Introduction

This is an interesting time to think about cosmology from a string theory perspective. The two subjects: cosmology and strings, complement each other in several ways. Cosmology needs an underlying theory to approach its basic questions such as the initial singularity, if there was any, the origin of inflation or any alternative way to address the problems that inflation solves, such as the horizon and flatness problems and, more importantly, the origin of the density perturbations in the cosmic microwave background (CMB). The successes of inflation Guth in this regard makes us sometimes forget that it is only a scenario in search of an underlying theory. There are many variations of the inflationary scenario Rev but at the moment there is no concrete derivation of inflation (or any of its alternatives) from a fundamental theory such as string theory.

On the other hand string theory, although lacking a full nonperturbative formulation, has been highly developed and understood in many respects and needs a way to be confronted with physics. One possibility is low-energy phenomenology, although it may be a long way before this can be tested, unless supersymmetry is discovered at low energies and its properties provide some hints to its high energy origin or we get lucky and the string scale is low enough to be probed in the future colliders such as LHC. Otherwise cosmology may turn out to be the main avenue to probe string theory. This has been reinforced in view of the recent observational discoveries that seem to indicate that the effective cosmological constant is not exactly zero and, furthermore, the accuracy of the CMB experiments COBE, cmbexp that have provided a great deal of information on the scalar density perturbations of the cosmic microwave background. Th

Moreover, based mostly on string theory ideas, the brane world scenario has emerged in the past few years offering dramatic changes in our view of the universe braneworld. The fact that we may be living on a hypersurface in higher dimensions does not only imply that the scale of string theory could be as small as 1TeV, but also provides completely new scenarios for the cosmological implications of string theory branecosmology, branecosmology2. Actually if the brane world scenario is realized, but with a string scale close to the Planck scale, the main place to look at its possible implications will be cosmology, rather than tabletop experiments or high energy accelerators.

Finally, the study of cosmological implications of string theory can shed some light into the better understanding of the theory itself. We already have the experience with the study of black hole backgrounds in string theory which has led to some of the main successes of the theory, namely the explicit calculation of the black hole entropy and the identification of the AdS/CFT correspondence adscft, which not only provides a concrete realisation of the holographic principle thooft but has also led to important results in field and string theories. Cosmology is the other arena where nontrivial string backgrounds can be explored, some of the ideas developed from other studies can be put to test in cosmology and probably new insights may emerge. In particular the recent realisation that our universe could be in a stage with a nonzero vacuum energy gives rise to an important challenge for string theory, we need to be able to understand string theory in such a background desitter. Also previous ideas about quantum cosmology in general may find new realisations in the context of string theory. In summary, we may say that cosmology presents probably the
most important challenges for string theory: the initial singularity, the cosmological constant, the definition of observables, the identification of initial conditions, realisation of de Sitter or quintessential backgrounds of the theory, etc.

It is then becoming of prime importance to learn the possible applications of string theory to cosmology. These lecture notes are an effort to put some of these ideas together for non-experts. Due to limitations of space, time and author’s knowledge, the discussion is at a superficial level and incomplete. They were originally given to review the basic ideas on the subject, including brane cosmology in static and time dependent backgrounds to conclude with D-brane inflation and tachyon condensation, together with some details about the ekpyrotic scenario. However, right after the lectures were given, several interesting developments have occurred related with the subject of the lectures that have to be briefly included for completeness (rolling tachyon, S-branes, time dependent orbifolds). Fortunately there are several good reviews on the first part of the lectures that can be consulted for deeper insights Rev, brandenberger, riotto, copeland, mcosm, carroll, easson, veneziano. There are hundreds of articles on brane cosmology and I cannot make justice to everybody working in the field. I do apologise for omissions of important references. The presentation tries to include only the brane cosmology ideas formulated in the context of string theory or that have connections to it, therefore, many interesting developments in brane cosmology, which are not clearly related to string theory are omitted.

I first give an overview of the standard big-bang cosmology that introduces the physical parameters, notation and problems. Then I describe briefly the main ideas discussed in the past (before the year 2000) in string cosmology. These include the Brandenberger-Vafa scenario BV where T-duality and winding modes could have an interesting implication for early universe cosmology, including the possible determination of the critical dimension of spacetime. Also the cosmology associated to the moduli fields, which could be candidates for inflaton fields moduliniflation , but also can cause a serious and generic cosmological problem once they get a mass, since they can either ruin nucleosynthesis by their decays, if they are unstable, or over close the universe, if they happen to be stable cmp (see also polony). This has been called the cosmological moduli problem. Finally we mention the main ideas behind the Gasperini-Veneziano ‘pre big-bang cosmology’ that during the years has become the string cosmology scenario subject to more detailed study (see veneziano for a very complete review on the subject with references to the earlier work).

In the third part of the lectures I concentrate on some recent developments. I will emphasise the role that string theory p-branes can play in cosmology. First we describe some of the interesting results coming out of a treatment of brane cosmology in the simple setting of 4D brane worlds moving in a 5D bulk branecosmology, branecosmology2, kraus, keki, ida, shiromizu, BCG, verlinde. Two points are emphasised: the Einstein’s equations in the 4D brane do not have the standard behaviour in the sense that the relation between the Hubble parameter and the energy density is different from the standard 4D cosmology. We also remark the interesting possibility for understanding cosmology in the brane world as just the motion of the brane in a static bulk. An observer on the brane feels his universe expanding while an observer in the bulk only sees the brane moving in a static spacetime, this is usually known as mirage cosmology.

Then I discuss the possibility that the dynamics of D-branes may have direct impact in cosmology, in particular considering a pair of D-branes approaching each other and their subsequent collision could give rise to inflation tye. For a D-brane/antibrane pair it is possible to compute the attractive potential from string theory and actually obtain inflation with the inflaton field being the separation of the branes quei,dvali. Furthermore, it is known from string theory that after the branes get to a critical distance, an open string mode becomes tachyonic thus providing an instability which is precisely what is needed to end inflation quei. Obtaining then a realisation of the hybrid inflation scenario hybridinflation with the two relevant fields having well defined stringy origin, i.e. the separation of the branes generates inflation and the open string tachyon finishes inflation and provides the mechanism to describe the process of thebrane/antibrane collision and annihilation. Natural extensions of these ideas to include orientifold models, intersecting branes at angles and related constructions, kali, queii, g-b, lusti will also be discussed, which illustrates that the realisation of hybrid inflation from the inter-brane separation as the inflaton field and the open string tachyon as the field responsible to end inflation and re-heat is very generic in D-brane models.

Tachyon condensation is one of the few physical process that has been studied in detail purely from string theory techniques banks,sen,lowerbranes,tseytlin and can have by itself important implications to cosmology, independent of its possible role in the brane inflation scenarios tacos. The rolling tachyon field has properties
that have been uncovered just recently senroll,garytac, such as resulting in a pressure-less fluid at the end of its relaxation towards the minimum of the potential. Furthermore, its potential includes the D-branes as topological defects which also play an important role in cosmology (providing for instance dangerous objects such as monopoles and domain walls and less dangerous ones like cosmic strings). Finally it has partially motivated the introduction of a new type of branes known as space-like or S-branes gutperle which can roughly be thought of as D-branes and domain walls. These geometries are related to black holes and then the mass, charge, Hawking temperature and entropy can be computed in a similar way. The bouncing behaviour can be interpreted analogous to the Schwarzschild wormhole or Einstein-Rosen bridge, but this time connecting past and future cosmologies instead of the two static, asymptotically flat regions bqrtz. Stability of these solutions may be a potential problem for their full interpretation.

We also briefly discuss the probably more ambitious proposal of the ekpyrotic universe ekpyrosis,kosst,cyclic, in the sense that with the same idea of colliding branes, this time in the context of Horava-Witten compactifications rather than D-branes, it may not lead necessarily to inflation but could provide an alternative to it, approaching the same questions as inflation does, especially the inhomogeneities of the cosmic microwave background. This scenario has also lead to two interesting developments. First, resurrecting the idea of the cyclic universe cyclicold, cyclic and second, the realisation of cosmological string backgrounds by just orbifolding flat spacetime hs, kosst, TDbackgrounds, hp. This process guarantees an exact solution of string theory and has opened the possibility to approach issues concerning a big bang-like singularity performing explicit string calculations. Possible problems with this approach to time dependent backgrounds are briefly mentioned.

Cosmology Overview

Standard FRW Cosmology

The standard cosmological model has been extremely successful given its simplicity. The starting point is classic Einstein equations in the presence of matter. The requirements of homogeneity and isotropy of the 4D spacetime determines the metric up to an arbitrary function of time $a(t)$, known as the scale factor, which measures the time evolution of the Universe and a discrete parameter $k = -1, 0, 1$ which determines if the Universe is open, flat or closed, respectively. The Friedmann-Robertson-Walker (FRW) metric describing the evolution of the Universe can then be written as:

$$\text{frw} \; ds^2 = -dt^2 + a^2(t) \left[ dr^2 + k r^2 + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \right].$$

The scale factor $a(t)$ is given by solving Einstein's equations

$$G_{\mu\nu} = R_{\mu\nu} - 8\pi G T_{\mu\nu}.$$

In natural units $\hbar = c = 1$, Newton’s constant $G$ can be written in terms of the Planck mass $8\pi G = 1/M^2_{\text{Planck}}$. The stress-energy tensor $T_{\mu\nu}$ is usually taken to correspond to a perfect fluid (latin indices are 3-dimensional):

$$T_{00} = \rho, \quad T_{ij} = p g_{ij}, \text{with the energy density } \rho \text{ and the pressure } p \text{ satisfying an equation of state of the form } \rho = w p. \text{ Here } w \text{ is a parameter which, for many interesting cases is just a constant describing the kind of matter dominating in the stress-energy tensor. We present in the table the values of } w \text{ for common cases corresponding to matter, radiation and vacuum domination.}$$

1.7 tabular—c—c—c—c— Stress Energy $w$ Energy Density Scale Factor $a(t)$

Einstein’s equations for the ansatz (frw) above reduce to the Friedmann’s equations The second equation is sometimes referred to as the Raychaudhuri equation. $H^2 = 8\pi G \rho - ka^2$

The conservation $\dot{\rho} = -3H (p + w)$ Therefore, after using the equation of state $p = w\rho$ we are left with two equations which we can take as the first Friedmann equation and the energy conservation, for $\rho$ and $a(t)$, which can be easily solved. Equation (conservation) gives immediately $\rho \sim a^{-3(1+w)}$ introducing this in Friedmann’s equation gives the solution for $a(t)$. We show in the table the behaviour of $\rho$ and $a(t)$ for chaudhuri equation.
typical equations of state. Notice that for the expressions for $a(t)$ we have neglected the curvature term $k/a^2$ from Friedmann’s equation and therefore we write the solution for the flat universe case. The exact solutions for the other cases can also be found.

For the more general case we may say several things without solving the equations explicitly. First of all, under very general assumptions, based mainly on the positivity of the energy density $\rho$, it can be shown that the FRW ansatz necessarily implies an initial singularity, the big-bang, from which the universe starts expanding. For instance if $\rho + 3p > 0$, which is satisfied for many physical cases, the acceleration of the universe measured by $\ddot{a}$ is negative, as seen from the second Friedmann’s equation. For $k = -1$, 0 the first equation tells us that, for positive energy density, the universe naturally expands forever whereas for $k = 1$ there will be a value of $a$ for which the curvature term compensates the energy density term and $\ddot{a} = 0$, after this time $a$ decreases and the universe re-collapses. A word of caution is needed at this point, which is usually a source of confusion. Because of the previous argument, it was often claimed that, for instance, a closed universe ($k = 1$) will re-collapse. However we can see from the second Friedmann equation, which is independent of $k$, that if $\rho + 3p < 0$ the universe will always accelerate. This happens for instance for vacuum domination ($w = -1$) where for $k = 1$ the solution is $a(t) \sim \cosh(\sqrt{\Lambda/3}t)$, which is clearly accelerating.

We can illustrate the structure of the big-bang model in terms of a spacetime diagram, known as Penrose or conformal diagram, see for instance Hawking. This diagram not only pictures the relevant parts of the spacetime, in this case the initial singularity, but it is such that by a conformal transformation, it represents the points at infinity in a compact region and furthermore, even though the spacetime is highly curved, especially close to the singularity, light rays follow lines at 45 degrees just as in standard Minkowski space. These diagrams are usually, but not always (depending on the symmetries of the metric) two-dimensional. In the FRW case it includes the $t - r$ plane, so each point in the diagram represents a 2-sphere for $k = 1$ or the 2D flat and hyperbolic spaces for $k = 0, -1$ respectively. The wiggled line of figure 1 is the spacelike surface at $t = 0$ representing the singularity. At this point the scale factor $a = 0$ and the radius of the sphere (for $k = 1$) is zero and $\rho \to \infty$. The Penrose diagrams extract in a simple way the causal structure of the spacetime. In this case we can see that extrapolating to the past from any point in the diagram necessarily hits the big-bang singularity.

A useful concept to introduce is the critical density $\rho_{\text{critical}} \equiv 3H^28\pi G$. Which, for the present time $H = H_0 \sim 65\text{Km/s/Mpc}^{-1}$ gives $\rho_c \sim 1.7 \times 10^{-29}\text{g/cm}^3$ (1 Mpc (mega-parsec) = 3 x 10^{22} meters). This allows us to define a dimensionless parameter $\Omega$ which corresponds to the ratio of the energy density of a given system to the critical density: $\Omega_i \equiv \rho_i/\rho_{\text{critical}}$, with the index $i$ labelling the different contributions to the energy density, and the total ratio is $\Omega = \sum_i \Omega_i$. With these definitions we can write the first Friedmann equation as: $\Omega = 1 + kH^2a^2$. From this we can see the clear connection between the curvature of the spatial sections given by $k$ and the departure from the early universe. A flat universe ($k = 0$) corresponds to a critical density ($\Omega = 1$) whereas open ($k = -1$) and closed ($k = 1$) universes correspond to $\Omega < 1$ and $\Omega > 1$ respectively.

With all this information in mind, we just assume that the early universe corresponds to an expanding gas of particles and, with the input of the standard model of particle physics, and some thermodynamics as long as gravity is weak we can safely define the concept of thermal equilibrium and therefore a temperature, to can trace the evolution of the system. We present in table 1 some of the important points through the evolution and refer to the standard literature for details Rev. There are few things to keep in mind: the gas is considered to be in equilibrium. The main two reasons for a particle to leave equilibrium is that its mass threshold is reached by the effective temperature of the universe and so it is easier for this particle to annihilate with its antiparticle than being produced again, since, as the universe cools down, there is not enough energy to produce such a heavy object. Also, if the expansion rate of the relevant reactions $\Gamma$ is smaller than the expansion rate of the universe, measured by $H$, some particles also get out of equilibrium. For instance, at temperatures above 1 MeV the reactions that keep neutrinos in equilibrium are faster than the expansion rate but at this temperature $H \geq \Gamma$ and they decouple from the plasma, leaving then an observable, in principle, trace of the very early universe. Unfortunately we are very far from being able to detect such radiation.

before a temperature, w
At the atomic physics scale, the universe is cold enough for atoms to be formed and the photons are out of equilibrium, giving rise to the famous cosmic microwave background. At approximately the same time also the universe changes from being radiation dominated ($w = 1/3$) to matter dominated ($w = 0$). After this, the formation of structures such as clusters and galaxies can start, probably due to the quantum fluctuations of the early universe, leading to our present time.

1.7 tabular—c—c—c—c— Temperature Time Particle Physics Cosmological Event

The standard cosmological model has strong experimental evidence which can be summarised as follows:

1. The original observation of Hubble and Slipher at the beginning of the 20th century, that the galaxies are all separating from each other, at a rate that is roughly proportional to the separation, is clearly realised for $H$ approximately constant at present, $H = H_0 > 0$. This has been overwhelmingly verified during the past few decades.

2. The relative abundance of the elements with approximately 75% Hydrogen almost 24% Helium, and other light elements such as Deuterium $D$ and helium-4 $^4He$, with small fractions of a percent, is a big success of nucleosynthesis, and at present is the farther away in the past that we have been able to compare theory and observation.

3. The discovery of the cosmic microwave background, signalling the time of last photon scattering, by Penzias and Wilson in 1964 was perhaps the most spectacular test of the model. Starting in the 1990’s with the discoveries of the COBE satellite and more recent balloon experiments such as BOOMERANG, Maxima and DASI, cosmology has been brought to the status of precision science. In particular, the confirmation of the black body nature of the CMB is known with excellent precision, but, more importantly, the fluctuations in the temperature $\delta TT$ signalling density fluctuations $\delta \rho\rho$ in the early universe provide a great piece of information about the possible microscopic origin of the large scale structure formation. The temperature fluctuations are analysed in terms of their spherical harmonics decomposition $\delta TT = \sum_{lm} a_{lm} Y_{lm}(\theta, \phi)$, with the power spectrum $C_l = \langle |a_{lm}|^2 \rangle$ showing a peak structure (see figure 2). The higher the multi-pole moment the smaller angular separation in the sky. The location and height of the peaks provides precise information about the fundamental parameters of FRW cosmology, such as $\Omega$, $\Omega_{baryon}$, the cosmological constant, etc. See for instance kamkos,triangle. In particular the first peak being at approximately $l = 200$ provides a very strong evidence in favour of a flat universe.