Hybrid Mesons Masses in a Quark-Gluon Constituent Model

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Abstract

QCD theory allows the existence of states which cannot be built by the naïve quark model; both theoretical arguments and experimental data confirm the hypothesis that gluons may have freedom degrees at the constituent level, and should be confined. In this work, we use a phenomenological potential motivated by QCD (with some relativistic corrections) to determine the masses and the wavefunctions of several hybrid mesons, within the context of a constituent $q\bar{q}g$ model. We compare our estimates of the masses with the predictions of other theoretical models and with the observed masses of candidates.

1 Introduction

Quantum Chromodynamics, acknowledged as the theory of strong interactions, allows that mesons containing gluons (as $q\bar{q}g$ hybrids) may exist. The physical existence of these “exotic” particles (beyond the quark model) is one of the objectives of experimental projects [1]. These programs would contribute significantly to the future investigation of QCD exotics, and should

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improve our understanding of hybrid physics and on the role of the gluon in QCD. From experimental efforts at IHEP \cite{2}, KEK \cite{3}, CERN \cite{4} and BNL \cite{5}, several hybrid candidates have been identified, essentially with exotic quantum numbers $J^{PC} = 1^{-+}$.

Hybrids have been studied, using the Flux-Tube Model \cite{6}, the Quark Model with a Constituent Gluon \cite{7,8,9}, the MIT Bag Model \cite{10}, the Lattice Gauge Theory \cite{11}, QCD Sum Rules \cite{12} and using Many-Body Coulomb Gauge Hamiltonian \cite{13}. These models predicted that the lightest hybrid meson will be the $J^{PC} = 1^{-+}$ meson, in $1.5-2.1 \text{ GeV}$ mass range; a charmed hybrid meson will have a mass around $4.0 \text{ GeV}$, and a bottom hybrid meson with a mass around $10.0 \text{ GeV}$.

We propose estimates for the masses of the hybrid mesons considering the five ($u$, $d$, $s$, $c$ and $b$) flavors, within the context of a Quark-Gluon Constituent Model using a QCD-inspired potential and taking into account some relativistic effects.

\section{The phenomenological potential model}

Although the Lagrangian of QCD is known, its theory is not able to describe unambiguously the strong-coupling regime. In this framework, the process requests alternative theories like Flux-Tube model, Bag model, QCD string model, Lattice QCD, or phenomenological potential models.

The potential model is essentially motivated by the experiment, and its wave functions are used to represent the states of the strong interaction and to describe the hadrons. The most used is the harmonic oscillator potential, which gives very simple calculations and which is qualitatively in good agreement with the experimental data; although its expression is not explicitly describing the QCD characteristics, namely the (linear) confinement and the asymptotic freedom.

The most usual kinds of potential models are using non relativistic kinematics, which is convenient to the heavy flavors systems, but cannot be suitable to the hadrons containing light flavors. In this work, we consider relativistic systems to adjust the situation and then extend the study to such bound states.

We introduce a model in which the gluon is considered as massive constituent particle. The constituent gluon mass ($m_g \simeq 800 \text{ MeV}$) is chosen as an order of magnitude, taking in account the mass of the glueball candi-
date (1.6 \, GeV). Furthermore, the authors of [21] are generating constituent quarks and gluon masses in the context of Dynamical Quark Model employing BCS vacuum, and obtained a constituent gluon mass of 800 \, MeV. As we will see below, the impact of the $m_g$ in the results of hybrid masses is weak and the order of magnitude remains the same.

The Hamiltonian is constructed, containing a phenomenological potential which reproduce the QCD characteristics; its expression has the mathematical “Coulomb + Linear” form, and we take into account also some additional spin effects.

The basic hypothesis is to use a relativistic Schrödinger-type wave equation\cite{14}:

$$
\left\{ \sum_{i=1}^{N} \sqrt{\vec{p}_i^2 + m_i^2 + V_{eff}} \right\} \Psi(\vec{r}_i) = E \, \Psi(\vec{r}_i). \tag{1}
$$

Another wave equation, more convenient for multiparticle systems, can be used\cite{15, 16}:

$$
\left\{ \sum_{i=1}^{N} \left( \frac{\vec{p}_{\lambda_i}^2}{2M_i} + \frac{M_i}{2} + \frac{m_i^2}{2M_i} \right) + V_{eff} \right\} \Psi(\vec{r}_i) = E \, \Psi(\vec{r}_i); \tag{2}
$$

where $M_i$ are some “dynamical masses” satisfying the conditions:

$$
\frac{\partial E}{\partial M_i} = 0; \tag{3}
$$

$V_{eff}$ is the average over the color space of chromo-spatial potential\cite{17}:

$$
V_{eff} = \langle V \rangle_{color} = \left\langle -\sum_{i<j=1}^{N} F_i \cdot F_j \, v(r_{ij}) \right\rangle_{color} = \sum_{i<j=1}^{N} \alpha_{ij} v(r_{ij}) ; \tag{4}
$$

where $v(r_{ij})$ is the phenomenological potential term.

For the hybrid meson, $i$ and $j$ representing the constituents:

$$
i, j = 1 \equiv q \n$$
$$i, j = 2 \equiv \bar{q} \n$$
$$i, j = 3 \equiv g. \n$$
We consider all the possible $S_i \cdot S_j$ combinations ($\alpha_{qg} = \frac{-1}{6}$; $\alpha_{qg} = \alpha_{qg} = \frac{1}{2}$).

We have chosen a QCD-motivated potential which has the form:

$$v(r_{ij}) = -\frac{\alpha_s}{r_{ij}} + \sigma r_{ij} + c; \quad (5)$$

the $\alpha_s$, $\sigma$, and $c$ may be fitted by experimental data or taken from lattice and Regge fits.

Whereas, for light quarks we should add the spin-dependent correction represented by the (smeared) hyperfine term of Breit-Fermi interaction$^{[18, 19]}$:

$$V_S = \sum_{i<j=1}^{N} \alpha_{ij} \frac{8\pi \alpha_h}{3M_i M_j \sqrt{\pi^3}} \sigma^3_b \exp(-\sigma^2_h r^2_{ij}) S_i \cdot S_j; \quad (6)$$

we neglect the tensor and spin-orbit terms which effects are supposed to be small$^{[14, 19]}$.

3 The hybrid mesons

3.1 The quantum numbers

For the classification of hybrid mesons in a constituent model we will use the notations of $[8]$:

$l_g$: is the relative orbital momentum of the gluon in the $q\bar{q}$ center of mass;

$l_{q\bar{q}}$: is the relative orbital momentum between $q$ and $\bar{q}$;

$S_{q\bar{q}}$: is the total quark spin;

$j_g$: is the total gluon angular momentum;

$L = l_{q\bar{q}} + j_g$.

Considering the gluon moving in the framework of the $q\bar{q}$ pair, the parity of the hybrid will be:

$$P = P(q\bar{q}) . P(g) . P(\text{relative})$$

$$= (-1)^{l_g + 1} . (-1) . (-)^{j_g};$$

$(-1)$ being the intrinsic parity of the gluon. Then the parity of hybrid meson will be:
\[ P = (-)^{l_q + l_g}. \]

The charge conjugation is given by:
\[ C = (-)^{l_q + S_{q} + 1}. \]

Some \( J^{PC} \) states are given in Table 1 where lower sign stands for the state with \( S_{q} = 0 \). Our attention is taking aim essentially to the \( 1^- + \) states, which are predicted to be the lightest hybrid states by the theoretical models; furthermore some \( 1^- + \) candidates have been observed.

Let us consider the lightest \( 1^- + \) hybrid mesons: \( S_{q} = 0, \ l_q = 1 \) and \( l_g = 0 \), which we shall refer as the quark-excited hybrid (QE), and \( S_{q} = 1, \ l_q = 0 \) and \( l_g = 1 \), which we shall refer as the gluon-excited hybrid (GE).

### 3.2 The Hamiltonian and the wavefunctions

We have to solve the wave equation relative to the Hamiltonian:
\[ H = \sum_{i=q, \bar{q}, g} \left( \frac{\vec{p}_i^2}{2M_i} + \frac{M_i}{2} + \frac{m_i^2}{2M_i} \right) + V_{\text{eff}}; \quad (7) \]

with, for the hybrid meson (4):
\[ \alpha_{qq} = -\frac{1}{6}; \]
\[ \alpha_{\bar{q}g} = \alpha_{qg} = \frac{3}{2}. \]

We define the Jacobi coordinates:
\[ \vec{\rho} = \vec{r}_q - \vec{r}_{\bar{q}}; \]
\[ \vec{\lambda} = \vec{r}_g - \frac{M_q \vec{r}_q + M_{\bar{q}} \vec{r}_{\bar{q}}}{M_q + M_{\bar{q}}}. \]

Then, the relative Hamiltonian is given by:
\[ H_R = \frac{\vec{p}_\rho^2}{2\mu_\rho} + \frac{\vec{p}_\lambda^2}{2\mu_\lambda} + V_{\text{eff}}(\vec{\rho}, \vec{\lambda}) + \frac{M_q}{2} + \frac{m_q^2}{2M_q} + \frac{M_{\bar{q}}}{2} + \frac{m_{\bar{q}}^2}{2M_{\bar{q}}} + \frac{M_g}{2} + \frac{m_g^2}{2M_g}; \quad (8) \]

with
\[ \mu_\rho = \left( \frac{1}{M_q} + \frac{1}{M_{\bar{q}}} \right)^{-1}; \]
\[ \mu_\lambda = \left( \frac{1}{M_q} + \frac{1}{M_{\bar{q}} + M_q} \right)^{-1}; \]

5
and
\[
V_{\text{eff}}(\vec{p}, \vec{\lambda}) = -\alpha_s \left( -\frac{1}{6\rho} + \frac{3}{2} \frac{1}{|\vec{\lambda} + \frac{\vec{p}}{2}|} + \frac{3}{2} \frac{1}{|\vec{\lambda} - \frac{\vec{p}}{2}|} \right) + \sigma \left( -\frac{1}{6\rho} + \frac{3}{2} \frac{1}{|\vec{\lambda} + \frac{\vec{p}}{2}|} + \frac{3}{2} \frac{1}{|\vec{\lambda} - \frac{\vec{p}}{2}|} \right) + \frac{17}{6} c + V_S .
\] (9)

We have chosen to develop the spatial wave function as follows:
\[
\psi_{l\bar{q}q\bar{q}}(\vec{p}, \vec{\lambda}) = \sum_{n=1}^{N} a_n \varphi_{l\bar{q}q\bar{q}}^{aqtg}(\vec{p}, \vec{\lambda}) ;
\] (10)

where \( \varphi_{l\bar{q}q\bar{q}}^{aqtg}(\vec{p}, \vec{\lambda}) \) are the Gaussian-type functions:
\[
\varphi_{l\bar{q}q\bar{q}}^{aqtg}(\vec{p}, \vec{\lambda}) = \rho_{l\bar{q}q\bar{q}} \lambda_l \exp \left( -\frac{1}{2}n \beta_N \left( \rho^2 + \lambda^2 \right) \right) Y_{lqq\bar{m}qq}(\Omega_{\rho}) Y_{lqgm}(\Omega_{\lambda}).
\] (11)

Thus we solve the eigenvalue problem:
\[
H_{nm} a_m = \epsilon_N N_{nm} a_m ;
\]
where:
\[
H_{nm} = \int d\vec{p} \, d\vec{\lambda} \, \varphi_{l\bar{q}q\bar{q}}^{aqtg}(\vec{p}, \vec{\lambda})^* H_R \varphi_{l\bar{q}q\bar{q}}^{aqtg}(\vec{p}, \vec{\lambda})
\]
\[
N_{nm} = \int d\vec{p} \, d\vec{\lambda} \, \varphi_{l\bar{q}q\bar{q}}^{aqtg}(\vec{p}, \vec{\lambda})^* \varphi_{l\bar{q}q\bar{q}}^{aqtg}(\vec{p}, \vec{\lambda}).
\]

This process yields \( \epsilon_N(\beta_N, M_q, M_{\bar{q}}, M_g) \) which is then minimized with respect to parameters \( \beta_N, M_q, M_{\bar{q}} \) and \( M_g \).

For the potential parameters, we must distinguish between the light and the heavy flavors.

In Table 2 we present the parameters, fitting to the low lying isovector S-, P- and D-wave states of the light meson spectrum (except the mass of the strange quark obtained by fitting to the mass of Kaons)[19].

In Table 3 we give the parameters fitting to \( J_{PC} = 1^- \), (c\bar{c}) and (b\bar{b}) spectrum.
4 Hybrid mesons masses

We present in Table 4 our estimates of hybrid mesons masses for different flavors without spin effects; we take 800 MeV for the mass of the gluon. We compare our results with the predictions by other models.

We find the masses of the hybrid mesons larger in the GE mode than the QE mode. Indeed, the strong force being proportional to the color charge, the exchange of a color octet does require an important energy. Then the GE hybrid meson will be heavier than the QE hybrid, which is lower attractive.

We note that the mass of the QE hybrid state exceeds the mass of the corresponding orbitally excited $q\bar{q}$ meson state (which is of mass around $\sim 1.2$ GeV like $f_1$ or $b_1$) by $\sim 0.1$ GeV, the mass of the added gluon being 0.8 GeV. But we cannot consider the $q\bar{q}$ system of the $q\bar{q}g$ hybrid meson like the corresponding meson of mass 1.2 GeV: the $q\bar{q}$ system of the excited meson being a color singlet, and the $q\bar{q}$ system of the hybrid meson being a color octet, the difference has an important impact on the interaction between $q$ and $\bar{q}$; then the $\alpha_{ij}$ color coefficients (equ.4) will be different, and we have:

$$\frac{V(q\bar{q})_{1}}{V(q\bar{q})_{8}} = -8.$$ 

For the orders of magnitude of the masses, our results are in good agreement with the masses obtained by QCD Sum Rules\cite{12}, Lattice QCD \cite{11}, Flux-Tube Model\cite{6}, Bag Model\cite{10}, (massless) Const. Ghon Model\cite{9} and with the observed masses of candidates (namely $J^{PC} = 1^- +$ at 1.4 and 1.6 GeV).

As mentioned above to check the impact of the gluon mass value on the results, we have calculated hybrid mesons masses taking different values of $m_g$ (Table 5): the order of magnitude of the hybrid mass remains the same.

5 The mixed-hybrid states and spin effects

The followed expansion representing the hybrid wave function in the cluster approximation:

$$\Psi_{JM}(\vec{p}, \vec{\lambda}) = \sum_{n, l_{qq}, l_g} q_{n}^{l_{qq}l_g} \sum_{j_g, L, (m), (\mu)} q_{n}^{l_{qq}l_g} (\vec{p}, \vec{\lambda}) e^{\mu}_{\gamma} \chi_{S_{qq}}^{\mu_{qq}} \langle l_g m_g 1_{\mu_g} | j_g M_g \rangle \times \langle l_{qq} m_{q\bar{q}j_g} M_g | Lm \rangle \langle Lm S_{qq} \mu_{qq} | JM \rangle.$$
For the $J^{PC} = 1^- +$ states, restricting ourselves to the first orbital excitations ($l_{q\bar{q}}$ and $l_g \leq 1$) we can expand a mixing of the two modes (QE and GE):

$$\Psi_{1^- +} (\vec{p}, \vec{\lambda}) \simeq \sum_{n=1}^{N} a_n^{QE} \varphi_n^{QE} (\vec{p}, \vec{\lambda}) + \sum_{n=1}^{N} a_n^{GE} \varphi_n^{GE} (\vec{p}, \vec{\lambda}).$$

For the spin states we chose $\{|S_q\bar{q}; s_g; S\rangle \}$ ($s_g = 1$ et $S = S_{q\bar{q}} + s_g$).

Numerical results

Our numerical results show that the QE-hybrid and the GE-hybrid mix very weakly:

$$|1^- + (u\bar{u}g)\rangle \simeq -0.999 |QE\rangle + 0.040 |GE\rangle; \quad E \simeq 1.34 \text{ GeV}$$
$$|1^- + (u\bar{u}g)\rangle \simeq - |GE\rangle; \quad E \simeq 1.72 \text{ GeV}$$

$$|1^- + (s\bar{s}g)\rangle \simeq -0.999 |QE\rangle + 0.050 |GE\rangle; \quad E \simeq 1.60 \text{ GeV}$$
$$|1^- + (s\bar{s}g)\rangle \simeq - |GE\rangle; \quad E \simeq 2.02 \text{ GeV}$$

$$|1^- + (c\bar{c}g)\rangle \simeq -0.999 |QE\rangle - 0.040 |GE\rangle; \quad E \simeq 4.10 \text{ GeV}$$
$$|1^- + (c\bar{c}g)\rangle \simeq -0.031 |QE\rangle - 0.999 |GE\rangle; \quad E \simeq 4.45 \text{ GeV}$$

In fact these results are not altered by the spin-spin interaction and then are true for a $J^{PC} = 1^- -$.

In Table 6 we present the $1^- +$ light hybrid mesons masses calculated within spin-spin corrections (6). Note that for the string parameter $\sigma = \frac{3}{4} 0.20$ the mass of the lightest $1^- +$ hybrid $M_{u\bar{u}g}$ do not exceed 1.56 MeV.

We note that the spin-spin terms give an < 9% correction for the light flavors; we have neglected the L-S and tensor potential, which we suppose to be small[14, 19]. But an explicit calculation can be made, to obtain a rigorous result; the study will be interesting, essentially to remove the degeneracy between the states of the Table 1.

6 Conclusion

In this work we use a QCD-motivated potential (Coulombic plus Linear) to estimate masses of both light and heavy hybrid mesons, in the context of a
constituent Quark-Gluon Model, taking into account the spin-spin interaction effects for light hybrids. Our results are in a good agreement with the other methods, like Lattice QCD, QCD Sum Rules, Bag Model, ... and with the experimental candidates (1−+ at 1400, 1600 and 2000 MeV). We find also that 1−+ hybrid mesons may exist in two weakly-mixed modes: QE (l_qq = 1, l_g = 0 and S_qq = 0) and GE (l_qq = 0, l_g = 1 and S_qq = 1), the later being much heavier.

A more rigorous description of the hybrid meson can be given by improving the potential, to obtain a classification of all the states in Table 1, adding tensor terms and spin-orbit terms (which can generate a more significative mixing between the two QE-GE modes[8]).

At this time, it is more interesting to focus essentially on the 1−+ hybrid states, taking in account the recent experimental observations of the 1−+ hybrid candidates: π_1(1400), π_1(1600).

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References


Table 1: Hybrid meson quantum numbers.

<table>
<thead>
<tr>
<th>$l_{qar{q}}$</th>
<th>$l_q$</th>
<th>$P$</th>
<th>$J^C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>+</td>
<td>$0^+, 1^+, 2^+$</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>−</td>
<td>$0^+, 1^+, 2^+, 3^−$</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>−</td>
<td>$0^−, 1^−, 2^−, 3^+$</td>
</tr>
</tbody>
</table>

Table 2: Light flavors potential parameters\(^{[19]}\).

<table>
<thead>
<tr>
<th>$\sigma_h$ (GeV)</th>
<th>$\alpha_h$</th>
<th>$\alpha_s$</th>
<th>$\sigma$ (GeV(^2))</th>
<th>$c$ (GeV)</th>
<th>$m_u = m_d$ (GeV)</th>
<th>$m_s$ (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>.70</td>
<td>.840</td>
<td>0.857</td>
<td>.4 0.151</td>
<td>−0.4358</td>
<td>.375</td>
<td>.650</td>
</tr>
</tbody>
</table>

Table 3: Heavy flavors potential parameters.

<table>
<thead>
<tr>
<th>$\alpha_s$</th>
<th>$\sigma$ (GeV(^2))</th>
<th>$c$ (GeV)</th>
<th>$m_c$ (GeV)</th>
<th>$m_b$ (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.36</td>
<td>0.144</td>
<td>−0.45</td>
<td>1.70</td>
<td>5.05</td>
</tr>
</tbody>
</table>

Table 4: Predicted hybrid mesons masses (in GeV).

<table>
<thead>
<tr>
<th>model</th>
<th>$m_u$ (GeV)</th>
<th>$m_d$ (GeV)</th>
<th>$m_s$ (GeV)</th>
<th>$m_c$ (GeV)</th>
<th>$m_b$ (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coul. + Lin. $l_g = 0; l_{qar{q}} = 1$; QE</td>
<td>1.31</td>
<td>1.57</td>
<td>4.09</td>
<td>10.34</td>
<td></td>
</tr>
<tr>
<td>(Our results) $l_g = 1; l_{qar{q}} = 0$; GE</td>
<td>1.70</td>
<td>2.00</td>
<td>4.45</td>
<td>10.81</td>
<td></td>
</tr>
<tr>
<td>QCD Sum Rules</td>
<td>1.5</td>
<td>1.6-1.7</td>
<td>4.1-5.3</td>
<td>10.6-11.2</td>
<td></td>
</tr>
<tr>
<td>Lattice QCD</td>
<td>1.5-1.8</td>
<td>2.1</td>
<td>4.19</td>
<td>10.81</td>
<td></td>
</tr>
<tr>
<td>Flux Tube Model</td>
<td>1.8-2.0</td>
<td>2.1-2.2</td>
<td>4.2-4.5</td>
<td>10.8-11.1</td>
<td></td>
</tr>
<tr>
<td>Bag Model</td>
<td>1.3-1.8</td>
<td>2.5</td>
<td>3.9</td>
<td>10.49</td>
<td></td>
</tr>
<tr>
<td>Massless Const. Gluon Model</td>
<td>1.7</td>
<td>2.0</td>
<td>4.1</td>
<td>10.64</td>
<td></td>
</tr>
</tbody>
</table>

\( m_{\pi^0} (GeV) \) | 700 | 800 | 900 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>$\pi^0$ $GE$</td>
<td>1.52</td>
<td>1.70</td>
<td>1.77</td>
</tr>
<tr>
<td>$\pi^0$ $QE$</td>
<td>1.27</td>
<td>1.31</td>
<td>1.42</td>
</tr>
<tr>
<td>$\eta^0$ $GE$</td>
<td>4.39</td>
<td>4.45</td>
<td>4.52</td>
</tr>
<tr>
<td>$\eta^0$ $QE$</td>
<td>4.02</td>
<td>4.09</td>
<td>4.17</td>
</tr>
</tbody>
</table>
Table 5. The impact of $m_g$ on the results.

<table>
<thead>
<tr>
<th>$S$</th>
<th>$\bar{u}u_g$</th>
<th>$u\bar{s}g$</th>
<th>$s\bar{s}g$</th>
<th>Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.32</td>
<td>1.45</td>
<td>1.58</td>
<td>QE Mode ($l_{qq} = 1; l_g = 0$ and $S_{qq} = 0$)</td>
</tr>
<tr>
<td>0</td>
<td>1.56</td>
<td>1.72</td>
<td>1.87</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.69</td>
<td>1.84</td>
<td>1.99</td>
<td>GE Mode ($l_{qq} = 0; l_g = 1$ and $S_{qq} = 1$)</td>
</tr>
<tr>
<td>2</td>
<td>1.75</td>
<td>1.89</td>
<td>2.04</td>
<td></td>
</tr>
</tbody>
</table>

Table 6: $1^- +$ Light hybrid mesons masses (GeV).