Abstract. It is known that the soft tail of the gamma-ray bursts’ spectra show excesses from the exact power-law dependence. In this article we show that this departure can be detected in the peak flux ratios of different BATSE DISCSC energy channels. This effect allows to estimate the redshift of the bright long gamma-ray bursts in the BATSE Catalog. A verification of these redshifts is obtained for the 8 GRB which have both BATSE DISCSC data and measured optical spectroscopic redshifts. There is good correlation between the measured and estimated redshifts, and the average error is \( \Delta z \approx 0.33 \). The method is similar to the photometric redshift estimation of galaxies in the optical range, hence it can be called as "gamma photometric redshift estimation". The estimated redshifts for the long bright gamma-ray bursts are up to \( z \simeq 4 \). For the faint long bursts - which should be up to \( z \simeq 20 \) - the redshifts cannot be determined unambiguously with this method.

Key words: Cosmology: large-scale structure of Universe – gamma-rays: bursts
1. Introduction

The gamma-ray bursts (hereafter GRBs) of the long sub-
group (Kouveliotou et al., 1993) detected by the BATSE
instrument (Meegan et al., 2000) are at high redshifts. The
highest directly measured redshift is at $z = 4.5$ (Ander-
sen et al., 2000; Mészáros, 2001), but there are indirect
considerations - based on BATSE data - predicting the
existence of redshifts up to $z \approx 20$ (Mészáros & Mészáros,
1995, 1996; Horváth et al., 1996; Balázs et al., 1998). This
result is based on distribution densities and deals with the
GRB redshifts in statistical sense only. This means that
one may obtain the fraction of GRBs being at a given red-
shift interval (see, for example, Schmidt (2001)), but one
cannot obtain the redshift of a given GRB event.

There are only a few cases, when the observation with
the BeppoSAX satellite (Piro et al., 2002) or other instru-
ments (Klose, 2000) made possible to detect the afterglows
and then the measurement of redshifts using optical spec-
troscopy. The Current BATSE Catalog (Meegan et al.,
2000) consists of more than 1200 long bursts, but for only
9 of them have redshift measurement (8 have redshifts
and there is one GRB with an upper redshift limit). This
result is based on distribution densities and deals with the
GRB redshifts in statistical sense only. This means that
one may obtain the fraction of GRBs being at a given red-
shift interval (see, for example, Schmidt (2001)), but one
cannot obtain the redshift of a given GRB event.

Hence, any method that could estimate the redshifts
from X-ray/gamma-ray observations alone would be a
great help.

In Ramirez-Ruiz & Fenimore (2000) and Reichart
et al. (2001) a linear relation between the intrinsic peak-
luminosities of GRBs and their so called ”variabilities”
was found. Similarly, Norris et al. (2000a) found a relation
between the so called spectral lag and the peak-luminosity
allowing to estimate the redshifts of long GRBs. These
relations were calibrated on a few cases of GRBs, when
GRBs were observed both by BATSE and other instru-
ments measuring the optical redshift from afterglows.
Then, having either the variabilities or the spectral lag of
a given GRB, one can estimate its redshift. The physical
meaning of the correlation between the variability (spec-
tral lag) and the peak-luminosity remains unclear. These
two methods can be combined (Schaefer et al., 2001) to
determine redshifts if all the needed input parameters are
available for the GRBs.

In this article we present a new method of the estima-
tion of the redshifts for the long GRBs. The situation is in
some sense similar to the optical observations of galaxies,
where the number of objects with broad band photomet-
ric observations is much larger than the number of objects
with measured spectroscopic redshifts. For galaxies and
quasars the growing field of photometric redshift estima-
tion (Koo, 1985; Connolly et al., 1995a; Gwyn & Hartwick,
1996; Sawicki et al., 1997; WangBahcall et al., 1998;
Fernández-Soto et al., 1999; Benítez, 2000; Csabai et al.,
2000; Budavári et al., 2000, 2001) achieved a great suc-
cess in estimating redshifts from photometry only. Here we
present a method that is quite similar to these methods;
hence we call it as gamma photometric redshift estimation
(GPZ for short). We utilize the fact that broadband fluxes
change systematically, as characteristic spectral features
redshift into, or out of the observational bands. Hence, contrary to the variability and spectral lag methods, this technique has a well defined physical meaning.

The article is structured as follows. First, using a spectral model for GRBs we deduce an expected relation between a measurable quantity (peak flux ratio) and the redshift (Sect. 2). Having this relation, we verify it on the existing sample of a few GRBs having measured redshifts (Sect. 3). Because both Sect. 2 and Sect. 3 suggest that this method is usable, Sect. 4 presents the estimated redshifts for hundreds of long GRBs. In Sect. 5 we discuss and summarize the results.

2. Gamma photometric redshift estimation

To understand our method in this Section we outline the general scheme of broadband observations. The method is generally the same both for the optical and gamma-ray ranges. The only major difference is that in the X-ray and gamma-ray range the extra- and intergalactic medium have negligible effects, but the optical photons are attenuated.

Let us take two different instrumental channels defined by $E_2 > E_3$ and $E_2 > E_1$. If one would know the rest-frame energy spectrum ($L(E)$) of the burst, for a perfect instrument that captures all the photons in the above energy channels, the observed luminosity (in units photons/s) for a burst at redshift $z$ would be the following:

$$L_{2,1} = \int_{(1+z)E_2}^{(1+z)E_1} L(E) dE, \quad L_{4,3} = \int_{(1+z)E_3}^{(1+z)E_1} L(E) dE,$$  

For the observed fluxes ($P$) similar equations can be used. Let us define the following flux ratio:

$$R(E_4, E_3, E_2, E_1, z) = \frac{L_{4,3} - L_{2,1}}{L_{4,3} + L_{2,1}} = \frac{P_{4,3} - P_{2,1}}{P_{4,3} + P_{2,1}},$$

which in general depends on the redshift, since $L_{2,1}$ and $L_{4,3}$ depend on the redshift.

Assume for the moment that one observes a pure power-law spectrum. This means that $L_E \propto E^{-\alpha}$ holds, where the exponent $\alpha$ is a real number. In this special case one could prove easily that for any redshift $z$

$$\frac{L_{4,3}^{z=0} - L_{2,1}^{z=0}}{L_{4,3}^{z=0} + L_{2,1}^{z=0}} = \frac{L_{4,3} - L_{2,1}}{L_{4,3} + L_{2,1}},$$

where

$$L_{2,1}^{z=0} = \int_{E_1}^{E_2} L(E) dE, \quad L_{4,3}^{z=0} = \int_{E_1}^{E_4} L(E) dE.$$  

This means that in this special case $R = R(E_4, E_3, E_2, E_1, z)$ is not depending on $z$.

Of course, in the real situation, the spectrum has got a more complicated form, and hence $R$ is depending on $z$; this will be the effect that we will use for redshift estimation.

In addition, in the real situation, the incident spectrum measured by the detector is convolved with the detector’s response function defined by response matrix (Pendleton et al., 1994) resulting the measured flux of the corresponding channel. For the channel with energy range $E_2 > E > E_1$ the measured flux $P_{1,2}$ is therefore given by

$$P_{1,2} = \int_{E_1}^{E_2} P(E) c(E) dE,$$  

where $c(E)$ is the detector’s response function. The similar holds for the second channel, too, with the same $c(E)$.

Hence, in general, $R$ is depending both on the spectrum and the response function.

If the rest-frame spectrum for a GRB is known, one is able to calculate the theoretical $R$ as a function on $z$. Then these values can be compared with the flux ratio obtained from the broadband measurements ($R_{\text{meas}}$). The redshift, where $(R - R_{\text{meas}})^2$ is minimal, could give the estimated gamma photometric redshift.

Regarding this gamma photometric redshift estimation, the major problem comes from the fact that the spectra are changing quite rapidly with time; the typical timescale for the time variation is $\approx (0.5 - 2.5)\, s$ (Ryde & Svensson, 1999, 2000). Hence, if possible, one should consider spectra which are defined for time intervals smaller than this characteristic time. Therefore, we will consider the spectra in the 320 ms time interval (i.e. in five 64 ms time intervals), with the peak-flux being at the center of this time interval.

In the following we will assume that the spectrum has the same shape around the time of the peak-flux for all long bursts. Unfortunately we do not have any deep theoretical or observational evidence for this assumption, instead we will test our assumption on GRBs, where spectroscopic redshifts are available (next Section). Because, this assumption seems to be acceptable, in Sect. 4 we will use $R$ to estimate $z$ for long GRBs.

3. Application on GRBs: Calibration

It is well-known (Band et al., 1994; Amati et al., 2002) that the time-integrated average spectra of GRBs can be approximated by a broken power-law; the break is at some energy $E_o$. The typical rest-frame energy for $E_o$ is above $\approx 500\, \text{keV}$ (Preece et al., 2000a; Preece et al., 2000b), but this might vary for different GRBs.

Of course this broken power-law spectrum is simply an approximation; first, because the break around $E_o$ may have a more complicated form (Preece et al., 2000a; Preece et al., 2000b), and, second, because at low rest-frame energies (around $\approx 80\, \text{keV}$) there may be essential departures from the power-law. This is the so called soft-excess, which is confirmed for $\approx 15\%$ of GRBs on the high confidence level (Preece et al., 1996, 2000a; Preece et al., 2000b); and for the remaining GRBs the soft-excess seems to occur, too (Preece et al., 1996).

Based on this, we construct our template spectrum that will be used in the GPZ process in the following manner: Let the spectrum be a sum of the Band’s function (Band et al., 1994), and of a low energy power-law function tak-
ing the form
for $E \leq E_0$, $L(E) = a(E/E_{cr})^{-\alpha} + a(E/E_{cr})^{-\beta}$ (6)
for $E \geq E_0$, $L(E) = a_3(E/E_0)^{-\gamma}$, (7)
where $a_3 = a[(E_0/E_{cr})^{-\alpha} + (E_0/E_{cr})^{-\beta}]$ comes from the normalization. In this spectrum there are six parameters, but the amplitude $a$ is for $R$ unimportant, and need not be specified. We fix all the above parameters according to the available literature, so there are no free tunable parameters in our method. The low energy cross-over is to the available literature, so there are no free tunable parameters in our method. The low energy cross-over is

As we remarked above, the spectrum is rapidly changing. But our assumption is that the spectrum has a characteristic shape around the instant of the peak. Now we have to chose a short time interval around the peak (maximum of the total counts), during which the change of spectrum is still negligible, but the number of photons allows good signal-to-noise ratio. To be able to cut out such time interval around the peak-flux, we need data with a reasonably good time resolution. In our study we will use the 64 ms resolution BATSE LAD DISCSC data from the public BATSE Catalog (Meegan et al., 2000). During this time interval the change of our template spectrum is still negligible (Ryde & Svensson, 2000).

We have also checked the robustness of the PFR against the integration time around the peak. Both the doubling of the integration time (for 640 ms) and its skewing around the peak did not change significantly the values of PFR. All this means that the template spectrum defined by Eqs.(6-7) seems to be a good approximation for the 320 ms time interval around the peak. This is in fact expectable from earlier studies of spectra (Band et al., 1994; Ryde & Svensson, 1999, 2000).

The 4 energies for the BATSE instrument are: $E_1 = 25$ keV, $E_2 = E_3 = 55$ keV, $E_4 = 100$ keV. Using the detector response matrices (Pendleton et al., 1994) one can calculate the observed counts and flux for any incoming spectrum. In Fig. 1 we show a typical response function $c(E)$. The response function is different for each burst, but using the BATSE DRM data one can use the actual response function for every burst. Fig. 1 also demonstrates the behaviour of the spectrum at different redshifts. Going from $z = 0$ to higher redshifts one can see that the soft-excess moves from the second channel to the first one and then leaves the range of this detector around $z \approx 4$.

Before starting the detailed investigation of the fluxes that one can get using the template spectra and the response matrices by Eq.(6), let us test the correctness of the template spectrum in a simple way. Let us introduce the peak flux ratio (PFR hereafter) in the following way:

$$\text{PFR} = \frac{l_{34} - l_{12}}{l_{34} + l_{12}}$$

where $l_{ij}$ is the BATSE DISCSC flux in energy channel $E_i < E < E_j$ integrated for 320 ms around the peak flux. (I.e. five 64 ms intervals are summed - the middle one is where the flux is the biggest. Even during this time interval the change of spectrum is still negligible (Ryde & Svensson, 2000)). Our theory says that for the above template spectrum this ratio should increase with $z$. On Fig. 2 we plot the theoretical PFR curves calculated from the above defined template spectrum using the average detector response matrices for the 9 bursts that have both BATSE data and measured redshifts. These bursts’ data are collected in Table 1.

![Fig. 1. The detector response function $c(E)$ and the behaviour of a template spectrum at different redshifts.](image1)

![Fig. 2. The theoretical PFR curves calculated from the template spectrum using the average detector response matrix.](image2)
Table 1. The redshifts of GRBs that have both BATSE triggers and measured spectroscopic redshifts. Data compiled by Klose (2000); see also Bloom et al. (2001).

<table>
<thead>
<tr>
<th>Burst</th>
<th>BATSE z</th>
<th>Remark</th>
</tr>
</thead>
<tbody>
<tr>
<td>980425</td>
<td>6707</td>
<td>0.00857 SN1998bw</td>
</tr>
<tr>
<td>970508</td>
<td>6225</td>
<td>0.8356</td>
</tr>
<tr>
<td>970828</td>
<td>6350</td>
<td>0.9578 no DISCSC data</td>
</tr>
<tr>
<td>980703</td>
<td>6891</td>
<td>0.9676</td>
</tr>
<tr>
<td>991216</td>
<td>7906</td>
<td>1.020</td>
</tr>
<tr>
<td>990123</td>
<td>7343</td>
<td>1.6196</td>
</tr>
<tr>
<td>990510</td>
<td>7560</td>
<td>3.4127</td>
</tr>
<tr>
<td>971214</td>
<td>6533</td>
<td>3.5 upper limit only</td>
</tr>
</tbody>
</table>

remaining 6 GRBs have a clearly increasing PFR with increasing z.

In the used range of z (i.e. for $z \lesssim 4$) the relation between z and PFR is invertable. Hence we can use it to estimate the gamma photometric redshift (GPZ) from a measured PFR. In Fig. 4 the measured spectroscopic redshifts are compared with GPZ values for 8 considered GRBs. The errorbars show the effect of counts’ Poisson noise only.

Leaving out GRB associated with the supernova and GRB having upper redshift limit only, the estimation error is $\Delta z = \sqrt{\sum_{i=1}^{6}(z_{i}^{spec} - z_{i}^{GPZ})^2/5} \approx 0.33$.

In order to test the reality of the correlation between the soft excess and the redshift we made the null hypothesis that there is no relationship between these quantities, i.e the computed correlation is purely random. Assuming no true correlation between the soft excess and redshift the probability density of the computed quantity can be given by

$$f(x) = \frac{1}{\sqrt{\pi}} \frac{\Gamma((N - 1)/2)}{\Gamma((N - 2)/2)} (1 - x^2)^{(N-4)/2},$$

where $N$ is number of data points (Spiegel & Stephens, 1999). Let $r_c$ be the calculated correlation. Then the $\beta$ error probability at rejecting the null hypothesis is given by

$$\beta = 1 - \int_{-1}^{r_c} f(x)dx.$$

In Table 2, the $N$, $r_c$ and the calculated level of significance are shown for various cases.

Although it seems that $z^{GPZ}$ for GRB 971214 (where $z = 3.4127$) fits very well the estimation error without it is better: $\Delta z \approx 0.29$. However the linear correlation coefficient here with $N = 5$ yields a much poorer $r_c = 0.66$ with a $p = 0.89$ significance.

We see that PFR (if calculable from observations for the given burst) is a quantity that may allow to determine redshift. Problems may arise from the fact that for any value of PFR two redshifts are possible - either below or above $z \simeq 4$ (see Fig. 2), further measurements are needed to exclude one of the redshift.
4. Application on GRBs: Estimation of the redshifts

To avoid the problems with the instrumental threshold we exclude the faintest GRBs from the BATSE data. Similarly to Pendleton et al. (1997) and Balázs et al. (1998) these events have a \( F_{256} \) peak-flux (i.e. on 256 ms trigger scale) smaller than 0.65 photon/(cm²s). These GRBs are not discussed in this article.

Further restriction comes from the fact that short GRBs are today taken as different phenomena (Horváth et al., 2000; Norris et al., 2000b). In addition, due to instrumental effects (Piro et al., 2002), no spectroscopic redshifts are known for this subgroup of GRBs. Hence, we do not apply our method for short GRBs.

The reality of the intermediate subgroup of GRBs (Horváth, 1998; Mukherjee et al., 1998; Hakki, et al., 2000; Balastegui et al., 2001; Rajaniemi et al., 2002; Horváth, 2002) having remarkable sky angular distribution (Mészáros et al., 2000a,b; Litvin et al., 2001) is unclear yet. In any case, no spectroscopic redshifts are known also here. Hence, we exclude this subgroup, too.

Therefore, we restrict ourselves to long GRBs defined by \( T_{90} \) (Meegan et al., 2000) with \( T_{90} > 10 \) s. There are 1241 GRBs in BATSE Catalog fulfilling this condition. Deleting GRBs having no \( F_{256} \) and having \( F_{256} < 0.65 \) photon/(cm²s) 838 GRBs remain. This sample is studied here.

Introducing an another cut \( F_{256} > 2.0 \) photon/(cm²s) we can investigate roughly the brighter half of this sample. We will discuss the sample \( F_{256} > 2.0 \) photon/(cm²s) (“bright half” sample having 343 GRBs) and \( F_{256} > 0.65 \) photon/(cm²s) (“all” sample having 838 GRBs), respectively.

As the soft-excess range redshifts out from the BATSE DISCSC energy channels around \( z \approx 4 \), the theoretical curves converge to a constant value. For higher \( z \) it starts to decrease. This is where the power-law breakpoint (\( E_\gamma \)) is redshifts into soft energy range. This means that the method is ambiguous: for the given value of PFR one may have two redshifts - below and above \( z \approx 4 \). Because for the bright GRBs the values above \( z \approx 4 \) are practically excluded, for them the method is usable. In other words, using only the 25–55 keV and 55–100 keV BATSE energy channels, this method can be used to estimate GPZ only in the redshift range \( z \lesssim 4 \); outside of this region the \( z \) vs. PFR relation is non-invertable (see Fig. 2). For high redshifts (above \( z \approx 4 \)) the method gives two possible values. For faint GRBs the estimation also usable (at least in principle), but one has to decide by other arguments that either the redshift below \( z \approx 4 \) or above \( z \approx 4 \) is the correct value.

Let us assume for a moment that all observed long bursts, we have selected above, have \( z < 4 \). Then we can simply calculate the \( z^{GPZ} \) redshift for any GRB, which has calculable PFR from BATSE DISCSC data. Fig. 5 shows the distribution of the measured PFRs of the long GRBs having DISCSC data. The fact that the number of objects beyond the minimal and maximal theoretical PFR values (\( \sim -0.15 \) and \( \sim 0.37 \), respectively) is relatively small, is reassuring.

Fig. 6 shows the distribution of the estimated derived redshifts under the assumption that all GRBs are below \( z \approx 4 \). The distribution has a clear peak value around PFR \( \sim 0.2 \), which corresponds to \( z \approx (1.5 - 2.0) \).

5. Discussion

Having the estimated redshifts shown on Fig. 6 one may ask: Are these redshifts really correct?

There can be two different problems here. First of all, the method is based on the assumption that around the peak flux, the spectrum is the same for all the selected long GRBs. Second, the method gives degenerate result, with two possible redshift values.

Concerning the first problem we could just hope that in the near future some theoretical or experimental ev-
idence will confirm our assumption, but the situation is not worse than in the studies of Norris et al. (2000a) and Reichart et al. (2001). These articles also suggest that despite the deeper understanding of the underlying physics, the procedure itself is usable. In addition, here the PFR-z relation is well supported by earlier independent observations.

Concerning the second problem we think that the great majority of values of $z$ obtained for the bright half are correct. This opinion may be supported by three independent arguments. First, the obtained distribution of GRBs in $z$ for the bright half on Fig. 6 is very similar to the obtained distribution of Schmidt (2001) (see Fig. 6 of that article). The luminosity-based redshift distribution (Schaefer et al., 2001) also suggest an uniformly rising GRB density out to $z \approx 5$. Second, as $z$ moves into $z < 4$ regime for the bright GRB, one would obtain extremely high luminosities. Using Eq.12 of Mészáros & Mészáros (1996), there is a lower limit for the isotropic luminosity of the GRBs a value $\simeq 10^{53}$ ergs/s. (Note here that the precise value is, of course, calculable and is depending on the chosen cosmology model and on the typical energy of emitted photons. For the purpose of this article this approximate value is enough.) This is an unacceptable high value for a lower limit, because typical luminosities are $\simeq 10^{51-52}$ ergs/s. (see, e.g., Table 1 of Reichart et al. (2001)). We cannot exclude that a few cases from bright GRB are at $z < 4$, but - we think - in the bright half this cases are rare. Thirds, as an additional statistical test we compared the redshift distribution of the 17 GRB with observed redshift with our reconstructed GRB $z$ distributions (limited to the $z < 4$ range). For the $F_{256} > 0.65$ photon/($\text{cm}^2\text{s}$) group the Kolmogorov-Smirnov test suggests a 38% probability, i.e. the observed $N(< z)$ probability distribution agrees quite well with the GPZ reconstructed function. Although the observed distribution suffers from strong selection effects this fact is nevertheless reassuring.

For the faint GRBs being between $F_{256} = 0.65$ photon/($\text{cm}^2\text{s}$) and $F_{256} = 2.00$ photon/($\text{cm}^2\text{s}$) the situation is different. From Fig. 6 it follows that for $z < 1.7$ GRBs should be dominated by faint objects. From this Figure one would obtain that GRBs are in average at smaller redshifts. This is clearly a wrong conclusion, which is caused by the false assumption that $z < 4$ for all the faint GRBs. We think that the majority of faint GRBs $z$ should be changed into the value $z < 4$. Unfortunately, we are not able to say, concretely which GRB has a great ($z > 4$), and which GRB has still a small ($z < 4$) redshift. In addition, for these faint GRBs also the error of estimated redshifts should probably be bigger than $\Delta z \approx 0.33$. Simply, we conclude that in the current form with the current data, our method is not applicable for the faint GRBs.

The results of this work may be summarized as follows.

1. Based on earlier observations of GRB spectra it is shown that the peak flux ratio (PFR) should be a well defined function of $z$.
2. The estimated redshifts from PFR are in good accordance with the known redshifts of the few GRBs in BATSE Catalog having spectroscopic redshifts.
3. All this allows us to calculate the redshifts of long GRBs. Unfortunately, due to the twofold character of the PFR curve, the method is usable only for bright GRBs.
4. Redshift distribution of 343 bright long GRBs are determined (Fig. 6).

Acknowledgements. The useful remarks with Drs. T. Budavári, S. Klose, D. Reichart, A.S. Szalay and the anonymous referees are kindly acknowledged. This research was supported in part through OTKA grants T024027 (L.G.B.), F029461 (I.H.) and T034549. Czech Research Grant J13/98: 113200004 (A.M.), NASA grant NAG5-9192 (P.M.).

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