Bounds on the Magnetic Fields in the Radiative Zone of the Sun

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ABSTRACT

We discuss bounds on the strength of the magnetic fields that could be buried in the radiative zone of the Sun. The field profiles and decay times are computed for all axisymmetric toroidal Ohmic decay eigenmodes with lifetimes exceeding the age of the Sun. The measurements of the solar oblateness yield a bound $\lesssim 7$ MG on the strength of the field. A comparable bound is expected to come from the analysis of the splitting of the solar oscillation frequencies. The theoretical analysis of the double diffusive instability also yields a similar bound. The oblateness measurements at their present level of sensitivity are therefore not expected to measure a toroidal field contribution.

Subject headings: Sun: magnetic fields — Sun: interior

1. Introduction

It has been speculated that the solar radiative interior may contain a buried magnetic field, which does not penetrate into the convective zone and hence is very difficult to observe. Such a field could have formed, for instance, during the early stages of the solar evolution, perhaps as a result of the differential rotation which could have stretched the lines of a weak primordial seed field (Dicke 1979). Once formed, the large scale field structures would survive until the present time because of the high conductivity of the plasma in the radiative zone.

The observation that the lifetime of the large scale field eigenmodes in the solar interior may exceed the solar age was first made by Cowling over half a century ago (Cowling 1945). Cowling's conclusion was confirmed by other authors, in particular by Wrubel (1952) who computed the lifetimes of the three longest living poloidal modes and by Bahcall & Ulrich (1971) who computed the lifetime of the lowest mode using an early solar model.

The motivation for discussing the hypothetical radiative zone (RZ) field changed with time. The RZ fields, with a characteristic strength of $10^8$ G, were invoked by several authors (Bartenwerfer
1973; Chitre, Ezer, & Stothers 1973) in the 1970’s as a possible explanation for the deficit of solar neutrinos reported by the Homestake experiment. The fields of comparable strength was postulated by Dicke (Dicke 1978, 1979, 1982) to explain the so-called Princeton solar oblateness measurements. Recently, Gough & McIntyre (1998) argued that a relatively weak, but nonzero, RZ poloidal field is necessary to explain the uniform rotation of the radiation zone. Separately, the fields of 30 MG strength were proposed as a way to improve the agreement between the solar model and the measurements of the $^8$B neutrino flux at the SNO and SuperKamiokande experiments (Couvidat, Turck-Chieze, & Kosovichev 2002). In a still separate development, it was found that the presence of a submegagauss toroidal field in the radiative zone could explain all presently available solar neutrino data, if neutrino possesses a relatively large transition magnetic moment (Friedland & Gruzinov 2002).

Regardless of the motivation, it is important to establish what bounds, both observational and theoretical, exist on the strength of the radiative zone fields. This is the goal of the present paper. Both Cowling and Wrubel in their analyses assumed a poloidal topology for the primordial field. Such field cannot be stronger than a few gauss at the top of the radiative zone, otherwise it would penetrate in the convective zone (CZ) and cause a polarity asymmetry between the two halves of the magnetic cycle (Levy & Boyer 1982; Boyer & Levy 1984; Boruta 1996). The absence of the asymmetry places a very strong bound ($B \lesssim 10^2 - 10^3$ G) on the strength of the poloidal field allowed in the solar core. At the same time, the field which is entirely confined to the radiative zone, in particular a toroidal axisymmetric field, is not subject to this bound, and may in fact be very difficult to observe.

Before presenting the constraints on the strength of the toroidal field, in Section 2 we compute the Ohmic decay eigenmodes for the axisymmetric toroidal field configurations and the corresponding lifetimes, and give the profiles and lifetimes for all modes whose lifetimes exceed the solar age. To the best of our knowledge, such a calculation has not been done previously. The problem was touched on by Dicke (1982), who presented the profile of the $l = 2$ mode (which is not the longest living mode), without specifying its lifetime. The discussion of the bounds on the field strength follows in Section 3. This is followed in Section 4 by the discussion of theoretical instabilities that may provide bounds on the strength of the field. In Section 5 we summarize our conclusions.

2. Eigenmodes

We will approximate the electric conductivity in the radiative zone of the Sun by the Spitzer formula (Spitzer 1962)

$$
\sigma = \frac{2^{5/2} (kT)^{3/2} \gamma_E}{\pi^{3/2} m_e^{1/2} e^2 Z_{\text{eff}} \lambda},
$$

where $m_e$ and $e$ are the electron’s mass and charge, $Z_{\text{eff}} = \sum_i n_i Z_i^2 / \sum_i n_i Z_i$ is the effective ion charge, $T$ is the temperature, $\gamma_E$ is an order one coefficient describing the deviation of the plasma
from the ideal Lorentz gas, and $\lambda$ is the Coulomb logarithm. Numerically, this yields
\[
\sigma = (2.37 \times 10^{17} \text{s}^{-1}) T_6^{3/2} \frac{\gamma_E}{Z_{\text{eff}} \lambda},
\]
where $T_6$ is the temperature in millions of Kelvins. The quantity $Z_{\text{eff}}$ can be computed by using the tables of the solar model; its dependence on the solar radius for the BP2000 solar model (Bahcall, Pinsonneault, & Basu 2001) is shown in Fig. 1(a). The Coulomb logarithm $\lambda$ for $T_6 \gtrsim 0.6$ is given by
\[
\lambda = 2.4 - 1.15 \log_{10} n + 2.3 \log_{10} T_6,
\]
where the number density of electrons $n$ is in units of Avogadro number per cm$^3$. Finally, the coefficient $\gamma_E$ for $Z_{\text{eff}}$ in the interval $1 < Z_{\text{eff}} < 2$ is well described by
\[
\gamma_E = 0.58 + 0.33 \log_{10} Z_{\text{eff}}.
\]
The radial dependence of the factor $\gamma_E/Z_{\text{eff}} \lambda$ in the Sun is shown in Fig. 1(b).

The principal approximation affecting the accuracy of Eq. (2) is using the Spitzer conductivity in the center of the Sun, where Coulomb logarithm is not large, $\lambda \approx 2.9$.

The corresponding magnetic diffusivity, $\eta = c^2/(4\pi \sigma)$ is given by
\[
\eta = (3.0 \times 10^2 \text{cm}^2 \text{s}^{-1}) \frac{Z_{\text{eff}} \lambda T_6^{-3/2}}{\gamma_E}.
\]
In the convective zone, the effective magnetic diffusivity is very large because of turbulence.

The Ohmic decay of the field is governed by the magnetic diffusion equation,
\[
\partial_t \mathbf{B} = -\nabla \times (\eta \nabla \times \mathbf{B}),
\]
which follows from $\partial_t \mathbf{B}/c = -\nabla \times \mathbf{E}$, $\mathbf{E} = \mathbf{j}/\sigma$, and $\nabla \times \mathbf{B} = 4\pi \mathbf{j}/c$. Eqs. (5,6) can be used to estimate the minimal size $l_{\text{min}}$ of the magnetic field features that survive over the age of the Sun, $l_{\text{min}} \sim \sqrt{\eta \tau_\odot}$, where $\tau_\odot = 4.6 \times 10^9$ years is the age of the Sun. Since $\eta$ varies with the distance from the center, so does $l_{\text{min}}$. At $r = 0.6 R_\odot$ one finds $l_{\text{min}} \sim R_\odot/9$, while at $r = 0.25 R_\odot$ $l_{\text{min}} \sim R_\odot/20$. Thus, field modes with the size comparable to the size of the radiative zone are expected to survive until the present time.

To make the preceding rough estimate more precise, we next compute the profiles and the lifetimes of the axisymmetric toroidal eigenmodes that are localized to the solar RZ. For an axisymmetric toroidal field $\mathbf{B} = B(r, \theta, t)\hat{e}_\phi$, Eq. (6) gives
\[
\partial_t B = \eta \left( \nabla^2 B - \frac{1}{r^2 \sin^2 \theta} B \right) + \eta' r^{-1} \partial_r (r B),
\]
where $' = d/dr$. The eigenmodes have the form $B = e^{-t/\tau_{n,l}} F_{n,l}(r) P_l^1(\cos \theta)$ where $n = 1, 2, ..., l = 1, 2, ...$. The function $F_{n,l}$ satisfies the equation
\[
\eta \left( F'' + 2r^{-1} F' - l(l+1)r^{-2} F \right) + \eta' (F' + r^{-1} F) = -\tau_{l}^{-1} F.
\]
The boundary conditions are $F = 0$ at $r = 0$ and $r = R_{\text{CZ}}$.

The lifetimes of various modes, computed for the BP2000 Sun (Bahcall, Pinsonneault, & Basu 2001), are tabulated in Table 1. The $n = 1$, $l = 1$ mode has the longest lifetime, $\tau_{1,1} = 24\text{Gyr}$. As can be seen from the table, there are eight modes whose lifetimes exceed the age of the Sun and three modes whose lifetimes exceed 10 billion years. Therefore, the toroidal field in the radiation zone of the Sun can in principle have complex structure. The profiles $F_{n,l}(r)$ of the eight longest living modes are shown in Figs. 2 and 3 and tabulated in Table 2.

Although the preceding calculation does not take into account the changes of the solar parameters with time, the effect of the solar evolution on the lifetimes is expected to be small. As a result of hydrogen burning, as the Sun evolves from $t = 0$ to $t = 4.7$ Gyr, the core contracts and becomes hotter, while the outer layers of the RZ expand and cool. Since the lifetime depends on the product $l^2 T^{3/2}$, there is partial cancellation, both in the core and in the outer part of the RZ, between the effects of changing temperature and changing size. Moreover, the crossover radius, at which the temperature and the radius distance do not change, is at about $0.15 R_\odot$ (Demarque & Guenther 1991), close to where the maximum of the $n = 1$, $l = 1$ mode.

3. Observational upper bounds on the magnetic field

In this Section we discuss observational upper bounds on the allowed strength of the magnetic field in the radiation zone. A variety of independent arguments rule out fields in excess of $\lesssim 10^8$ G. We briefly summarize the arguments from helioseismological measurements of the sound speed and from the measurements of the $^8\text{B}$ neutrino flux. A much stronger bound on the axisymmetrical fields comes from the measured oblateness of the Sun. We derive a bound on the strength of the field specializing to the longest living $n = 1$, $l = 1$ mode. We also point out that a comparable bound could be obtained from the helioseismological measurements of the mode splitting.

3.1. Bounds from helioseismology and $^8\text{B}$ neutrino flux

The magnetic field of strength $B$ adds a contribution to pressure $p_m = B^2 / 8\pi$, correspondingly decreasing the gas pressure $p_g$. By estimating the change in the gas temperature as $|\delta T / T| \sim p_m / p_g$, one obtains a bound on the allowed strength of the field in the core and in the RZ from the observed flux of the $^8\text{B}$ neutrinos and the helioseismological measurements of the sound speed. The two constraints are complimentary, because the $^8\text{B}$ neutrinos are produced very close to the center of the Sun ($r < 0.09 R_\odot$), where the accuracy of the helioseismological measurements is relatively poor.

The flux of the $^8\text{B}$ neutrinos neutrinos is not directly tied to the solar luminosity and has a strong dependence on the core temperature, $\phi(^8\text{B}) \propto T^\beta$, where $\beta \sim 24$ (Bahcall & Ulmer 1996).
Therefore, the flux $\phi(^{8}\text{B})$ can be used as a probe of the central temperature. The flux $\phi(^{8}\text{B})$ has been measured by the SNO experiment using a neutral current reaction $\nu + d \rightarrow \nu + p + n$ and assuming oscillations into active neutrinos. The measured value, $5.09 \pm 0.64 \times 10^6$ cm$^{-2}$ s$^{-1}$, is in good agreement with the predictions of the standard solar model, $5.05^{+1.0}_{-0.8} \times 10^6$ cm$^{-2}$ s$^{-1}$ (Bahcall, Pinsonneault, & Basu 2001). Combining the above ingredients, we obtain an order of magnitude bound on the strength of the fields in the core,

$$B \lesssim p_g^{1/2} \left( \frac{\delta \phi(^{8}\text{B})}{\phi(^{8}\text{B})} \right)^{1/2} \sim 2 \times 10^8 \text{G}. \quad (9)$$

A similar order of magnitude bound follows from the measurements of the sound speed. Taking the uncertainty on the sound speed to be $\delta c_s/c_s \sim 10^{-3}$ we find for $r = 0.2R_\odot$,

$$B \lesssim (8\pi p_g)^{1/2} \left( \frac{\delta c_s}{c_s} \right)^{1/2} \sim 0.4 \times 10^8 \text{G}. \quad (10)$$

To obtain more accurate bounds in both cases requires incorporating magnetic fields in the solar model calculations. Recent analysis by Couvidat, Turck-Chieze, & Kosovichev (2002) finds the limit $B \lesssim 3 \times 10^7$ G from the measurements of the sound speed.

### 3.2. Constraints from Solar Oblateness

It is well known that the rotation distorts the shape of the Sun, making it oblate. The theoretical value of the solar rotational oblateness $\epsilon = 1 - R_{\text{pole}}/R_{\text{equator}}$ is $\epsilon_{\text{rot}} = 9 \times 10^{-6}$ (Godier & Rozelot 2000). The magnetic fields will further distort the equilibrium figure of the Sun. If we take the recently measured oblateness of the Sun to be $(10 \pm 3) \times 10^{-6}$ (Godier & Rozelot 2000), we must require that the contributions to the oblateness from the magnetic fields not exceed the purely rotational oblateness by over $\sim 30\%$, which translates into a bound on $B_{\text{max}}$.

The distortion is obtained from the equilibrium equation

$$\vec{\nabla} (p + p_m) + \frac{2p_m}{R} \hat{R} + \rho \vec{\nabla} \phi = 0. \quad (11)$$

Here $p$ is the gas pressure, $p_m = B^2/(8\pi)$ is the pressure of the toroidal magnetic field, $R = r \sin \theta$ is the cylindrical radius, $\hat{R}$ is the unit vector along cylindrical radius, $\rho$ is the density, $\phi$ is the gravitational potential. We can exclude $p$ by taking the curl, and then linearize, that is leave only first order terms in $p_m$:

$$\phi_0' \partial_\theta \delta \rho - \rho_0' \partial_\theta \delta \phi = 2 \cot \theta \partial_r p_m - 2r^{-1} \partial_\theta p_m. \quad (12)$$

Here $\delta \rho$ and $\delta \phi$ are density and potential perturbations due to magnetic pressure, $\rho_0$ and $\phi_0$ are the unperturbed density and potential of the Sun, and prime is the derivative with respect to $r$. 

We now specialize to the longest living mode, \( p_m(r, \theta) = B_{\text{max}}^2[F_{1,1}(r)]^2 \sin^2 \theta / 8\pi \), with \( F_{1,1}(r) \) tabulated in Table 2. Then \( \delta \rho \) and \( \delta \phi \) are \( \propto P_2(\cos \theta) \) and, using the Poisson equation \( \nabla^2 \phi = 4\pi G \rho \), we obtain

\[
(4\pi G)^{-1} \phi'' + 2r^{-1} \phi' - 6r^{-2} \phi \rho' \delta \phi = -(4/3)B_{\text{max}}^2 F_{1,1}(F_{1,1}' - r^{-1}F_{1,1}).
\] (13)

This equation was solved numerically, and the calculated oblateness of the Sun was found to be

\[
\epsilon_{\text{magn}} = -6.4 \times 10^{-8} \left( \frac{B_{\text{max}}}{1 \text{ MG}} \right)^2.
\] (14)

By requiring that \( \epsilon_{\text{magn}} \lesssim 3 \times 10^{-6} \), we get an upper bound

\[
B_{\text{max}} \lesssim 7 \text{ MG}.
\] (15)

Thus the bound from oblateness is much stronger than the ones considered earlier.

The effect of the 12 MG field would be to completely cancel out the solar rotational oblateness, making the Sun into a perfect sphere. The fields of the strength \( 3 \times 10^7 \) G, that were invoked by Couvidat, Turck-Chieze, & Kosovichev (2002) as a possible way to improve the agreement between the observed and predicted \(^8\text{B}\) fluxes, would make the figure of the Sun strongly prolate and hence are not allowed.

### 3.3. Splitting of Solar Oscillation Frequencies

The observations of the splitting of solar oscillation frequencies can provide another way to constrain the strength of the magnetic field. Antia, Chitre, & Thompson (2000) analyzed the observed splittings and derived an upper bound of \( B < 0.3 \text{ MG} \) for a toroidal field of radial extent \( d = 0.04 R_\odot \) near the base of the convective zone at \( r = 0.7 R_\odot \). We can write their upper bound in a form \(^1\)

\[
\frac{d}{r} \frac{B^2}{8\pi p(r)} < 3 \times 10^{-6}.
\] (16)

If we assume that the error bars on the measured splitting coefficients do not increase too much as we move deeper into the Sun, we obtain an upper bound on the strength of the lowest field mode that is comparable to the one derived from oblateness. To make this bound more precise requires repeating the full analysis of Antia, Chitre, & Thompson (2000) with the magnetic field profile composed of the modes found in Sect. 2.

\(^1\)This is a much stronger upper bound than the upper bound \( B^2/(8\pi p) < 10^{-3} \), considered in Sect. 3.1.
4. Theoretical Considerations

In this section we describe theoretical bounds on the magnitude of the radiative zone magnetic field. The bounds are based on the observation that a sufficiently strong field will rise to the surface as a result of thermal diffusion in the solar plasma. We first consider the simplified scenario in which the field rises as a whole and then address the possibility that the field may fragment and the fragmented flux tubes rise to the surface. The latter analysis gives a particularly stringent bound on the field strength. We also comment on the role of the ideal magnetohydrodynamic instability that may exist for the field configuration in question.

4.1. Constraint from Magnetic Buoyancy

A robust bound on the strength of the primordial magnetic field follows from the consideration of the rise time in the stably stratified RZ (Parker 1974, 1979). The essence of the effect is that a flux tube in hydrostatic equilibrium in a stably stratified medium has a lower temperature than the temperature of the surrounding medium. The resulting flow of heat into the tube causes it to rise to assume a new equilibrium position. The rise velocity \( u \) thus depends on the field strength (through temperature gradient) and the heat conduction properties of the plasma,

\[
u \approx \frac{1}{\delta} \frac{B^2}{8\pi p} \left( \frac{\lambda}{R} \right)^2 \frac{I}{p}.
\]

(17)

Here \( p \) is the equilibrium pressure, \( I = L_\odot/4\pi r^2 \) is the heat flux, \( R \) is the radius of the flux tube, \( \lambda \equiv d\ln T/dr \) is the temperature scale height of the medium, and \( \delta \equiv (d\ln n/d\ln T) - 1/(\gamma - 1) \) is a parameter determining the stability of the medium (for an adiabatic medium \( \delta = 0 \)).

Numerically, the rise time from \( r = 0.2R_\odot \) to the top of the RZ is

\[
t_{\text{rise}} \sim 3 \times 10^7 \text{yr} \left( \frac{B^2}{8\pi p} \right)^{-1},
\]

(18)

so that the field \( B \sim 10^8 \text{ G} \) would rise in the time comparable to the age of the Sun. For the field \( B \sim 0.5 \times 10^6 \text{ G} \) considered in Friedland & Gruzinov (2002) the distance traveled over the age of the Sun is only about 8 km and the effect can be safely ignored.

The above mechanism can be used to make an argument that the primordial RZ field could not have formed with the strength of \( 10^8 \text{ G} \). This is possible using the abundance of Beryllium at the solar surface. Be is destroyed at \( T = 3 \times 10^6 \text{ K} \), which is the temperature at \( r = 0.61R_\odot \). Since Beryllium observed at the solar surface is not noticeably depleted, the material at \( r \sim 0.7R_\odot \) and the material at \( r \sim 0.6R_\odot \) have not been mixed during the lifetime of the Sun (except for a brief period during the pre-main sequence convection stage). The rise of the \( 10^8 \text{ G} \) field would have forced such mixing, and therefore could not have happened.
4.2. Double Diffusive Instability

The preceding analysis showed that the rise velocity of a flux tube is inversely proportional to the diameter of the tube squared. The simple reason for this is that small flux tubes are heated faster than large ones. It is therefore conceivable that instead of rising as a whole, the magnetic field might fragment into small flux tubes, which would rise to the surface faster. This instability is counteracted by the diffusion of the magnetic field lines. Superficially, it may appear that, since the heat diffusivity of the solar plasma,

$$\chi = \frac{16 \sigma T^3}{3 \kappa \rho c_v},$$

(19)
is some four orders of magnitude greater than the magnetic diffusivity \(\eta\), given in Eq. (5), any field configuration will be unstable. However, a more careful consideration of the effect shows that the relevant parameter controlling the instability is the combination \((\chi/\eta)(p_m/p_g)\). The stability analysis was carried out by Schubert (1968), who found that for the field configuration to be stable, it is necessary that

$$\frac{\partial \ln(pp^{-\gamma})}{\partial r} > -\frac{\chi B^2}{\eta 4 \pi p_g} \frac{\partial \ln(B\phi/\rho r)}{\partial r}.$$ 

(20)

Substituting in this condition the profile of the longest living toroidal eigenmode \((l = 1, n = 1)\), we find that if the normalization of the field, \(B_{\text{max}}\), exceeds \(\sim 2.1 \text{ MG}\), the field configuration becomes unstable at \(r \sim 0.6R_\odot\). Therefore, this argument provides the most stringent bound on the toroidal magnetic field.

4.3. Ideal magnetohydrodynamic instability

An arbitrarily weak purely toroidal magnetic field is unstable to ideal magnetohydrodynamic (MHD) perturbations (Tayler 1973; Goossens & Veugelen 1978; Goossens & Tayler 1980). However, one can show (Gruzinov & Friedland 2002) that the perturbation considered by Goossens & Veugelen (1978) is stabilized by a poloidal field \(B_p\) which is much weaker than the destabilizing toroidal field \(B_t\) \((B_p^2/(8\pi p) \lesssim (B_t^2/(8\pi p))^2)\).

The general problem of ideal MHD stability of toroidal plus poloidal magnetic field in spherical geometry with stable stratification is unsolved. One also notes that toroidal fields satisfying observational upper bounds are very weak \((B_t^2/(8\pi p) \lesssim 3 \times 10^{-6})\), and the unstable perturbations have a very small wavelength (Goossens & Veugelen 1978). Then even a trace of differential rotation in the RZ can affect stability in an unknown manner. It is therefore difficult to extract a concrete bound on the strength of the magnetic field from the analysis of the MHD instability with the present level of understanding of the problem.
5. Discussion

We have discussed a variety of constraints on the allowed strength of the magnetic fields in the solar radiative zone. We have shown that the strongest constraints come from the measurements of the solar oblateness, from the splitting of the oscillation frequencies, and from the theoretical analysis of the double diffusive instability of the field. All three constraints point to maximal field values of less than a few megagauss. That very similar bounds emerge from the analysis of such physically different phenomena is in itself quite remarkable. Alternatively, our results show that magnetic field that would measurably distort the figure of the Sun would cause an instability in the Sun, and hence is not allowed. This explains why the oblateness measurements at their present level of sensitivity have not seen a toroidal field contribution.

Our analysis excludes the possibility of perturbing the solar model with the fields of the order of several tens of megagauss that have been invoked to obtain a better agreement with the solar neutrino data (Couvidat, Turck-Chieze, & Kosovichev 2002).

The bounds obtained here have a direct implication for the analysis of the neutrino survival probability discussed in Friedland & Gruzinov (2002). The survival probability depends on the value of the product of the neutrino transition moment, $\mu$, and the magnetic field normalization, $B_{\text{max}}$. The value of the product, which was found to give a good fit to the solar neutrino data, is $\mu B_{\text{max}} \sim 0.5 \times 10^{-11} \text{MG} \times \mu_B$ ($\mu_B$ is the Bohr magneton). The upper bound on the magnetic field strength therefore yields a lower bound on the size of the transition moment that is necessary to fit the solar neutrino data. The bound from oblateness yields $\mu \gtrsim 7 \times 10^{-13} \mu_B$, while the limit from the double diffusive instability gives $\mu \gtrsim 2 \times 10^{-12} \mu_B$.

Among the theoretical issues that require further study is the problem of ideal MHD stability described in Section 5 and the possibility that the symmetry axis of the magnetic field may become tilted as a result of the dissipative motions of the plasma (Spitzer 1958; Mestel, Nittmann, Wood, & Wright 1981).

In summary, the bounds on the strength of the hypothetical magnetic fields in the radiative zone of the Sun described here may be useful for searches for such fields and also for the discussions of their physical implications.

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Fig. 1.— The radial dependence of the effective ion charge $Z_{\text{eff}}$ (left) and the quantity $\gamma_E/Z_{\text{eff}}\lambda$ (right), computed for the BP2000 Sun.
Fig. 2.— The radial dependence $F_{n,l}(r)$ of the $l = 1$ eigenmodes (left) and $l = 2$ eigenmodes (right) whose lifetimes exceed the age of the Sun.

Fig. 3.— The radial dependence $F_{n,l}(r)$ of the $l = 3$ (left) and $l = 4$ (right) eigenmodes whose lifetimes exceed the age of the Sun.
Table 1: The lifetimes of the field eigenmodes in billions of years as a function of $n$ (vertical) and $l$ (horizontal).

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Table 2. The radial dependence $F_{l,n}(r)$ of the eigenmodes whose lifetimes exceed the age of the Sun.

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REFERENCES


