Cosmological Effects of a Class of Fluid Dark Energy Models

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We study the impact of a generalized Chaplygin gas as a candidate for dark energy on CMB anisotropies. The generalized Chaplygin gas is a fluid component with an exotic equation of state, \( p = -A/\rho^\alpha \) (a polytropic gas with negative constant and exponent). Such component interpolates in time between dust and a cosmological constant, with an intermediate behaviour as \( p = \alpha \rho \). Perturbations of this fluid are stable on small scales, but behave in a very different way with respect to standard quintessence. Moreover, a generalized Chaplygin gas could also represent an archetypal example of a phenomenological unified models of dark energy and dark matter. The results presented here show how the CMB anisotropies induced by this class of models differ from a ΛCDM model.

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1. According to the present recipe to explain observational data, two dark components seem to fill the universe up to 95% of the total content. The baryon density is indeed very small, \( \Omega_\mu h^2 \approx 0.02 \) [1], assuming a flat Universe. This situation has been slowly reached over decades. In addition to cold dark matter (CDM), in the 90s a cosmological constant term \( \Lambda \) was called into play to explain the recent acceleration of the universe indicated by the Supernova data [2], but then confirmed by other observations.

Avoiding to explain on theoretical grounds the embarrassing smallness of a cosmological constant \( \Lambda \) constrained by observations, a scalar field \( Q \) [3], dubbed quintessence, was suggested in order to explain the recent acceleraring of the universe indicated by the Supernova data [2], but then confirmed by other observations.

An alternative to quintessence for modelling a dark energy component could be a perfect fluid (henceforth PF) with a generic pressure \( p = \rho(\phi) \) (not being linear in energy density \( \rho \) as for barotropic fluids) whose energy momentum tensor is:

\[
T_{\mu\nu} = \rho g_{\mu\nu} + (\rho + p) u_\mu u_\nu
\]

where \( g_{\mu\nu} \) is the metric and \( u_\mu \) is the fluid velocity \((u_\mu u^\mu = -1)\). From a theoretical point of view a scalar field description would be preferable in the logic to relate an accelerating universe with a fundamental quantum. From a phenomenological point of view the reasons to prefer one over the other are less obvious. An exotic fluid capable to develop a negative pressure at late times may represent as well the effective degree of freedom which drives the present acceleration of the universe. In particular, a PF model with this property would allow to explore the possibility that dark energy clusters on small scales. Indeed, it is important to note that, by parametrizing dark energy with an uncoupled scalar field with ordinary kinetic term, one does implicitly assume no clustering of dark energy on scales smaller than the Hubble radius.

Among the class of PF models which could work as a dark energy component - in principle one could design suitable pressure profiles \( p(\rho) \) instead of potentials \( V(\phi) \) for a quintessence field \( \phi \) - the Chaplygin gas [6] has recently received a lot of attention [7]. A Chaplygin gas is characterized by a pressure \( p_X \) related to the energy density \( \rho_X \) in the following way:

\[
p_X = -\frac{A}{\rho_X^{\alpha}}
\]

with \( \alpha = 1 \). Even if such exotic fluid was proposed in the context of aerodynamics [6], there are interesting connections with particle physics and d-branes [8]. A Chaplygin gas is also equivalent to a tachyon field with a constant potential [9] and, at the homogeneous level, to a complex scalar field [10] or to a quantum scalar field [11] in the bottom of a potential (called Thomas-Fermi approximation in [10]). In this paper, we study a generalized version of the Chaplygin gas (henceforth GCG) [12] by considering \( 0 < \alpha \leq 1 \) in Eq. (2).

2. In a Robertson-Walker metric

\[
ds^2 = -dt^2 + a(t)^2 \left( \frac{dr^2}{1-Kr^2} + r^2 d\Theta^2 \right),
\]

where \( K = 0, \pm 1 \) is the curvature of the spatial sections and \( \Theta \) is the solid angle, the energy conservation equation for a GCG

\[
\dot{\rho}_X + 3H(\rho_X + p_X) = 0
\]

can be immediately integrated [7,12]:

\[
\rho_X = \left( A + \frac{B}{a^{4(1+\alpha)}\dot{a}} \right)^{\frac{1}{\alpha}}.
\]

where \( A, B \) are constants with dimensions \([M^{4(1+\alpha)}]\). We note that a GCG reduces to a ΛCDM model for \( \alpha = 0 \).
and to a sCDM for $A = 0$. We see that this fluid with an exotic equation of state behaves like dust for small $a$ (when $B/A \gg a^{3(1+\alpha)}$, assuming $a = 1$ at present time) and a cosmological constant given by $A^{-\alpha}$ in the opposite limit ($B/A \ll a^{3(1+\alpha)}$). By Taylor expanding in this limit [7,12] we obtain from Eqs. (5) and (2):

$$
\rho_X \simeq A^{-\alpha} \left( 1 + \frac{B}{1 + \alpha} a^{3(1+\alpha)} + O \left( \frac{B^2}{A^2} \right) \right),
$$

$$
p_X \simeq A^{-\alpha} \left( -1 + \frac{\alpha B}{1 + \alpha} a^{3(1+\alpha)} + O \left( \frac{B^2}{A^2} \right) \right). \quad (6)
$$

Therefore, in this limit a GCG behaves as a sum of a cosmological constant and a perfect fluid characterized by $p = \alpha \rho$, shining light upon the physical meaning of the parameter $\alpha$. We note that the solution (5) to Eq. (4) is valid for any $\alpha > -1$, but also for $\alpha < -1$ (a standard polytropic gas). In this latter case the behaviour of such a PF interpolates between a cosmological constant and dust. For $\alpha = -1$ Eq. (2) describes the usual barotropic perfect fluid. According to Eq. (2), the equation of state $w_X$ defined as:

$$
w_X = \frac{p_X}{\rho_X} = -\frac{A}{\rho_X^{1+\alpha}} \quad (7)
$$

decreases from the value 0 to $-1$. An example of a background evolution for a GCG as dark energy is given in Fig. (1).

The presence of a GCG as a dark energy component could be distinguishable from a quintessence component because of the parametric form of the pressure $p = p(\rho)$. The time derivative of the equation of state of the dark energy component is important in the program of reconstructing the total equation of state [13]. Indeed, the time derivative of $w_X$ for a GCG is

$$
\dot{w}_X = 3H(\alpha + 1)w_X(1 + w_X) \quad (8)
$$

while for a scalar field $\phi$ with potential $V = V(\phi)$ is:

$$
\dot{w}_\phi = 3H(1 + w_\phi)(w_\phi - 1) - 2 \frac{\dot{V}}{\rho_\phi}. \quad (9)
$$

SN Ia observations can constrain the GCG as a candidate for dark energy, as recently studied by different authors [14]. The purpose of this letter is to show how CMB data could be more selective.

3. The behaviour of perturbations is a very interesting aspect of the model. When trying to build a model for an accelerating universe with a barotropic (constant equation of state $w$) perfect fluid, one runs in the problem of instabilities on short scales because of a negative sound speed for perturbations. Infact, the sound speed is equal to the equation of state $w$ and this must be negative ($w < -1/3$) to explain the acceleration. This is the usual problem for a fluid description of domain walls and cosmic strings. Quintessence models with scalar fields with a standard kinetic term do not have this problem. The sound speed for a scalar field is equal to 1. K-essence models [15] based on scalar fields with a non-standard kinetic terms are different in this respect [16], but still have positive sound speed. For a GCG the sound speed $c^2_X$ for perturbations is

$$
c^2_X = \frac{\partial p}{\partial \rho} = \alpha \frac{A}{\rho_X^{1+\alpha}} = -\alpha w_X. \quad (10)
$$

Therefore, because of its non-barotropic nature, perturbations of a GCG are stable on small scales even in an accelerating phase, and behave similarly to dust perturbations when the gas is in the dust regime. When the behaviour of the background Chaplygin gas is of $\Lambda$-type, the sound-speed is $\alpha$. In order to avoid causality issues, we shall consider $\alpha \leq 1$. This latter constraint, with the requirement of positive sound speed ($\alpha > 0$), marks the interesting physical range of $\alpha$. We note that the possibility of having a non negative sound speed and a negative equation of state, as happens for a GCG, could open a new development in modelling a domain wall or cosmic string network which is not plagued by short scale instability for perturbations.

The equations for the energy density contrast $\delta_X = \delta \rho_X / \rho_X$ and the velocity potential $\theta_X$ in the synchronous gauge are, according to Ref. [17] and by using Eqs. (8,10)

$$
\delta'_X = -(1 + w_X) \left( \theta_X + \frac{h'}{2} \right) + 3H(w_X - c^2_X)\delta_X
$$

$$
\theta'_X = -H(1 - 3c^2_X)\theta_X + \frac{c^2_X}{1 + w_X} \frac{k^2}{h^2} \delta_X, \quad (11)
$$

where $h$ is the trace of the metric perturbations in the synchronous gauge [17]. This set of equations agrees with those used in [18]. In order to study the Jeans instability for a GCG it is useful to study the equation for the (gauge invariant) comoving density contrast $\Delta_X = \delta_X + 3(1 +
to CDM in the dust limit (when \( w_\text{X} \approx 0 \)). However, because of the time dependence of \( w_\text{X} \sim c_\text{X}^2 \sim 0 \). We confirm numerically this behaviour, also when other fluids are present. In Fig. (2) we show the comparison of the evolution of cosmological perturbations and on CMB anisotropies (without CDM). We have implemented Eqs. (5) and (11) in a modified version of CMBFAST [19]. We have compared the code against the sCDM model obtained by setting \( A = 0 \) in Eq. (5) and we have considered an initial adiabatic scale invariant spectrum for perturbations. Instead, by setting \( B = 0 \) in Eq. (5) one obtains a ΛCDM model in the background, but with not-trivial perturbations in the dark energy sector (see Eq. (12)).

In Fig. (3) we show the dependence of the spectrum of temperature anisotropies \( \delta T/T \) on the ratio \( B/A \) and \( \alpha \). The three parameters \( A, B, \alpha \) are in direct relation with the physical quantities of dark energy at present time, as \( \Omega_{X_0}, w_{X_0}, \dot{w}_{X_0} \). Therefore these parameters can be constrained by maximizing the likelihood function with the present data [20]. In particular, from the left panel of Fig. (3) one can see how spectra can be sensibly different for a GCG model which differs from a ΛCDM model by less than 10% in the background evolution.

Because of its early dust behaviour, a GCG may also represent a prototypical unified model of dark matter and dark energy (see [21] for a similar proposal, but with a scale dependent equation of state). In Figs. (4,5) we compare GCG models without CDM (\( h = .7, \Omega_{X_0} = .95 \)) with the BOOMERANG [22], MAXIMA [23] and DASI [24] data, when varying \( B/A \) and \( \alpha \). In particular, in Fig (4) we can see how all the models lie below the limiting case of sCDM (\( A = 0 \)) and a Λ-baryon model (very similar, but not equivalent to \( B = 0 \)). The resulting spectrum of CMB anisotropies constrain these GCG models [20] much more than the SN Ia data [14].

4. We have studied the implications on the evolutions of cosmological perturbations and on CMB anisotropies of a GCG as a candidate for dark energy. This GCG covers all the interesting possible cases of a dark energy model from a polytropic gas. A GCG is more distinguishable from a ΛCDM model than a QCDM model [5] since both GCG background and perturbations are im-
important not only at late redshift, when the GCG behaves like a cosmological constant. Infact the GCG plays a role of dust component before turning in a cosmological constant, modifying not only the positions of the peaks, but the overall shape of the $C_l$ because of a big Integrate Sachs-Wolfe (ISW) effect. In this sense, the QCDM models with standard scalar fields [3,4] are the most economic way to modify a ACDM model in a dark energy model. The dark component sector(s) may be much more obscure and less simple [25], and the GCG models are an example of this. The next CMB experiments [26,27] and LSS data will be be very hepful in constraining the physical properties of the dark component sector(s).

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Note added: While this paper was in preparation a preprint [28] by Bento et al., which studies the location of the CMB peaks in presence of a unified GCG model within an analytic approximation, appeared. We have checked their analytic results with our code and we have found a systematic overestimation of the peak positions (in particular for the third peak).

[27] http://astro.estec.esa.nl/Planck