Matching the Observed Star Formation Intensity Distribution with Empirical Laws

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ABSTRACT

This letter matches the shape of the star formation intensity distribution function to empirical laws such as the Schmidt law. The shape of the distribution at a redshift of one is reproduced from the empirical Schmidt law with a critical density, a Schechter distribution of galaxy masses and the assumption that star formation occurs mainly in exponential disks. The shape of the distribution depends primarily on two values, the characteristic mass $m^*$ in the Schechter mass distribution and the characteristic radius $r_e$ in the exponential disk. As these characteristic values evolve they will affect the shape of the distribution function. The expected direction of evolution of the parameters partially cancels each other leaving the distribution shape relatively invariant.

Subject headings: stars: formation - galaxies: evolution - galaxies: high redshift

1. Introduction

This letter has two goals. The first is to determine if the observed star formation intensity distribution is a natural consequence of empirical laws such as the Schmidt law and the Schechter mass function. The second is to determine which parameters in the empirical law most influence the shape of the distribution. Galaxy modelers can then predict the evolution of the distribution with redshift and compare it against future observations. The star formation intensity distribution developed by Lanzetta et al. (1999) has received significant attention both as a constraint on models of galaxy formation (Barkana 2002) and as a method of correcting for star formation missed by the effects of surface brightness diming at high redshifts (Thompson, Weymann and Storrie-Lombardi 2001; Lanzetta et al. 2002). At present, the distribution is only empirically derived from the observations by fitting the distribution at low redshift. Correction of higher redshift observations for surface brightness dimming is achieved by matching the bright end of the fitted distribution to the bright end of the observations at higher redshift. In particular Thompson, Weymann and
Storrie-Lombardi (2001) used the empirical fit to the distribution function at a redshift of one to correct observations at higher redshifts. In that case it was simply postulated that the shape of the distribution function was invariant with redshift. The purpose of this work is to see if the shape of the distribution function is governed by basic physical parameters and empirical laws such as the Schmidt law (Kennicutt 1998). Once the primary parameters influencing the distribution shape are found estimates can be made on whether the distribution is invariant or evolves with redshift. This is particularly important when the distribution is used to correct for the effects of surface brightness dimming.

2. Observations

A combination of observations in the Northern Hubble Deep Field with WFPC2 (Williams et al. 1996) and with NICMOS (Dickinson 2000) define the empirical form of the distribution. These observations were analyzed in essentially the same manner as described in Thompson, Weymann and Storrie-Lombardi (2001). The present paper utilizes just the star formation intensity distribution at a redshift of one. The full analysis of the star formation history of the entire Northern HDF will be published in a subsequent paper. The width of the \( z = 1 \) redshift bin is from \( z \) of 0.5 to 1.5, comprising 437 galaxies and \( 2.34 \times 10^6 \) pixels. When known, spectroscopic redshifts from Cohen (2001) replace the photometric redshifts in determining the SFR intensity distribution. This affects only a very small percentage of galaxies. Fig. 1 shows the empirical distribution function defined from the NHDF observations at \( z = 1 \).

Primarily due to variations in sensitivity over the field of the NICMOS Camera 3 Lanzetta et al. (2002) developed a selection function measuring the probability of a pixel with redshift \( z \) and star formation intensity \( x \) being detected by the source extraction technique. For consistency only, the technique was also used in this analysis but its effect on the data at redshift 1 is very negligible. For each pixel detected by the source extraction program the fraction \( f \) of the detector area that it could be detected in was calculated. If the fraction was less than 1 the proper area that the pixel contributed to \( h(x) \) was increased by \( 1/f \). Fractions less than 0.1 were set to 0.1 even though very few detected pixels have fractions below 0.1. Inclusion of the selection function correction only eliminates a slope inflection at a \( \log(x) \) value of -1.5.
3. Computed Distribution Function

The star formation intensity $x$ is defined as the star formation rate in solar masses per year per proper square kiloparsec. The intensity is calculated for each pixel that is part of a galaxy. Within a given redshift interval the distribution function, $h(x)$, is defined as the sum of all the proper areas in a interval of star formation intensity, divided by the interval and by the comoving volume in cubic megaparsecs defined by the field and redshift interval (Lanzetta et al. 1999). Defined in this manner the values of $h(x)$ determine the star formation rate per cubic comoving megaparsec through eqn. 1.

$$sfr = \int_0^\infty x h(x) dx$$

(1)

The star formation rate is then the first moment of the distribution function. It is important to note that the value of $h(x)$ for any interval of $x$ is comprised of pixels from a very large number of galaxies. This attribute makes it an excellent vehicle for assessing general laws of star formation. The smoothness of the $h(x)$distribution over almost 8 decades of $x$ is quite remarkable. Barkana (2002) points out that although $h(x)$ is dependent on cosmology through the comoving volume, $x$ is independent of cosmology when it is calculated from the UV surface brightness. He also points out that calculations of galaxy formation and evolution should be constrained to match the observed distribution. Barkana then utilizes a hierarchical galaxy formation and evolution code to predict the distribution and matches it with the distribution function given in Lanzetta et al. (2002).

It is important to note that the distribution function, $h(x)$, in fig. 1 utilizes extinction corrected star formation intensity values and is therefore different than the distribution functions in Lanzetta et al. (2002) which have not been corrected for extinction. The extinction corrected distribution reflects the basic physical parameters of the galaxies rather than the variance of extinction values. Thompson, Weymann and Storrie-Lombardi (2001) give a detailed account of the star formation rate calculation and extinction correction. Briefly the 6 galaxy fluxes in a 0.6" diameter aperture are matched via chi-squared analysis against SED template fluxes that have been numerically redshifted and extincted by the obscuration law of Calzetti et al. (1994) to give a single extinction for the galaxy. The total star formation rate for the galaxy is calculated from the 1500˚A flux of the selected template with zero extinction, giving the extinction corrected star formation rate. The star formation rate assigned to a pixel in a galaxy is the total star formation rate for the galaxy times the fraction of extinction corrected 1.6 $\mu$m total luminosity represented by the pixel. 1.6 $\mu$m was picked because it is the wavelength least affected by dust.
The extinction correction is the major correction to the observed data. Extinction correction moves pixels horizontally to the right in figure 1 as can be seen by the difference between the corrected and uncorrected points. The corrected \( h(x) \) must be a lower limit on the true \( h(x) \) at low values of \( x \) for two reasons. First, pixels in a galaxy that are below our detection limit due to extinction will not be included and second, entire galaxies that fall below our detection limit due to high extinction values will not be included. The break in the corrected data at \( \log(x) = -3.5 \) is most probably due to the first effect, although failure of the selection function at this level may also contribute. The faintest detected pixels lie at \( \log(x) = -5.0 \), 30 times fainter than the break value. The difference between the faintest and the break value corresponds to an \( E(B-V) \) value of 0.4 which is toward the higher end of the extinctions found for the sample. Values of \( h(x) \) past \( \log(x) = -3.5 \) should not be considered as accurate, however, the character of both the power law slope and the exponential fall off are well established by the distribution at \( \log(x) \) values greater than -3.5.

The observed distribution has two main components, a power law component at low \( x \) values and an exponential fall off at high \( x \) values. This Schechter like distribution (Press and Schechter 1974) is very common in astronomical phenomena. If either the power law were extended to high values of \( x \), or the exponential to low values of \( x \), the total star formation rate would diverge. Given this familiar form of the distribution it is interesting to see if it arises naturally from known general empirical laws regarding mass distribution in galaxies and relations between gas density and star formation. Formulation in terms of these general relations can also give insight into whether and how the distribution function might evolve with redshift.

The derivation of the distribution shape utilizes two general empirical laws and an assumption. The two empirical laws are the Schmidt law with a critical density relating star formation intensity to gas surface density and a Schechter law for the distribution of galaxy masses. The assumption is that star formation occurs predominantly in exponential disks. The following uses a dimensionless mass variable \( y = m/m^* \) where \( m^* \) is the mass parameter in the Schechter mass distribution. \( \phi^* \) is given in \( m^* \) per comoving Mpc\(^3\) in eqn. 2. In the following mass always refers to the gas mass in the galaxy since we are computing star formation.

\[
\phi(y) = \phi^* y^\alpha \exp (-y)
\]  

(2)


\[
\Sigma_{sfr} = a_o \left( \frac{m^*}{10^6 \Sigma_{gas}} \right)^{1.4} = a'_o \Sigma_{gas}^{1.4}
\]  

(3)
where $a_o = 2.5 \times 10^{-4}$ in the appropriate units and $m^*$ is in solar masses. The extra term $\left(\frac{m^*}{10^6}\right)^{1.4}$ comes from working in mass units of $y$ and gas densities in terms of square kpc. Studies (Kennicutt 1989; Martin and Kennicutt 2001) indicate that the Schmidt law does not extend to very low densities. There appears to be a critical density below which star formation is severely curtailed. The critical density is a function of the dynamics of the system and therefore not a constant density for all galaxies. To account for this phenomena we multiply the star formation efficiency $a'$ by a function of the form

$$a' = a'_o \left(1 + \frac{0.1}{x}\right)^{-1.5} \tag{4}$$

reducing the star formation rate at low star formation intensities which is equivalent to low surface densities. From eqn. 3 a star formation intensity of 0.01 $M_\odot$ per year per kpc$^2$ corresponds to a surface density of 14 $M_\odot$ per pc$^2$ which marks the surface density where the star formation rate begins to deviate from the Schmidt law in this formulation. The critical density is discussed further in sec. 4.

For a galaxy of mass $y$ in units of $m^*$ the assumption of an exponential disk with a characteristic radius $r_e$ gives a gas surface density as shown in eqn. 5.

$$\Sigma_{\text{gas}}(r) = \Sigma_o \exp\left(\frac{-r}{r_e}\right) = \frac{y}{2\pi r_e^2} \exp\left(\frac{-r}{r_e}\right) \tag{5}$$

where

$$r_e = r_o y^n \tag{6}$$

Setting the area integral of the density equal to $y$ determines the value of $\Sigma_o$. In 6 both $r_o$ and $n$ are adjustable variables. The final values of $r_o = 1.4$ kpc and $n = 5$ appear to be within a reasonable range.

For a galaxy with a mass of $y$ in units of $m^*$ each radius $r$ corresponds to a particular star formation intensity $x$.

$$\left(\frac{y^{1.4}}{2\pi r_o^2} \exp\left(\frac{-r}{r_o y^n}\right)\right)^{1.4} = \frac{x}{a'} \tag{7}$$

This leads to an equation for the radius $r(x,y)$ at which the sfr intensity is $x$ in a galaxy of mass $y$. 
\[ r(x, y) = r_0 y^{\frac{1}{2}} ((1 - \frac{2}{n}) \ln(y) - K(x)) \]  
\[ (8) \]

where \( K(x) \) is given by
\[ K(x) = \ln(2\pi r_0^2) + \frac{1}{1.4} \ln\left(\frac{x}{a}\right) \]  
\[ (9) \]

The simple requirement that the radius \( r(x,y) \) be greater than or equal to 0 puts a lower limit on the mass \( y \) of a galaxy that can contribute to a star formation intensity \( x \). That minimum mass \( y_m \) is given by
\[ y_m^{1 - \frac{2}{n}} = 2\pi r_0^2 \left( \frac{x}{a} \right)^{1.4} \]  
\[ (10) \]

A given star formation intensity interval \( \Delta x \) corresponds to a radius interval \( \Delta r \) in a galaxy equal to \( \Delta x \frac{dr}{dx} \). The 0.25 logarithmic intervals of \( x \) used in fig 1 correspond to an interval of 0.584x which is a \( \Delta r \) of \( \frac{0.584}{1.4} r_e \). The area corresponding to intensity interval is simply \( 2\pi r(x,y) \Delta r \). The total area for the intensity interval is obtained by integrating over all masses \( y \) weighted by the probability of galaxies with that mass given by eqn. 2.
\[ area(x) = 2\pi \frac{0.584}{1.4} r_0^2 \phi^* \int_{y_m}^{\infty} ((1 - \frac{2}{n}) \ln y - K(x)) y^{\frac{1}{2}} \exp(-y) dy \]  
\[ (11) \]

Integration of the second term of eqn. 11 yields \( K(x) \) times the incomplete gamma function \( \Gamma(\alpha + 1 + \frac{2}{n}, y_m) \). The integration of the first term was performed numerically with Mathematica. The area is then
\[ area(x) = 2\pi \frac{0.584}{1.4} r_0^2 \phi^* \left( \int_{y_m}^{\infty} ((1 - \frac{2}{n}) y^{\frac{1}{2}} \ln y \exp(-y)) dy - K(x) \Gamma(\alpha + 1 + \frac{2}{n}, y_m) \right) \]  
\[ (12) \]

Equation 12 is appropriate for a galaxy viewed face on but the observations include galaxies at all inclinations. Inclination of a galaxy by an angle \( \theta \) to the line of sight increases the observed \( x \) value by \( (\cos(\theta))^{-1} \) over the face on \( x \) value. This has three effects on equation 12. The value of \( x \) in \( K(x) \) should be \( x \cos(\theta) \) since a value \( x \cos(\theta) \) in the face on frame appears to be \( x \) in the observed frame. Next the radius \( \Delta r \) of the ring spanning \( \Delta x \) is now \( \Delta r \cos(\theta) \) since \( \Delta x \) is now \( \Delta x \cos(\theta) \) in the face on coordinates. Finally the projected area is also reduced by \( \cos(\theta) \). This transforms equation 12 to
\[
\text{area}(x) = 2\pi \frac{0.584(\cos(\theta))}{1.4}^2 r_0^2 \phi^* \left( \int_{y_m}^{\infty} \left( (1 - \frac{2}{n})y^{\alpha+\frac{2}{n}} \ln y \exp(-y) \right) dy - K(\cos(\theta)x)\Gamma(\alpha+1+\frac{2}{n}, y_m) \right)
\]

(13)

Under the assumption that all inclinations are equally probable the range between 0 and 90\(^\circ\) is divided into 100 equally spaced angles and equation 13 is summed over all angles. The last 20 angles in the angle distribution are set equal to the 80th value to recognize the galaxies have significant thickness which also avoids the singularity at 90\(^\circ\) inclination.

The areas found for each intensity value are divided by the intensity interval to determine the calculated \(h(x)\) shown as the solid line in fig. 1. Free parameter values of \(\phi^* = 0.007m^*\) per comoving cubic Megaparsec, \(m^* = 4 \times 10^{10} M_\odot\), \(r_o = 2.0\) kpc, \(\alpha = -1.2\) and \(n = 5\) produce a good fit to the observed \(h(x)\). It could be asked whether with 5 free parameters can you always get a good fit? On the other hand the empirical laws governing the distribution contain these parameters and their values must be set. That the values are consistent with values that one would set a priori in an attempt to model the distributions gives a good indication that the distribution shape is determined by the empirical laws and assumptions employed. Experiments setting one of the parameters to a mildly nonphysical value indicated that no rearrangement of the remaining parameters could produce an acceptable fit.

4. Critical Density

The diamonds in fig. 1 indicate the computed distribution if the correction for critical density is omitted. It has a steeper slope than the observed distribution at low x values. No physical combination of free parameters resulted in a shallower slope than the one shown by the diamonds. Lowering \(\alpha\) in the Schechter mass equation to values that result in a divergent mass in galaxies lowered the slope slightly but was considered non-physical. The discussions in Kennicutt (1989) and Martin and Kennicutt (2001) indicate that there is a critical density that depends on dynamical factors through the Toomre Q factor and possibly the amount of shear in the galactic rotation. The form of the reduction of star formation efficiency utilized in eqn. 4 is a simple way of reducing efficiency at low surface density, i.e. low x, regions. The range of critical densities discussed in Martin and Kennicutt (2001) and the observed fact that there is star formation in regions that are below critical densities indicated a softer reduction of the star formation efficiency than a simple truncation. The value of the exponent (-1.5) produces a good match to the observed slope. That such a reduction is required to match the observations is further evidence for a reduction of star formation efficiency below the rate predicted by the simple Schmidt law at low surface densities.
The soft roll off of the Schmidt Law used in this work does not imply that there is not a local critical density below which star formation does not occur. Even though the average density in the roughly 1 square kiloparsec area covered by a single pixel may be below the critical density, there may be several areas where the local density is higher than the critical density. This is probably the best physical interpretation of the soft roll off on the kpc scale.

5. Evolution with Redshift

The main parameters that control the shape of the distribution function are $r_e$ and $m^*$. The fit is relatively insensitive to $\alpha$ and the value of $\phi^*$ mainly scales the distribution rather than change its shape. In hierarchical models of galaxy formation and evolution both the value of $m^*$ and $r_e$ will decrease as the redshift increases. Lowering $m^*$ decreases the value of $x$ where the transition from power law to exponential occurs. The effect of reducing $r_e$ is just the opposite, it increases the value of $x$ where the transition occurs. Both of these effects make physical sense. The exact evolution of the intensity distribution will then depend on the evolution of these two parameters. In an attempt to determine the effect of evolution the computation was performed for an epoch where the characteristic mass $m^*$ is 1/10 of the $3 \times 10^{10} \, M_\odot$ found from the matching at $z = 1$. The value of $r_e$ was then reduced to $(1/10)^{1/5}$ of its value at $z=1$. The result is the dashed line in fig. 1. The main effect is a transition to the exponential function at a lower of $x$. If we reduce $r_e$ by a larger amount a distribution very close to the $z = 1$ distribution is obtained. The HST observations at redshifts greater than 1.5 do not adequately define the shape of $h(x)$ to provide an observational test of the evolution of $h(x)$. Galaxy evolution modeler’s predictions of the free parameters and the resulting $h(x)$ can be compared at high redshift when NGST becomes operational.

6. Conclusions

We have shown that the star formation intensity distribution shape is a natural consequence of a Schechter galaxy mass distribution, the Schmidt law with a critical density and star formation occurring in exponential disks. The primary parameters that control the shape of the distribution are the value of $m^*$ in the Schechter function and the value of $r_e$ in the exponential surface density distribution. In a hierarchical galaxy formation scenario both of these values would expect to be reduced at higher redshifts. Reduction of the values of these two parameters have opposite effects on the shape of the function and may cancel each other out in part.
The author would like to acknowledge the very helpful comments of an anonymous referee. This work is supported in part by NASA grant NAG 5-10843. This work utilized observations with the NASA/ESA Hubble Space Telescope, obtained at the Space Telescope Science Institute, which is operated by the Association of Universities for Research in Astronomy under NASA contract NAS5-26555.

REFERENCES

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Fig. 1.— The values of $h(x)$ at a redshift of one from the Northern HDF in logarithmic intervals of 0.25 in $x$. The extinction corrected observed values are denoted by * and observed values not corrected for extinction by plus signs. The solid line is the computed fit to the function as described in the text. The dashed line is the expected evolution of the distribution to an epoch when $m^*$ is 1/10 the value at $z = 1$. The diamonds show the fit if a critical density for star formation is not invoked.