Quantum cryptography, or more precisely quantum key distribution (QKD) is the only physically secure method for the distribution of a secret key between two distant partners, Alice and Bob [1]. Its security comes from the well-known fact that the measurement of an unknown quantum state modifies the state itself: thus an eavesdropper on the quantum channel, Eve, cannot get information on the key without introducing errors in the correlations between Alice and Bob. In equivalent terms, QKD is secure because of the no-cloning theorem of quantum mechanics: Eve cannot duplicate the signal and forward a perfect copy to Bob.

In the last years, several long-distance implementations of QKD have been developed, that use photons as information carriers and optical fibers as quantum channels [1]. Most often, although not always [2], Alice sends to Bob a weak laser pulse in which she has encoded the bit. Each pulse is a priori in a coherent state $|\sqrt{\mu}e^{i\theta}\rangle$ of weak intensity, typically $\mu \approx 0.1$ photons. However, since no reference phase is available outside Alice’s office, Bob and Eve have no information on $\theta$. Consequently, they see the mixed state $\rho = \int d\theta |\sqrt{\mu}e^{i\theta}\rangle \langle \sqrt{\mu}e^{i\theta}|$. This state can be re-written as a mixture of Fock states, $\sum_n n^{\prime} p_{n^{\prime}} (n^{\prime}) = e^{-\mu} \mu^n / n!$. Because two realizations of the same density matrix are indistinguishable, QKD with weak pulses can be re-interpreted as follows: Alice encodes her bit in one photon with frequency $p_1$, in two photons with frequency $p_2$, and so on, and does nothing with frequency $p_0$. Thus, in weak pulses QKD, a rather important fraction of the non-empty pulses actually contain more than one photon. For these pulses, Eve is then no longer limited by the no-cloning theorem: she can simply keep some of the photons while letting the others go to Bob. Such an attack is called photon-number splitting (PNS) attack. Although PNS attacks are far beyond today’s technology [3], if one includes them in the security analysis, the consequences are dramatic [4,5].

In this Letter, we present new QKD protocols that are secure against the most powerful PNS attack up to significantly longer distances, and that can thus lead to a secure implementation of QKD with weak pulses. These protocols are much better tailored than the ones studied before, to exploit the correlations that can be established using $\rho$. The basic idea is that Alice should encode each bit into a pair of non-orthogonal states belonging to two or more suitable sets.

The structure of the paper is as follows. First, we review the PNS attack on the first and best-known QKD protocol, the BB84 protocol [6], in order to understand why this attack is really devastating when the bit is encoded into pairs of orthogonal states. Then we discuss in general the benefits of using non-orthogonal states. Finally, we analyze in detail a protocol using four states, which is a simple modification of the BB84.

PNS attacks on the BB84 protocol. Alice encodes each bit in a qubit, either as an eigenstate of $\sigma_x$ (|+\rangle coding 0 or |−\rangle coding 1) or as an eigenstate of $\sigma_y$ (|+y\rangle or |−y\rangle, with the same convention). The qubit is sent to Bob, who measures either $\sigma_x$ or $\sigma_y$. Then comes a classical procedure known as "sifting" or "basis-reconciliation": Alice communicates to Bob through a public classical channel the basis, $x$ or $y$, in which she prepared each qubit. When Bob has used the same basis for his measurement, he knows that (in the absence of perturbations, and in particular in the absence of Eve) he has got the correct result. When Bob has used the wrong basis, the partners simply discard that item. Consider now the implementation of the BB84 protocol with weak pulses. Bob’s raw detection rate is the probability that he detects a photon per pulse sent by Alice. In the absence of Eve, this is given by

$$R_{\text{raw}}(\delta) = \sum_{n \geq 1} p_n \left(1 - (1 - \eta_{\text{det}} \eta_b)^n\right) \simeq \eta_{\text{det}} \eta_b \mu^\ell,$$

where $\eta_{\text{det}}$ is the quantum efficiency of the detector (typically 10% at telecom wavelengths), and $\eta_b$ is attenuation due to the losses in the fiber of length $\ell$:

$$\eta_b = 10^{-\delta / 10}, \quad \delta = \alpha \ell [\text{dB}].$$

Below, when we give a distance, we assume the typical value $\alpha = 0.25 \text{ dB/km}$. The approximate equality in (1) is valid if $\eta_{\text{det}} \eta_b p_n n \ll 1$ for all $n$, which is always the case in weak pulses QKD.
If we endow Eve with unlimited technological power within the laws of physics, the following attack is in principle possible: (I) Eve counts the number of photons, using a photon-number quantum non-demolition (QND) measurement; (II) she blocks the single photon pulses, and for the multi-photon pulses keeps one photon in a quantum memory; she forwards the remaining photons to Bob using a perfectly transparent quantum channel, \( \eta_b = 1 \) [7]; (III) she waits until Alice and Bob publicly reveal the used bases and correspondingly measures the photons stored in her quantum memory; she has to discriminate between two orthogonal states, and this can be done deterministically. This way, Eve has obtained full information about Alice’s bits, thence no processing can distill secret keys for the legitimate users; moreover, Eve hasn’t introduced any error on Bob’s side.

The unique constraint on PNS attack is that Eve’s presence should not be noticed; in particular, Eve must ensure that the rate of photons received by Bob (1) is not modified [8]. Thus, the PNS attack can be performed on all pulses only when the losses that Bob expects because of the fiber are equal to those introduced by Eve’s storing and blocking photons, that is, when the attenuation in the fiber is larger than a critical value \( \delta_c^{BB84} \) defined by

\[
R_{\text{raw}}(\delta_c^{BB84}) = \sum_{n \geq 2} p_n (1 - \eta_{\text{det}})^{n-1} \approx \eta_{\text{det}} p_2 . \tag{3}
\]

For \( \mu = 0.1 \), we find \( \delta_c^{BB84} = 13 \) dB, that is \( \ell_c^{BB84} \approx 50 \) km. For shorter distances, Eve can optimize her attack, but won’t be able to obtain full information; Alice and Bob can therefore use a privacy amplification scheme to retrieve a shorter secret key from their data.

In conclusion, for \( \delta \geq \delta_c^{BB84} \), the weak-pulses implementation of the BB84 protocol becomes in principle insecure. We present here a family of protocols whose implementation with weak pulses are more robust.

**Non-orthogonal states.** The extreme weakness of the BB84 protocol against PNS attacks is due to the fact that whenever Eve can keep one photon, she gets all the information, because she has to discriminate between two eigenstates of a known Hermitian operator. Let us thus modify the protocol as follows. Alice still encodes each bit in the state of a qubit, belonging either to the set \( \mathcal{A} = \{ |0_a\rangle, |1_a\rangle \} \) or to the set \( \mathcal{B} = \{ |0_b\rangle, |1_b\rangle \} \); but now \( |0_a|1_a\rangle = \chi_a \neq 0 \) and \( |0_b|1_b\rangle = \chi_b \neq 0 \) — we consider \( \chi_a = \chi_b = \chi \) for simplicity (Fig. 1, left).

Bob wants to be perfectly correlated with Alice. Although the two states are not orthogonal, one can construct a generalized measurement that unambiguously discriminates between the two; the price to pay is that sometimes one gets an inconclusive result [9]. Such a measurement can be realized by a selective filtering, that is a filter whose effect is not the same on all states, followed by a von Neumann measurement on the photons that pass the filter [10]. In the example of Fig. 1, the filter that discriminates between the elements of \( \mathcal{A} \) is given by

\[
F_A = \frac{1}{\sqrt{1 + x}} (|+x\rangle \langle 1_a^+| + |-x\rangle \langle 0_a^+|), \quad \text{where } |\psi^\perp\rangle \text{ is the state orthogonal to } |\psi\rangle.
\]

When the photons are prepared in a state of the pair \( \mathcal{A} \), a fraction \( 1 - \chi \) of them pass this filter, and in this case the von-Neumann measurement of \( \sigma_x \) achieves the discrimination. It is then clear how the cryptography protocol generalizes BB84: Bob randomly applies on each qubit one of the two filters \( F_A \) or \( F_B \), and measures \( \sigma_x \) on the outcome. Later, Alice discloses for each bit the set \( \mathcal{A} \) or \( \mathcal{B} \): Alice and Bob discard all the items in which Bob has chosen the wrong filter.

Of course, since not all the qubits will pass the filter even when it was correctly chosen, there is a small nuisance on Bob’s side because the net key rate is decreased. This is compensated by increasing \( \mu \) by a factor \( 1/(1 - \chi) \). However, the nuisance is by far bigger on Eve’s side, even when the increased mean number of photons \( \mu \) is taken into account, because Eve can obtain full information only if (i) she can block all the pulses containing one or two photons, and (ii) on the pulses containing three or more photons, she performs a suitable unambiguous discrimination measurement (see below) and obtains a conclusive outcome, which happens only with probability \( p_{\text{ok}} < 1 \). Consequently, the critical attenuation on this protocol is defined by

\[
R_{\text{raw}}(\delta_c) \approx \eta_{\text{det}} p_3 (\mu^2) p_{\text{ok}}.
\]

For typical values, one expects \( \delta_c - \delta_c^{BB84} \approx 10 \) dB, which means an improvement of some 40km in the distance.

All this analysis will be made quantitative by studying a given protocol. Before turning to it, we want to elaborate on Eve’s attacks.

**PNS attacks on non-orthogonal states.** In the case of a protocol with two sets \( \mathcal{A} \) and \( \mathcal{B} \), an attack based on single-photon unambiguous discrimination (1UD) gives Eve full information when she can always keep two or more photons. In fact, Eve can perform \( F_A \) on the first photon, \( F_B \) on the second photon [11]; only if both have passed the filter — which happens with probability \( p_{\text{ok}} = p_{\text{ok}} A p_{\text{ok}} B \) — she sends the remaining photons to Bob. This attack does not require a quantum memory, since Eve completes her measurements immediately; in its simplest implementation [12], photons can be split by a beam-splitter and the QND measurement is not required either (but in this case \( p_{\text{ok}} \) is lower).

We will not study the 1UD attack in detail because there exist a more powerful attack, against which any protocol using four states becomes completely insecure for the three-photon pulses [13,14]. Here is the argument: Alice sends to Bob one state belonging to the set \( \{ |\psi_k\rangle \}, \ \ k = 1, \ldots, 4 \). A pulse containing three photons is necessarily in one of the states \( |\psi_k\rangle^\otimes 3 \), that is, in the symmetric subspace of 3 qubits. The dimension of this subspace is 4, and it can be shown that all the \( |\psi_k\rangle^\otimes 3 \) are linearly independent [15]. Therefore, there exist a generalized unambiguous measurement \( \mathcal{M} \) that distinguishes
between them. Eve can then perform an attack based on multi-photon unambiguous discrimination: (I) she measures the number of photons; (II) she discards all pulses containing less than 3 photons; (III) on the pulses containing at least 3 photons, she performs $\mathcal{M}$, and if the result is conclusive she sends a new photon prepared in the good state to Bob. We shall refer to this attack as to intercept-resend (I-R) attack. As it was the case for the 1UD attack, this I-R attack does not need the quantum memory; in addition, it does not even need any lossless channel, since the new state can be prepared by a friend of Eve located close to Bob.

For attenuations $\delta < \delta_c$, the attacks just described are generally not those that give Eve the largest information. For instance, when she can block always one photon but generally not those that give Eve the largest information. For attenuations $\delta < \delta_c$, the attacks just described are generally not those that give Eve the largest information.

To describe the I-R attack, define $|\Psi_1\rangle = |+\rangle^{\otimes 3}$, $|\Psi_2\rangle = |+\rangle^{\otimes 3}$, $|\Psi_3\rangle = |\pm\rangle^{\otimes 3}$, and $|\Psi_4\rangle = |\mp\rangle^{\otimes 3}$. One can find four orthogonal states of three qubits, $|\Phi_k\rangle$, $k = 1,...,4$, such that $\langle \Phi_i | \Phi_j \rangle = \frac{\delta_{ij}}{\sqrt{2}}$ [16]. The unambiguous measurement $\mathcal{M}$ is then simply the von-Neumann measurement of $\sum_k |\Phi_k\rangle \langle \Phi_k|$, that gives a conclusive outcome with probability $p_{ok} = \frac{1}{4}$, which is indeed optimal [13,15]. Therefore, Eve will have full information as soon as $\eta_1 = p_{ok} p_{3}(\mu)$; for $\mu = 0.2$, this gives $\delta_c = 25.6 \text{ dB} \approx 2\delta_{BB84}$. The ultimate limit of robustness (in the case of zero errors) is thus shifted from $\sim 50\text{ km}$ to $\sim 100\text{ km}$ by using our simple modification of the BB84 protocol (Fig. 2). To further increase this limit, which is already beyond the one imposed by the fibers and detectors [1,3,17] at the moment of writing, one can move to protocols using six or more non-orthogonal states [13].

Effect of a QBER. In real experiments, dark counts in the detectors and misalignment of optical elements always introduce some errors; so we must ensure that the protocol does not break down if a small amount of error on Bob’s side is allowed. Note that our new protocol has a nice feature, namely: Alice and Bob can evaluate the QBER after the sifting procedure, without disclosing any bit [18]. Attacks at non-zero QBER are described in detail in Ref. [13]. Here we give just the two ideas.

Eve’s optimal individual attack is known: Eve must use the phase- covariant cloning machine [19], as for the BB84; but since Alice does not disclose a basis but a set of non-orthogonal states, Eve can extract less information from her clones. Indeed, the condition $I_{Bob} = I_{Eve}$ is fulfilled up to QBER=15%, a value which is slightly higher than the 14.67% obtained for the standard BB84 protocol. So our new protocol, designed to avoid PNS attacks in a weak-pulses implementation, is also robust against individual eavesdropping.

In addition, on this protocol a new kind of attacks must be taken into account, that we call PNS+cloning attacks. Focus on the range $\delta \simeq 10 - 20 \text{ dB}$ (see Fig. 2), where one-photon pulses can be blocked and the occurrence of three or more photons is still comparatively rare.
Because for the BB84 Eve has already full information in this range, such attacks have never been considered before. Eve could take the two photons, apply an asymmetric $2 \rightarrow 3$ cloning machine and send one of the clones to Bob; she keeps two clones and some information in the machine. By a suitable choice of the cloning machine, the QBER at which $I_{\text{Bob}} = I_{E}$ is lowered down to $\sim 9\%$.

Security of long-distance QKD. In the light of these results, the experimental data of Ref. [17] demonstrate that secure quantum key distribution over 67 km can be achieved. In that experiment, $\mu = 0.2$ and $\delta = 15\text{ dB};$ the total QBER was $5\%$ [20], smaller than the critical value of $9\%$. A secret key could thus be recovered by applying our sifting procedure and a suitable privacy amplification protocol to those data.

In summary, we have shown that by encoding a classical bit in sets of non-orthogonal qubit states, quantum cryptography can be made significantly more robust against photon-number splitting attacks. Protocols using four states allow to beat the most powerful PNS attack — and not only the so-called ”realistic attacks” [3,5] — up to distances that already exceed the limits imposed by present-day technology. These ideas can be implemented in a protocol, which is identical to the BB84 protocol for all the manipulations at the quantum level and differs only in the classical sifting procedure. Even longer distances can be reached by using more than four states [13]. This work stresses the importance of searching for the optimal exploitation of the correlations that quantum mechanics offers. For example, we showed that the correlations obtained by weak-pulses implementation of the BB84 protocol are better exploited using our new sifting procedures.

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[7] Throughout our analysis, we suppose that Eve cannot modify the detection probability of Bob. If Eve has an access to Alice’s and Bob’s labs, she could as well read the data on their computers!
[8] We assume, which is a further advantage for Eve, that Bob has only access to the average detection rate, and not to the statistics of the photons that he receives.
[11] A single filter, followed by a suitable measurement after the basis reconciliation, is enough to discriminate (i) any two states, as in the B92 protocol: Ch.H. Bennett, Phys. Rev. Lett. 68, 3121 (1992); but also (ii) the four states of the protocol described in: B. Huttner et al., Phys. Rev. A 51, 1863 (1995). Thus, for these protocols, the condition (3) essentially holds (one should multiply the r.h.s by the probability $p_{ok}$ of passing the filter).
[16] With $| + \rangle = |0\rangle$, $| - \rangle = |1\rangle$: $|\Phi_1\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$, $|\Phi_2\rangle = \frac{1}{\sqrt{2}}(|10\rangle + |01\rangle + |10\rangle + |11\rangle)$, $|\Phi_3\rangle = \frac{1}{\sqrt{2}}(|11\rangle - |00\rangle)$, $|\Phi_4\rangle = \frac{1}{\sqrt{2}}(|10\rangle - |01\rangle + |10\rangle + |11\rangle)$.
[18] Suppose that Alice sends $|+\rangle$ and declares the set $A_{++}$. After Eve’s intervention, the noisy state reaching Bob is of the form $(1 - D)|+\rangle + D|x\rangle$, with $D = \text{QBER}$. When Bob measures $\sigma_x$, he accepts the right result half of the times; when he measures $\sigma_z$, he accepts the wrong result with probability $D$. All in all, the fraction of the bits kept after sifting is $\frac{1}{2} + \frac{1}{2}D$, whence $D$ is immediately available to both Alice and Bob.
[20] If Eve cannot decrease Bob’s dark counts [7], privacy amplification must be performed only on the optical QBER, which was $1\%$.

FIG. 1. Two pairs of non-orthogonal states on the equator of the Poincaré sphere.
FIG. 2. PNS attacks with QBER=0 on the BB84 protocol for $\mu = 0.1$ as a function of the attenuation $\delta = \alpha \ell$. 