DETERMINING TANGENTIAL PECULIAR VELOCITIES OF CLUSTERS OF GALAXIES USING GRAVITATIONAL LENSING

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ABSTRACT

We propose two new methods for measuring tangential peculiar velocities of rich clusters of galaxies. Our first method is based on weak gravitational lensing and takes advantage of the differing images of background galaxies caused by moving and stationary gravitational potentials. Our second method is based on measuring relative frequency shifts between multiple images of a single strongly lensed background galaxy. We illustrate this method using the example of galaxy cluster CL 0024+1654.

Subject headings: galaxies: clusters: general– gravitational lensing

1. INTRODUCTION

The distribution of peculiar velocities of galaxies and clusters of galaxies is sensitive to the overall matter density in the Universe on large scales, and can be used to study structure formation. Peculiar velocities of galaxies have been used to reconstruct matter density fields (Colberg et al. 2000; Dekel & Ostriker 1999; Peacock 1999). Peculiar velocities of clusters could be used to determine large-scale bulk motion (Kashlinsky & Atrio-Barandela 2000) and check the reconstructed matter density fields. Direct methods to measure peculiar velocities of clusters of galaxies suggested so far are based on the frequency changes that moving clusters generate in the background photons of the CMBR due to inverse Compton scattering or gravitational lensing.

Measurements of cluster radial peculiar velocities are based on inverse Compton scattering of CMBR photons off hot electrons in the intra-cluster gas, the Sunyaev-Zel’dovich effect (Sunyaev & Zel’dovich 1980). On average the CMBR photons gain energy from this scattering and a static and stationary cluster induces a decrement in the Rayleigh-Jeans part of the spectrum (the static thermal SZ effect). If the cluster is moving relative to the CMBR, an additional kinematic SZ effect is generated because of the bulk motion of the intra-cluster gas. The ratio of the kinematic to static thermal SZ effect, \( \Delta T_{\text{KSZ}}/\Delta T_{\text{SSZ}} \), in the Rayleigh-Jeans spectral region may be expressed as

\[
\frac{\Delta T_{\text{KSZ}}}{\Delta T_{\text{SSZ}}} = 0.085 \left( \frac{v_r}{1000 \text{ km s}^{-1}} \right) \left[ kT_e/10 \text{ keV} \right]^{-1},
\]

where \( v_r \) is the radial peculiar velocity, \( T_e \) is the electron temperature, and \( k \) is the Boltzmann constant, so that in moderate- or high-temperature clusters of galaxies the static effect will dominate. The kinematic SZ effect can be separated from the static effect using its different frequency distribution, even though their spatial distribution is the same (for recent reviews see Birkinshaw 1999 and Rephaeli 1995). The accuracy to which this radial velocity effect can be measured in a cosmological context was analyzed by Aghanim, Górski & Puget (2001). At present only upper limits exist on radial velocities of clusters based on the kinematic SZ effect. Holzapfel et al. (1997) determined limits of peculiar radial velocities for two clusters, A2163 and A1689, with \( \pm 1 \sigma \) ranges: \(-390 \text{ km s}^{-1} \leq v_r \leq 1860 \text{ km s}^{-1} \) and \(-460 \text{ km s}^{-1} \leq v_r \leq 985 \text{ km s}^{-1} \).

Tangential peculiar velocities of clusters can be determined using the moving cluster effect (Birkinshaw & Gull 1983; see also Pyne & Birkinshaw 1993 and Gurvits & Mitrofanov 1986). This effect is due to the fact that gravitational bending of light deforms the dipolar CMBR temperature anisotropy in the rest frame of the cluster, and thus when this pattern is transformed back to the CMBR rest frame (the observer’s frame) we do not recover the original, uniform, CMBR distribution. This effect is only of order \( 5 \mu \text{K} \), far less than the static or kinematic SZ effects from prominent clusters (400 \( \mu \text{K} \) and 50 \( \mu \text{K} \)). Measurements of such small fluctuations in the CMBR on the few arcminute scale of clusters is beyond our current capability, and so no attempt has yet been made to measure this effect. Sensitivity improvements of a factor \( \sim 100 \) are needed if this effect is to be measured, and this will be difficult in the presence of primary CMBR structures with the same spectrum.

In this letter, we propose two new methods for measuring tangential peculiar velocities of rich clusters of galaxies without recourse to CMBR observations. Our first method is based on weak gravitational lensing. The distortions of images of background objects caused by a moving gravitational lens are different from those caused by a static lens. We calculate the resulting weak lensing signal and discuss how this effect could be used to measure tangential velocities of clusters using current technology. Our second method is based on frequency shifts generated by moving gravitational lenses, but instead of using the CMBR as a reference, we suggest using multiple images of a single strongly lensed background galaxy. We illustrate this second method using the example of galaxy cluster CL 0024+1654.

2. THEORY

In the following derivation we use the thin lens approximation for gravitational lensing and assume that the deflection angle is small (\( \ll 1 \) radian). This approximation is well justified for gravitational lensing caused by most clusters of galaxies (see for example Schneider, Ehlers & Falco 1992). In this approximation, the lens equation becomes

\[
\theta = \theta^s + \alpha,
\]

where \( \theta^s \) and \( \theta \) are the source and image coordinates, and \( \alpha \) is the observer’s frame bend angle (Schneider et al. 1992). We choose coordinate axes \( \mathbf{x} \) and \( \mathbf{y} \) to lie in the plane of the sky. In the rest frame of the cluster, the bending angle is
\( \delta(x, y) \equiv (\delta_x, \delta_y) = \nabla \psi(x, y), \) \hfill (3)

where \( x \) and \( y \) are angular coordinates in the sky and \( \psi(x, y) \) is the 2-dimensional gravitational deflection potential. The change in the photon four-momentum in the observer’s frame is

\[
\Delta = \left[ L^{-1}(\beta) R(\delta) L(\beta) - I \right] \gamma^{in} = D \gamma^{in}; \hfill (4)
\]

where \( L, R, I \) and \( D \), are the four-dimensional Lorentz transformation, rotation (caused by gravitational lensing), identity and distortion matrices, \( \gamma^{in} \) is the incoming photon four-momentum, and \( \beta \equiv v/c \) is the peculiar velocity of the moving lens in units of the speed of light, \( c \).

First we briefly discuss the case when the gravitational lens is moving radially, i.e., \( \beta = v_T/c \). In this case, for small bending angles and peculiar velocities (\( \delta \ll 1 \) and \( \beta \ll 1 \)), from Equation (4), we obtain the deflection angle

\[
\alpha = (1 + \beta) \delta. \hfill (5)
\]

This result agrees with the results of Frittelli, Kling & Newman (2002; Equation 99).

In theory this correction to the bending angle should be taken into account when using weak lensing to determine masses of clusters of galaxies. Cluster masses are determined from the bending angle. If the lens is moving radially, the bending angle changes because of its radial velocity, and the correct cluster mass, \( M \), is related to the mass determined by ignoring the radial motion of the lens, \( M_0 \), by

\[
M = \frac{M_0}{1 + \beta}. \hfill (6)
\]

However, since we expect that the peculiar velocities of clusters are less than 3000 km s\(^{-1}\), this effect is less than one percent, and is negligible relative to other errors even if we take it into account that linear structure formation theories underestimate peculiar velocities of clusters by about 40\% (see Colberg et al. 2000). Therefore we do not discuss this case further.

We now turn to the effect caused by tangential peculiar velocities. For simplicity, we assume that the peculiar velocity of the gravitational lens, \( \beta = v_T/c \), is entirely in the plane of the sky. We choose \( \mathbf{x} \) parallel to \( \beta \). The bending angle in the observer’s frame can be read from the spatial components of \( \Delta \) (Equation 4, assuming \( \delta \ll 1 \) and \( \beta \ll 1 \)) as

\[
\alpha = \begin{pmatrix} \gamma & 0 \\ 0 & 1 \end{pmatrix} \delta, \hfill (7)
\]

where \( \gamma = 1/\sqrt{1 - \beta^2} \). This gives \( \alpha = \delta \) when \( \beta = 0 \), as expected. The Jacobian, \( J \), of the transformation from the source to the image plane can be seen from Equations (2) and (7) to be

\[
J = J^{2D} + D^{2D}, \hfill (8)
\]

where \( J^{2D} \) is the two-dimensional identity matrix, and the two-dimensional matrix

\[
D^{2D} \equiv \begin{pmatrix} \psi_{xx} & \psi_{xy} \\ \psi_{yx} & \psi_{yy} \end{pmatrix}, \hfill (9)
\]

(where \( \psi_{ij} \) are second partial derivatives of the deflection potential with respect to variables \( i \) and \( j \)) describes the distortion. The distortion matrix can be decomposed as

\[
\mathcal{D}^{2D} = \left( \begin{array}{cc} -\kappa - \hat{\gamma}_1 & \hat{\gamma}_2 \\ \hat{\gamma}_2 & -\kappa + \hat{\gamma}_1 \end{array} \right) + \mathcal{R}^{2D}(\varphi_0), \hfill (10)
\]

where the convergence, \( \kappa \), and shear, \( (\hat{\gamma}_1, \hat{\gamma}_2) \), are determined by the deflection potential, \( \psi \), and the second term, \( \mathcal{R}^{2D}(\varphi_0) \), is a rotation of the source image by an unmeasurable orientation angle \( \varphi_0 \), from the arbitrary assignment of axes \( \mathbf{x} \) and \( \mathbf{y} \) (see for example Peacock 1999). From Equations (9) and (10) we conclude that the shear is characterized by the two-dimensional vector field, \( \hat{\gamma} \), with components

\[
\hat{\gamma}_1 = \frac{1}{2} \left[ -\gamma \psi_{xx} + \psi_{yy} \right], \hfill (11)
\]

\[
\hat{\gamma}_2 = \frac{1}{2} \left[ \gamma \psi_{xy} + \psi_{yx} \right]. \hfill (12)
\]

Whenever these second derivatives of the deflection potential exist, we should be able to determine the tangential velocity (which enters here through \( \gamma \)) and the deflection potential separately since the potential is a scalar, continuous field. Unless the deflection potential is such that \( \partial^2 (\nabla^2 \psi) / \partial x \partial y \) vanishes identically, \( \gamma \) can be determined as

\[
\gamma = \frac{\hat{\gamma}_{2,xy} - \hat{\gamma}_{1,xy}}{\hat{\gamma}_{2,xx} + \hat{\gamma}_{1,xy}}, \hfill (13)
\]

where \( \hat{\gamma}_{1,ij} \) and \( \hat{\gamma}_{2,ij} \) are the double partial derivatives of \( \hat{\gamma}_1 \) and \( \hat{\gamma}_2 \).

The frequency shift in the spectrum of a lensed background galaxy caused by the gravitational field of a tangentially moving cluster of galaxies can be read from the time component of the four-vector \( \Delta \) (Equation 4, again assuming \( \delta \ll 1 \) and \( \beta \ll 1 \)) as

\[
\frac{\Delta \nu}{\nu_0} = \gamma \beta \delta_1, \hfill (14)
\]

where \( \beta = v_T/c \), and \( \delta_1 \) is the \( x \) component of the rest frame gravitational bend angle (Equation 3). In the case of a gravitational potential with spherical symmetry, this reduces to

\[
\frac{\Delta \nu}{\nu_0} = \gamma \beta \delta \cos \varphi, \hfill (15)
\]

where \( \varphi \) is the angle measured from the direction of the tangential velocity of the moving lens, which is identical to the expression derived by Birkinshaw & Gull (1983, their Equation 9) and to first order in \( \beta \), to Equation (2) of Gurvits & Mitrofanov (1986).

Since there is a time delay between the two images of the strong lensed background galaxy, there is a small additional redshift difference, \( \Delta z \), between the lines of the two images due to the expansion of the Universe. This can be approximated as

\[
\Delta z = (1 + z_{\text{los}}) H(z_{\text{los}}) \Delta t, \hfill (16)
\]

where \( H(z_{\text{los}}) \) is the Hubble parameter evaluated at the redshift of the lens, \( z_{\text{los}} \), and \( \Delta t \) is the time delay difference between the two images. However, for a typical cluster, this redshift, \( \Delta z \approx 5 \times 10^{-11} (1 + z_{\text{los}}) (H_0/50 \text{ km s}^{-1} \text{ Mpc}^{-1}) (\Delta t/1 \text{ yr}) \), yields a velocity difference only of order 20 m s\(^{-1}\) and is negligibly small. The frequency shift in Equation (14) is not measurable unless one can find a reference frame, or measure differences in frequency shifts in multiple images of a single background object.
3. DETERMINING TANGENTIAL VELOCITIES

3.1. Weak Lensing

We carried out simulations to quantify the observability of the tangential peculiar motion on the shear field as revealed by images of background galaxies (Equations 11 and 12). The shear parameter, \( \gamma \), is directly related to the measurable ellipticity parameter, \( \epsilon = (\epsilon_1, \epsilon_2) \), \( \gamma \approx \epsilon \), since \( \kappa \ll 1 \). The ellipticity parameter as a function of position can be directly measured from the images of lensed galaxies using their isophotes (e.g., Schneider et al. 1992). We assumed that ellipsitcity could be measured for 40 galaxies per arcmin\(^2\), over an area of \( 50 \times 50 \) arcmin\(^2\), and that the ellipticity parameters of the background galaxies can be measured with 20% error. We average over 0.5 arcmin\(^2\) cells, containing 20 galaxies each to obtain 5% ellipticity measurements in \( N = 5000 \) samples. We choose an elliptical gravitational deflection potential, \( \psi \), which approximates the potential of a singular isothermal mass distribution at large radii (see for example Schneider and Bartelmann 1997)

\[
\psi = \psi_0 \left[ \frac{r^2}{r_{\text{core}}^2} + \frac{x^2}{(1-\eta)^2} + \frac{y^2}{(1+\eta)^2} \right]^\frac{1}{2},
\]

with \( \eta = 0.1 \), \( r_{\text{core}} = 1' \), and \( \psi_0 = 2.455 \times 10^{-3} \) to show that the shape parameters of the potential \( \psi_0, r_{\text{core}}, \eta \) and the tangential peculiar velocity of the cluster decouple, i.e., can be determined separately. We evaluated the errors using the \( \Delta \chi^2 \) statistic, where we minimize

\[
S(\beta, \psi_0, r_{\text{core}}, \eta) = \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \frac{[\gamma_i(x_i, y_i) - \epsilon_j(x_j, y_j)]^2}{\sigma_{ij}^2}. \quad (18)
\]

\( \gamma \) is calculated using Equations (11) and (13), and the ellipticity parameters, \( \epsilon \) are obtained by means of Monte Carlo simulations assuming errors \( \sigma_{ij} \). \( S \) should be distributed like \( \Delta \chi^2 \) with \( 2N-2 \) degrees of freedom. Our simulations show that \( S \) is a non-degenerate four-dimensional ellipsoid near its minimum, so that \( \beta, \psi_0, r_{\text{core}}, \eta \) are independently determined, and that a tangential peculiar velocity of \( v_T = 1500 \) km s\(^{-1}\) can be measured to \( \pm 300 \) km s\(^{-1}\) (1\( \sigma \), see Figures 1 and 2).

3.2. Frequency Shift

The method of measuring the tangential peculiar velocity of a cluster of galaxies proposed by Birkinshaw & Gull (1983) and corrected by Gurvits & Mitrofanov (1986) and Pyne & Birkinshaw (1993) relied on observing the brightness change of the CMBR arising from the frequency effect, Equation (15). However, it would also be possible to measure this frequency effect directly if there were a background with a sharp line feature. Such a background is available in the case of strongly-lensed multiple images of a single background galaxy.

We illustrate this method using the well-studied, rich cluster of galaxies, CL 0024+1654 (Shapiro & Iliev 2000; Tyson et al. 1998; Dressler, Gunn & Schneider 1985), at a redshift of 0.395, which produces multiple images of a single background located at a redshift of 1.675 (Broadhurst et al. 2000). The surface mass density of the cluster was found to be

\[
\Sigma(r) = K \left[ 1 + \frac{r^2}{r_{\text{core}}^2} \right] \left[ 1 + \frac{r^2}{r_{\text{core}}^2} \right]^{-\frac{1}{2}},
\]

with \( K = 7900 M_{\odot} \text{ pc}^{-2} \), \( r_{\text{core}} = 35 \) kpc, and \( \eta = 0.57 \) (Shapiro & Iliev 2000). The deflection angle for a spherical potential can be expressed as

\[
\delta(r) = \frac{4GM(r)}{c^2 r},
\]

where \( M \) is the mass included within cylindrical radius, \( r \). The relative frequency shift between arcs in the direction of, and opposite to the direction of tangential motion of the cluster becomes twice as large as the single-image frequency shift in Equation (14). Assuming that the tangential velocity \( v_T = 1000 \) km s\(^{-1}\), and is aligned with two of the lensed images, we obtain a frequency shift in emission (or absorption) lines equivalent to a Doppler shift of 1 km s\(^{-1}\). To measure this effect we must find narrow lines from the same spatial location of the imaged background galaxy in each arc, and measure the frequency difference between them. Narrow forbidden emission lines of [OIII] at \( \lambda \lambda 4959, 5007 \), [OIII] at \( \lambda \lambda 3727, 3729 \), and [NII] at \( \lambda 6584 \), shifted into the infra-red (IR) band should be suitable.

High signal/noise spectra of such lines should be able to fix their central wavelengths to a few per cent of their widths. The precision of the measurement will be dependent on the stability of the spectrograph (which is better than 1 km s\(^{-1}\) for stable
systems such as Phoenix on Gemini South; Hinkle et al. 2000) and the intrinsic widths of the lines being used. Narrow-line emitting regions within galaxies should have line widths of under 50 km s$^{-1}$, and so spectra with signal/noise $10^3$ would be needed to fix the line centroids precisely enough to measure the cluster tangential velocity to 1000 km s$^{-1}$ or better. Such a high signal/noise cannot currently be achieved in the near IR at the magnitudes of interest for CL 0024+1654, but would be possible with 50-m class telescopes.

4. CONCLUSION

In this letter we proposed two new methods for measuring tangential peculiar velocities of clusters of galaxies. Our first method is based on weak gravitational lensing. We carried out simulations to estimate how well we could determine tangential velocities of clusters from distortions of the shapes of background lensed galaxy images. We assumed that the ellipticities of background galaxies can be measured with 20% error. With today's technology, obtaining such a large-scale and accurately measured shear field is possible only for a limited area. In order to measure this effect one would need to reduce the scatter in the intrinsic ellipticities of the background galaxies so that this noise source does not swamp the effect of tangential motion. One possible way of doing this would be to select the spherical components of lensed background galaxies, perhaps observing the bulge components in the IR for these galaxies using next generation space telescopes.

Our second method is based on measuring frequency shifts between multiple images of a single strongly lensed background galaxy. This method allows us to measure components of tangential velocities of clusters in the direction of pairs of images of strongly-lensed galaxies by measuring their relative redshift. We used cluster CL 0024+1654 to demonstrate that the needed velocity resolution is achievable with present-day spectrographs. However, the signal/noise requirements are not achievable with existing near-IR instruments and telescopes, with improvements of a factor 10 in throughput needed to make the measurement possible in this band.

We might expect clusters of galaxies situated in superclusters to show the largest tangential velocities, since these clusters seem to move faster than "field clusters" (Colberg et al. 2000). While the differential redshift technique should still be feasible for such clusters, the shear technique may be compromised by the presence of the large-scale shear field of the supercluster.

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