Hyperon Polarization from Unpolarized \( pp \) and \( ep \) Collisions

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Abstract: Cross section formulas for the \( \Lambda \) polarization in \( pp \rightarrow \Lambda^\uparrow(\ell_T)X \) and \( ep \rightarrow \Lambda^\uparrow(\ell_T)X \) are derived and its characteristic features are discussed.

In this report we discuss the polarization of \( \Lambda \) hyperon produced in unpolarized \( pp \) and \( ep \) collisions relevant for the ongoing RHIC-SPIN, HERMES and COMPASS experiments. According to the QCD factorization theorem, the polarized cross section for \( pp \rightarrow \Lambda^\uparrow X \) consists of two twist-3 contributions:

\[
(A) \quad E_a(x_1, x_2) \otimes q_b(x') \otimes \delta \hat{q}_c(z) \otimes \hat{\sigma}_{ab \rightarrow c},
\]

\[
(B) \quad q_a(x) \otimes q_b(x') \otimes \hat{G}_c(z_1, z_2) \otimes \hat{\sigma}'_{ab \rightarrow c},
\]

where the functions \( E_a(x_1, x_2) \) and \( \hat{G}_c(z_1, z_2) \) are the twist-3 quantities representing, respectively, the unpolarized distribution in the nucleon and the polarized fragmentation function for \( \Lambda^\uparrow \). \( \delta \hat{q}_c(x) \) is the transversity fragmentation function for \( \Lambda^\uparrow \).

The \( A \) contribution for \( pp \rightarrow \Lambda^\uparrow X \) has been analyzed in \([1]\), where it was shown that \( A \) gives rise to growing \( P_{\Lambda} \) at large \( x_F \) as observed experimentally. Here we extend the study to the \( B \) term (see also \([2]\)) at RHIC energy and also for the \( ep \) collision.

The unpolarized twist-3 distribution \( E_{F,D}(x_1, x_2) \) is defined in \([1]\). Likewise the twist-3 fragmentation function for a polarized \( \Lambda \) (with momentum \( \ell \)) is defined as the lightcone correlation function as \((w^2 = 0, \ell \cdot w = 1)\)

\[
\frac{1}{N_c} \sum_X \int \frac{d\lambda}{2\pi} \int \frac{d\mu}{2\pi} e^{-iw(\lambda + \mu)} \langle 0|\psi_i(0)|\pi X\rangle \langle \pi X|gF^\alpha\beta(\mu w)w_\beta\bar{\psi}_j(\lambda w)|0\rangle
\]

\[
= \frac{M_N}{2z_2} \langle \ell \rangle_{ij} e^{a_\ell w_{\perp}} \hat{G}_F(z_1, z_2) + \frac{iM_N}{2z_2} \langle \gamma_5\ell \rangle_{ij} \gamma^a S_{\perp} \hat{G}_F(z_1, z_2) + \cdots. \tag{1}
\]
Note that we use the nucleon mass $M_N$ to normalize the twist-3 fragmentation function for $\Lambda$. There is another twist-3 fragmentation functions which are obtained from (1) by shifting the gluon-field strength from the left to the right of the cut. The defined functions $\tilde{G}_{FR}(z_1, z_2)$ and $\tilde{G}_{FR}^\alpha(z_1, z_2)$ are connected to $\tilde{G}_{F}(z_1, z_2)$ by the relation $\tilde{G}_{F}(z_1, z_2) = \tilde{G}_{FR}(z_2, z_1)$ and $\tilde{G}_{FR}^\alpha(z_1, z_2) = -\tilde{G}_{FR}^\alpha(z_2, z_1)$, which follows from hermiticity and time reversal invariance. Unlike the twist-3 distributions, the twist-3 fragmentation function does not have definite symmetry property. Another class of twist-3 fragmentation functions $\tilde{G}_{D}^\alpha(z_1, z_2)$ is also defined from (1) by replacing $gF^\alpha\beta(\mu w)w_\beta$ by $D^\alpha(\mu w) = \partial^\alpha - igA(\mu w)$. Note, however, this is not independent from the above (1).

Following the method of [3] we present the analysis of the (C) term. The detailed analysis shows $\tilde{G}_{F}(z, z)$ as soft-gluon-pole contribution ($z_1 = z_2 = z$), while $\tilde{G}_{D}(z_1, z_2)$ appears as a soft fermion pole ($z_1 = 0$ or $z_2 = 2$). Physically, the latter is expected to be suppressed, and we include only the former contribution. This observation also applies to $E_{F,D}(x_1, x_2)$ relevant for the (A) term. In the large $x_F$ region, the main contribution comes from large-$x$ and large-$z$ (and small $x'$) region. Since $E_F$ and $\tilde{G}_{F}$ behaves as $E_F(x, x) \sim (1 - x)\beta$ and $\tilde{G}_{F}(z, z) \sim (1 - z)\beta'$ with $\beta, \beta' > 0$, $\left| (d/dx)E_F(x, x) \right| \gg \left| E_F(x, x) \right|$, $\left| (d/dz)\tilde{G}_{F}(z, z) \right| \gg \left| \tilde{G}_{F}(z, z) \right|$ at large $x$ and $z$. In particular, the valence component of $E_F$ and $\tilde{G}_{F}$ dominates in this region. We thus keep only the valence quark contribution for the derivative of these soft-gluon pole function (“valence-quark soft-gluon approximation”) for the $pp$ collision. For the $ep$ case, we include all the soft-gluon pole contribution, since the calculation is relatively simple compared to the $pp$ case.

In general $P_\Lambda$ is a function of $S = (P + P')^2 \simeq 2P \cdot P'$, $T = (P - \ell)^2 \simeq -2P \cdot \ell$ and $U = (P' - \ell)^2 \simeq -2P' \cdot \ell$ where $P$ and $P'$ are the momenta of the two nucleons, and $\ell$ is the momentum of $\Lambda$. In the following we use $S, x_F = \frac{2k_{B}}{\sqrt{S}} = \frac{T-U}{S}$ and $x_T = \frac{2k_{T}}{\sqrt{S}}$ as independent variables. The polarized cross section for the (B) term reads

$$E_{\Lambda} \frac{d^3\Delta \sigma(S_\perp)}{d^3l} = \frac{2\pi M_N\alpha_s^2}{S} \sum_{a} \int_{z_{\min}}^{1} \frac{dz}{z^2} \int_{x_{\min}}^{1} \frac{dx}{x} \int_{0}^{1} \frac{dx'}{x'} \delta \left( x' + \frac{xT}{xS + U} \right) \times \delta \left( x' + \frac{xT}{xS + U} \right) \times \left\{ \sum_{b,c} q^a(x) q^b(x') \left[ -z_1^2 \frac{\partial}{\partial z_1} \tilde{G}_{F}^a(z_1, z) \right] \right\}_{z_1 = z} \left( \frac{-2p_{\alpha}}{T} \tilde{\sigma}_{ab\rightarrow c} + \frac{-2p'_{\alpha}}{U} \hat{\sigma}_{ab\rightarrow c} \right) \right.
$$

$$+ \sum_{b,c} q^a(x) q^b(x') \left[ -z^2 \frac{d}{dz} \tilde{G}_{F}^a(z, z) \right] \frac{xp_{\alpha} + xp_{\alpha}'}{|xT + x'U|} \left( \tilde{\sigma}_{ab\rightarrow c} + \hat{\sigma}_{ab\rightarrow c} \right)$$
\[ +q(x)G(x)\left[-z^2 \frac{\partial}{\partial z} \tilde{G}_F^a(z, z)\right]_{z_1=\tau} \left(\frac{-2p_s}{T} \bar{\sigma}_{ag-a} + \frac{-2p_s}{U} \bar{\sigma}_{ag-a}^{II}\right) \]

\[ +q(x)G(x)\left[-z^2 \frac{d}{dz} \tilde{G}_F^a(z, z)\right] \frac{xp_s + x'p_s'}{x'T + x'U} \left(\bar{\sigma}_{ag-a} + \bar{\sigma}_{ag-a}^{II}\right), \]  

(2)

where the lower limits for the integration variables are \(z_{\text{min}} = -(T + U)/S = \sqrt{x_T^2 + x_T^3}\) and \(x_{\text{min}} = -U/z(S + T/z)\). The partonic hard cross sections are written in terms of the invariants in the parton level, \(s = (xp + x'p')^2 = xx'S\), \(t = (xp - \ell/z)^2 = xT/z\) and \(\hat{u} = (x'p' - \ell/z)^2 = x'U/z\). They read

\[
\bar{\sigma}_{ab-c}^{I} = -\frac{1}{36} \frac{s^2 + \hat{u}^2}{t^2} \delta_{ab} + \frac{7}{36} \frac{s^2 + \hat{u}^2}{\hat{u}^2} \delta_{bc} + \frac{1}{54} \frac{s^2}{\hat{u}^2} \delta_{ab} \delta_{ac},
\]

\[
\bar{\sigma}_{ab-c}^{II} = \frac{7}{36} \frac{s^2 + \hat{u}^2}{t^2} \delta_{ab} - \frac{1}{36} \frac{s^2 + \hat{u}^2}{\hat{u}^2} \delta_{bc} + \frac{1}{54} \frac{s^2}{\hat{u}^2} \delta_{ab} \delta_{ac},
\]

\[
\hat{\sigma}_{ab-c}^{I} = -\frac{1}{36} \frac{s^2 + \hat{u}^2}{t^2} \delta_{ab} + \frac{7}{36} \frac{\hat{u}^2 + \hat{t}^2}{s^2} \delta_{ac},
\]

\[
\hat{\sigma}_{ab-c}^{II} = \frac{1}{18} \frac{s^2 + \hat{u}^2}{\hat{t}^2} \delta_{ab} + \frac{1}{18} \frac{\hat{u}^2 + \hat{t}^2}{s^2} \delta_{ac},
\]

\[
\hat{\sigma}_{ag-q}^{I} = -\frac{1}{8} \left(1 \frac{s + \hat{u}}{\hat{t}^2}\right) + \frac{1}{288} \left(-\frac{\hat{u}}{\hat{s}} + \frac{s}{\hat{u}}\right) - \frac{\hat{s}}{16\hat{t}} - \frac{\hat{u}}{16\hat{t}},
\]

\[
\hat{\sigma}_{ag-q}^{II} = \frac{9}{16} \left(1 - \frac{s + \hat{u}}{\hat{t}^2}\right) + \frac{\hat{u}}{32\hat{s}} - \frac{s}{4\hat{u}} + \frac{9}{16}\frac{\hat{u}}{\hat{t}}.
\]  

(3)

Among these partonic cross sections, \(\hat{\sigma}^I\) becomes more important at large \(x_F\) because of the \(1/T\) factor in (2).

To estimate the above contribution, we introduce a model ansatz as \(\tilde{G}_F^a(z, z) = K_a \bar{q}_a(z)\) with twist-2 unpolarized fragmentation function \(\bar{q}_a(z)\), noting that the Dirac structure of \(G_F^p(z, z)\) and \(\bar{q}_a(z)\) is the same [3]. \(K_a\)'s are taken to be \(K_u = -K_d = 0.07\) which are the same values used in the relation \(G_F(x, x) = K_a q_a^a(x)\) to reproduce \(A_{N\pi}\) in \(p/p \rightarrow \pi X\) observed at E704 [4]. As noted before, \(G_F(z_1, z_2)\) does not have definite symmetry property unlike the twist-3 distribution \(E_F(x_1, x_2)\). Nevertheless we assume \([\partial/\partial z_1 \bar{E}_F(z_1, z\frac{\partial}{\partial z})\]_{z_1=z} = (1/2)(d/dz) \(\bar{E}_F(z, z)\). The result for the \(\Lambda\) polarization \(P_{\Lambda}^{pp}\) at \(\sqrt{S} = 62\) GeV is shown in Fig. 1 together with the R608 data. There (A) (chiral-odd) contribution studied in [1] is also shown for comparison. (For the adopted distribution and fragmentation functions, see [1].) One sees that the tendency of \(P_{\Lambda}^{pp}\) from the (B)(chiral-even) contribution is quite similar to the R608 data. Rising behavior of \(P_{\Lambda}^{pp}\) at large \(x_F\) comes from (i) the large partonic cross sections in (3) \((\sim 1/\hat{t}^2\) term\) and (ii) the derivative of the soft-gluon pole functions. With these parameters \(K_a\), \(P_{\Lambda}^{pp}\) at RHIC energy \((\sqrt{S} = 200\) GeV) is shown in Fig. 2 at \(l_T = 1.5\) GeV. Fig. 3 shows the \(l_T\) dependence of \(P_{\Lambda}^{pp}\) of
the (B) term, indicating large $\ell_T$ dependence at $1 \leq \ell_T \leq 3$ GeV. Experimentally, $P^{pp}_\Lambda$ grows up as $\ell_T$ increases up to $\ell_T \sim 1$ GeV and stays constant at $1 \leq \ell_T \leq 3$ GeV. So the $P^{pp}_\Lambda$ observed at R608 can not be wholly ascribed to the twist-3 effect studied here which is designed to describe large $\ell_T$ polarization.

![Figure 1: $P^{pp}_\Lambda$ at $\sqrt{S} = 62$ GeV.](image1)

![Figure 2: $P^{pp}_\Lambda$ at $\sqrt{S} = 200$ GeV.](image2)

![Figure 3: $\ell_T$ dependence of $P^{pp}_\Lambda$.](image3)

![Figure 4: $P^{ep}_\Lambda$ at $\sqrt{S} = 20$ GeV.](image4)

Using the twist-3 distribution and fragmentation functions used to describe $P^{pp}_\Lambda$, we show in Fig. 4 the obtained $P^{ep}_\Lambda$ corresponding to (A')(chiral-odd) and (B')(chiral-even) contributions. Remarkable feature of Fig. 4 is that in both chiral-even and chiral-odd contributions (i) the sign of $P^{ep}_\Lambda$ is opposite to the sign of $P^{pp}_\Lambda$ and (ii) the magnitude of $P^{ep}_\Lambda$ is much larger than that of $P^{pp}_\Lambda$, in particular, at large $x_F$, and it even overshoots one. (In our convention, $x_F > 0$ corresponds to the production of $\Lambda$ in the forward hemisphere of the initial proton in the $ep$ case.) The origin of these features can be traced back to the color factor in the dominant diagrams for the twist-3 polarized cross sections in $ep$ and $pp$ collisions.
Of course, the $P_\Lambda$ can not exceeds one, and thus our model estimate needs to be modified. First, the applied kinematic range of our formula should be reconsidered: Application of the twist-3 cross section at such small $\ell_T$ may not be justified. Second, our simple model ansatz of $E_F^a(x,x) \sim \delta q^a(x)$ (in (A) term) and $\hat{G}_F^a(z,z) \sim \hat{q}^a(z)$ should be modified at $x \to 1$ and $z \to 1$, respectively. The derivative of these functions, which is important for the growing $P_{ppp}$ at large $x_F$, eventually leads to divergence of $P_\Lambda$ at $x_F \to 1$ as $\sim 1/(1 - x_F)$.

As a possible remedy for this pathology we tried the following: As an example for the (B) (chiral-even) contribution we have a model $\hat{G}_F^a(z,z) \sim \hat{q}^a(z) \sim z \to 1 (1 - z)^\beta$ where $\beta = 1.83$ in the fragmentation function we adopted. Tentatively we shifted $\beta$ as $\beta \to \beta(z) = \beta + z^\delta$, which suppresses the divergence of $P_\Lambda$ at $x_F \to 1$ but still keeps rising behavior of $P_\Lambda$ at large $x_F$. This avoids overshooting of one in $P_{ep}$ but reduces $P_{pp}$ seriously. The result obtained by this modification is shown in Figs. 5 and 6.

To summarize we have studied the $\Lambda$ polarization in $pp$ and $ep$ collisions in the framework of collinear factorization. Our approach includes all effects for the large $\ell_T$ production. One needs to be cautious in interpreting the available $pp$ data at relatively low $\ell_T$ in terms of the derived formula. Determination of the participating twist-3 functions requires global analysis of future $pp$ and $ep$ data.

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References