r - D_{orb}$ plane with $|\Delta V_r| < 60$ km s$^{-1}$ and $D_{orb} < 18$ kpc is also populated by particles that become unbound more than 3 Gyr ago. Thus it has to be considered the possibility that some of the OHS clusters found in that region drifted into the Sgr Stream in more ancient times. For instance, this can be the case for NGC 5634, as we argue elsewhere (Bellazzini, Ferraro & Ibata 2002). On the other hand, the discussion of the individual OHS clusters is beyond the scope of the present analysis.

### 2.1. Proper Motions

Thanks to the painstaking effort of a few teams of astronomers, estimates of the proper motions (hereafter PMs) of 41 Galactic globular clusters are now available (see Dinescu, Girard & van Altena 1999; Palma, Majewski & Johnston 2002, for complete lists and references). Most of these measures have quite sizeable uncertainties, ranging from $\sim 0.2 \text{ mas/yr}$ to $\sim 2 \text{ mas/yr}$. Moreover, subtle and unaccounted - systematics may still affect them, as suggested by the large differences between independent estimates of the motion of the same cluster. In the list by Palma, Majewski & Johnston (2002) there are 14 cluster with more than one PM estimate. The average absolute difference among independent estimates for the same cluster is $1.72 \pm 1.29 \text{ mas/year in } \mu_\alpha \cos \delta$ and $2.90 \pm 2.50 \text{ mas/year in the } \mu_\beta$ component. Differences much larger than the quoted errors (up to $\sim 10$ times) are common and a reasonable assumption of the minimum uncertainty of the proper motion estimates is probably $\sim 1 \text{ mas/year in each component}$. Hence, while available PMs may be very valuable, for instance, to characterize the kind of orbit followed by a given globular, they are expected to have a modest constraining power in the application presented in this paper. Such constraining power is also lowered by the fact that PMs predicted by the model we are comparing
Fig. 3.— The distribution of the OHS globular clusters about the predicted orbit of Sgr. The parameter $D_{\text{orb}}$ is the distance of the cluster from the nearest point of the orbit, while $V_r(\text{obs}) - V_r(\text{orb})$ is the difference between the observed radial velocity and the prediction of the computed orbit (at the nearest point in the orbit). The observed distribution (large full circles) is compared with the distribution of 10000 points randomly drawn from our assumed halo model (upper panel) and with the distribution of the Sgr debris as computed by Ibata et al. (2001a) (lower panel). This N-body simulation displayed in the lower panel is the model by Ibata & Lewis (1998) that gives the best fit to the positions and velocities of the carbon stars observed in the Sgr stream, while retaining a bound center (their dwarf galaxy model “D1”, and halo model “H2”, integrated in a Galactic potential with circular velocity $v_c = 200 \text{ km s}^{-1}$ at 50 kpc). The boldface dots show particles that remain bound to Sgr or were bound less than 3 Gyr ago, whereas the ordinary dots were already unbound at that time. Clearly, several clusters in the OHS follow the expected trend of the recently disrupted Sgr debris. The encircled points represent the clusters similarly put in evidence in Figure 1. The horizontal dashed-dotted lines enclose the range $-60 \text{ km s}^{-1} \leq \Delta V_r \leq 60 \text{ km s}^{-1}$, while the continuous vertical segments are located at $D_{\text{orb}} = 6, 12$ and 18 kpc.
Fig. 4.— Results from the random drawing of 10000 synthetic GC samples (equivalent to the observed one) from the assumed halo model.
with carry their own, non negligible, uncertainty (typically \( \sim 0.85 \text{ mas/yr} \)) in each component, which we estimate from the intrinsic dispersions in distance, position and velocity in the N-body stream models, assuming a 20% uncertainty in the Galactic circular velocity at 50 kpc. Radial velocities, being unaffected by uncertainties in the distance, are more reliably predicted by the model and are easily, and much more accurately, measured. For these reasons we preferred to limit the bulk of our analysis to the the \( (X,Y,Z,V_r) \) phase-space.

Yet it may be interesting to compare the PMs predicted by the IL98 model for the clusters that are candidate members of the Sgr Stream (listed in Table 1, see below), with the observational estimates. The orientations (directions) of the PM vectors are expected to be less sensitive to systematics with respect to the actual moduli. In Fig. 5 we compare the predicted and observed directions of the PM vectors of the seven clusters listed in Tab. 1 for which PM estimates are available. Four of these seven clusters have two independent PM estimates: in these cases we report the comparison with both values (see caption). We stress that this kind of comparison cannot have a serious impact on our previous conclusions except if both the following conditions were simultaneously fulfilled: (a) if accurate PM estimates were available for all the considered clusters (or the large majority of them), and (b) if less than 3-4 clusters were found to have observed PMs in agreement with the predictions. This case would imply that the correlation among OHS clusters and the Sgr orbit we detect in the \( (X,Y,Z,V_r) \) phase-space is largely due to a very unlikely (but not impossible) chance alignment in four dimensions. On the other hand some (up to \( \sim 10 \), actually) of the clusters listed in Table 1 are expected not to belong to the detected structure.

In panel (a) the predicted proper motions include a correction for the Solar reflex motion (hereafter S.r.m.), assuming a Galactocentric distance of 8 kpc, a circular velocity of 220 km s\(^{-1}\) and the peculiar motion of the Sun as derived by Dehnen & Binney (1998). The predicted PM directions of all the seven clusters agree with the observed values to within \( \pm 45^\circ \). The adoption of alternative PM estimates does not change significantly the observed correlation. However, the result of this comparison may be affected by the inclusion of the Solar reflex motion. Hence, in panel (b) we present the comparison between the plain (uncorrected for the S.r.m) model predictions and the observed PM directions after correction for the S.r.m. The result is very similar to that presented in panel (a) despite of the larger errors involved. The only cluster whose predicted and observed PM directions differ by more than \( 45^\circ \) is Pal 12, e.g. the only one for which independent evidences of membership to the Sgr Stream are already available (Martinez-Delgado et al. 2002).

In conclusion, it is found that the available proper motion estimates are fully compatible with the results presented in §2. Given the high degree of correlation among predicted and observed PM directions shown in both panels of Fig. 5 one may also be tempted to draw firm conclusion on the actual membership of individual clusters. Nevertheless, in our view, this may still be an hazard. If one looks at the amplitude of the individual corrections it is found that (a) all the considered clusters have at least one component of the PM vector that is corrected by more than 50% of the observed value, (b) 4 out of seven clusters have at least one component whose S.r.m correction is larger that 100% of the observed values and, (c) corrections as large as 200 - 400% are indeed applied in some cases. Given the remarkable uncertainties in the observed PMs and the uncertainties involved in the S.r.m correction it is easy to conclude that currently measured PMs are not good tracers of the actual motion of the considered clusters and are instead predominantly measures of the Solar reflex motion. These considerations fully support the approach followed in §2, e.g. using only positions and radial velocity measurements in the search for statistical structures in the globular cluster system of the Milky Way.

### 2.2. Comparison with previous analysis

The possible association of Galactic globulars with the Sgr dSph galaxy has been considered before (Irwin 1999; Dinescu, Girard & van Altena 1999; Dinescu et al. 2000; Siegel et al. 2001; Palma, Majewski & Johnston 2002). All these studies strongly rely on PM estimates. Furthermore they are centered on testing the actual association of individual clusters to the Sgr galaxy and its remnants. On the other hand our approach and our results are statistical in nature. We find a statistically significant clustering of OHS clusters along the Sgr orbit but we have no possibility to check individual membership, except for the limited test shown in §2.1. We can only provide a list of possible members, ranked according to their distance to the Sgr orbit in the \( (X,Y,Z,V_r) \) phase-space (e.g., Tab. 1). Hence, the comparison with the above quoted studies can be made only in a broadly general sense.

Dinescu, Girard & van Altena (1999) performed a thorough study of the orbits of the Galactic globulars for which PM estimates are available. In doing this, they identify three metal-poor clusters having orbits typical of the thick disc (instead that of the halo, as expected) and they check the hypothesis that the peculiar orbits of these clusters (namely NGC 6254, NGC 6626 and NGC 6752) could be due to their former membership to the Sgr dSph, concluding that this possibility is unlikely. All these clusters have \( R_{GC} < 10 \), and are thus are not included in the OHS considered here and we cannot comment further on the Dinescu, Girard & van Altena (1999) results.

Following the suggestion of Irwin (1999), Dinescu et al. (2000) modeled the (PM based) orbits of the Sgr dSph and of Pal 12 and concluded that the available observations are compatible with the possibility that the cluster was torn from the galaxy \( \sim 1.4 - 1.7 \) Gyr ago. Our analysis confirms this result.

Siegel et al. (2001) obtained an estimate of the PM of Pal 13 and compared the general characteristics of the integrated orbit of the cluster (total energy, angular momentum, eccentricity, perigalactic and apogalactic radii) to those of five satellite galaxies of the Milky Way, including Sgr. They conclude that a common origin for Pal 13 and Sgr is unlikely. However we note that (a) Sgr provides (by far) the best match to the orbital parameters of Pal 13 with respect to any other considered galaxy (see Siegel et al.’s Tab. 8), the maximum discrepancy being \( \sim 40\% \) in angular momentum, and (b) the orbital parameters are provided without any uncertainty, as if the adopted PMs were...
Fig. 5.— The predicted direction of proper motions are compared with the observed ones for the subset of candidate Sgr Stream cluster for which estimates of proper motion are available (from the list by Palma, Majewski & Johnston 2002). When two independent estimates of proper motion are available for the same cluster we report both: one as a large filled circle and the other as a small filled circle, and the two symbols are joined by a long dashed line. The large dot of Pal 5 is the estimate quoted by Palma, Majewski & Johnston (2002) as Cudworth et al. (2001), the small dot is from Schweitzer et al. (1993). The solid line is the locus of perfect agreement between predictions and observation, the dotted lines are displaced by ±45 deg. The error-bars are derived from the propagation of the uncertainties in the PM estimates of clusters as reported by Dinescu, Girard & van Altena (1999); Palma, Majewski & Johnston (2002) and by the uncertainties in the model predictions, as described in §2.1. Panel (a): comparison between PMs including the S.r.m. Panel (b): comparison between PMs corrected for the S.r.m.
perfectly known. In our view this shortcoming seriously weakens the conclusion by these authors. Here we adopt the same PM estimate obtained by Siegel et al. (2001), reaching the conclusion that Pal 13 is a possible member of the Sgr Stream by direct comparison with a model that includes the Sgr Stream itself.

Palma, Majewski & Johnston (2002) (hereafter PMJ02) performed a global search for structures made of Galactic satellites, adopting the technique of the “orbital poles”, originally introduced by Lynden-Bell & Lynden-Bell (1995). In this kind of analysis one searches for intersections among the great circles described by the family of possible poles of the orbit of any given satellite. As stated by PMJ02 “any set of objects having common origin and maintaining a common orbit, no matter how spread out in the sky, will have great circle pole families (GCPFs) that intersects at the same pair of antipodal points on the celestial sphere”.

Adding the information from PMs, PMJ02 limit the valid pole family of each satellite to an arc of its great circle of orbital poles (arc segment pole family, ASPF). With this technique they select a set of candidate-former-members of the Sgr galaxy (Ter 7, Ter 8, Arp 2, M 54, M 53, NGC 5053, Pal 5, M 5, Pal 12 and NGC 6356) by requiring that their GCPF come within 5 deg from the Sgr ACPF and 6 \( \leq R_{GC} \leq 36 \) Kpc. The first four clusters in their list are the well known present members of the galaxy. They rise doubts of the clear association of each of the other candidate members. For instance, they exclude NGC 5053 and M 53 because their metal content is lower than any of the known Sgr globulars. With the same arguments adopted by Siegel et al. (2001) for Pal 13 they argue that Pal 5 and Pal 12 are unlikely members of the family, but they cannot rule out the possibility. Finally, they conclude that despite the counter-arguments presented they maintain their original list of candidates, with the exception of NGC 5053. Elsewhere in the paper, they also note that NGC 5466 has energy and angular momentum within 1-\( \sigma \) of those of the Sgr galaxy. NGC 6356, M 5 and M 53 have \( R_{GC} < 10 \) kpc, hence are not included in our sample. The possible association of NGC 5466 with the Sgr dSph is confirmed by our analysis. The possible association of NGC 5053, Pal 5 and Pal 12 is also confirmed by the present work. PMJ02 also suggested a possible association of NGC 4147 with a group of clusters whose orbits may be related to those of the Magellanic Clouds, but they state that the result is very uncertain. Furthermore we note that on the same basis the possible association of M 53 (elsewhere classified as a candidate for association with Sgr) with the Small Magellanic Cloud and Ursa Minor is also suggested by PMJ02. We conclude that there is no serious disagreement between the present analysis and the one by PMJ02.

The present contribution, however, provides the first proof that the orbit of Sgr is a preferential subset of phase-space for globular clusters inhabiting the outer halo of the Milky Way, and we place this result on a sound statistical basis, a result not accomplished by any of the previous studies. The success of the present analysis is due to the direct comparison of the positional and kinematical properties of the OHS clusters with a realistic model of the orbit of the Sgr dSph and its relics, that has been previously tested against independent observations (Ibata et al. 2001a,b).

Finally, we shall shortly comment on “non-kinematic” criteria that have been used to assess the association of a given cluster to the Sgr galaxy. Many authors (see, e.g., Dinescu, Girard & van Altena 1999; Palma, Majewski & Johnston 2002; and references therein) have discussed the likeliness of the membership of their candidates on the basis of the similarity of their metallicity, Horizontal Branch (HB) morphology and/or structural parameters with those of the four known Sgr clusters. In our opinion this approach may not be very useful. It is quite clear that Sgr was a much larger and complex system in the past and its present status (as well as its present GC system) may be not fully representative of its original range of properties. If, by chance, the cluster Ter 7 had been lost by the Sgr galaxy in the previous perigalactic passage, this kind of criteria would have erroneously classified it as a more metal rich ([Fe/H] ~ -0.5) than any of the other clusters that are still present in the main body of Sgr dSph ([Fe/H] < -1.5). Moreover, the existence of population and/or metallicity gradients may have driven the preferential loss of metal poor populations (see Alard 2001; Newberg et al. 2002; Bellazzini, Ferraro & Ibata 2002, and references therein), making the actual low metallicity limit of the Sgr stellar population quite uncertain. The judgement of the likelihood of membership on the basis of the HB morphology is even less justified since it is well known that clusters of different morphologies do actually co-exist in real galaxies (see, e.g.,) and references therein, for the case for Fornax).

A much more reliable discriminant may be provided, in the future, by the detailed abundance patterns of the clusters. For instance, the behaviour of \( \alpha \) elements as a function of [Fe/H] is determined by the star formation history of the parent galaxy (see references therein) and references therein) that was (probably) not the same in the Sgr dSph and in the Galactic Halo. In this context it is interesting to note that Pal 12, Ru 106 (Brown Wallerstein & Zucker) and Pal 5 (Smith, Sneden & Kraft 2002) have been found to be less \( \alpha \)-enhanced than Galactic Halo globulars of similar metallicity.

For the above reasons, we rely only on phase-space parameters in our analysis, recalling that by searching the lost relics of the Sgr system we are trying to reconstruct its original properties, not the present day ones.

3. Conclusions

We have demonstrated that there is a coherent structure in the phase-space distribution of the outer globular cluster of the Milky Way, strongly correlated with the orbital path of the Sgr dSph galaxy. This correlation cannot have originated by chance, as a random realization of an unstructured halo, if such halo has the same distribution of galactocentric distance and radial velocity as the considered OHS clusters.

Several of the OHS globular clusters have spatial positions and radial velocities compatible with the hypothesis

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3 This is not completely true since M 53 has the same metallicity as Ter 8, i.e. [Fe/H] = -1.99 (see Montegriffo et al. 1998; Da Costa & Armandroff 1995). See also below, for further discussion about non-kinematic criteria of selection of candidate Sgr members.
that they are following the same orbit as the Sgr dSph, i.e. they probably belong to the Sgr Stream. Furthermore the spread in phase space around the Sgr orbit of such clusters is similar to that predicted for the Sgr Stream population, according to the numerical simulations by Ibata et al. (2001a).

We conclude, with 95% confidence, that at least 4 (out of 32) of the OHS clusters are physically associated with the Sgr Stream and were former members of the Sgr dSph. It should be noted however, that the analysis presented in this contribution is best suited to identify clusters that became unbound relatively recently. In principle, more OHS globular clusters could be associated with the Sgr stream.

In Table 1 we report the list of the selected globulars having $|\Delta V_o| < 60$ Km/s and $D_{orb} < 18$ kpc, in order of increasing $D_{orb}$. The proper motions predicted by the IL98 model are also reported in Table 1. Since the adopted analysis is statistical in nature it is clear that some of the clusters listed in Tab. 1 may be just occasional interlopers of the Sgr orbit (up to 10 over 14 in the worst case, according to our estimate). The comparison with the available proper motion estimates shown in §2.1 does not provide any strong indication in this regard. While the actual membership to the Sgr Stream of each individual cluster can be firmly established only with very accurate proper motion measurements (as will be achieved by dedicated space missions like GAIA or SIM), it is reasonable to assume that the clusters with the smaller $D_{orb}$ are the ones for which the membership is more likely (see §2.1). We make the hypothesis that the six clusters more closely associated with the Sgr orbital path (i.e. those put in evidence in Figures 1 and 3) are former members of the Sgr dSph. In this case the number of globular clusters originally in the Sgr galaxy is $N_0 = 10$. If we adopt for the Sgr dSph the same globular cluster specific frequency ($?, S_N = N_010^{0.4(M_V+15)}$) of the other Galactic dSphs that has a globular cluster system, i.e. Fornax $S_N \simeq 26$, we obtain an estimate of the total absolute magnitude of Sgr before the occurrence of any tidal stripping, $M_V \simeq -14$. Newberg et al. (2002) estimated that in the Sgr Stream there are as many stars as in the present undisrupted body of the galaxy, thus the $ab initio$ total luminosity of the Sgr dSph was roughly two times the present value. Hence the total absolute magnitude at that time was $M_V - \log(2) \simeq -14.1$ (? , where $M_V = -13.4$, from])M98. The excellent agreement between the two independent estimates of the initial $M_V$ fully supports the plausibility of the proposed scenario.

According to the results presented, it emerges that the Sgr dSph was not only an important contributor of halo field stars (Ivezic et al. 2000; Yanny et al. 2000; Newberg et al. 2002) but it also had a significant role in the building-up of the globular clusters system of the Milky Way.

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REFERENCES

Lynden-Bell, D., 1982, The Observatory, 102, 201
Table 1

<table>
<thead>
<tr>
<th>Name</th>
<th>$D_{\text{orb}}$ [kpc]</th>
<th>$\Delta V_r$ [km/s]</th>
<th>$\mu_{\alpha} \cos \delta$ [mas/year]</th>
<th>$\mu_{\delta}$ [mas/year]</th>
<th>$\mu_l \cos b$ [mas/year]</th>
<th>$\mu_b$ [mas/year]</th>
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<tr>
<td>Pal 12</td>
<td>3.00</td>
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</table>

Note. — Distance from the nearest point in the orbital path followed by the Sgr dSph during the last Gyr ($D_{\text{orb}}$) and difference in radial velocity between the considered cluster and prediction of the computed orbit at that point ($\Delta V_r$) for the OHS clusters having $|\Delta V_r| < 60$ km/s and $D_{\text{orb}} < 18$ kpc. The clusters are ordered with growing $D_{\text{orb}}$. The known members of the Sgr system are also reported at the end of the table, for comparison. The first six clusters of the list are the ones put in evidence in Fig. 1 and Fig. 3. The four known members of the Sgr globular cluster system are also reported for comparison in the last four rows of the table. The proper motions predicted by the IL98 model [under the hypothesis that (a) the clusters belong to the Sgr stream, and (b) they have been lost during the last orbit of Sgr, see §2] are also reported, in both the equatorial and galactic reference system. The intrinsic dispersion of the modelled stream, combined with estimated 20% uncertainties in the Galactic mass model, translate to uncertainties of $\sim 0.85$ mas/yr on each PM component.

$^a$Bellazzini, Ferraro & Ibata (2002) argue that NGC 5634 was likely lost before the last peri-centric passage of Sgr, so the proper motion predictions listed here may be substantially in error.