Abstract

December 3, 2002

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This work is based on data from the SNO Collaboration and the chlorine experiments. The solar neutrino signal is seen in the chlorine experiments, while the non-solar signal is not seen in the SNO experiment. The best-fit model is the Super-Kamiokande model, which includes a non-zero atmospheric neutrino flux.
I. INTRODUCTION

Results of the SK and SNO experiments [1, 2] provide model-independent evidence of electron-neutrino oscillation into other flavors. A recent global fit of the Super-Kamiokande (SK) Collaboration [3] that includes data of SNO, Gallex/GNO and SAGE shows that the Large Mixing Angle (LMA) solution is preferred at the 98.9% confidence level. The corresponding mass-square difference $\Delta m^2$ is within the range $3 \cdot 10^{-5} \text{eV}^2 \leq \Delta m^2 \leq 19 \cdot 10^{-5} \text{eV}^2$, and the mixing angle $\theta$ within the range $0.25 \leq \tan^2 \theta \leq 0.65$.

The $^8\text{B}$ flux resulting from the SK best fit, $(5.33 \pm 0.36) \cdot 10^6 \, \text{cm}^{-2}\text{s}^{-1}$, is in substantial agreement with the SSM value [4], $(5.05^{+1.01}_{-0.81}) \cdot 10^6 \, \text{cm}^{-2}\text{s}^{-1}$. Also the $^8\text{B}$ flux measured by SNO [5] assuming the standard $^8\text{B}$ energy spectrum, $(5.09^{+0.44}_{-0.43}) \cdot 10^6 \, \text{cm}^{-2}\text{s}^{-1}$, or the one obtained using a distorted spectrum, $(6.42 \pm 1.57) \cdot 10^6 \, \text{cm}^{-2}\text{s}^{-1}$, are in agreement with the SSM value and with the SK fit. On the contrary, the $hep$ flux resulting from the SK best fit,

$$\Phi_{hep}^{SK} = 36 \cdot 10^3 \, \text{cm}^{-2}\text{s}^{-1}$$

which corresponds to the LMA solution, is about four times higher than the one predicted in the SSM, $9.3 \cdot 10^3 \, \text{cm}^{-2}\text{s}^{-1}$ [4], if one uses $S_{hep}(0) = 10.1 \cdot 10^{-20} \, \text{keV barn}$ [6], or

$$\Phi_{hep}^{SSM} = 7.9 \cdot 10^3 \, \text{cm}^{-2}\text{s}^{-1}$$

if one uses the more recent $S_{hep}(0) = (8.6 \pm 1.3) \cdot 10^{-20} \, \text{keV barn}$ [7]. The second best fit (LOW) gives also a $hep$ flux considerable larger than the one predicted in the SSM, $23 \cdot 10^3 \, \text{cm}^{-2}\text{s}^{-1}$. Only the Quasi-VAC and SMA solutions, which appear disfavored from present data, give a $hep$ flux compatible with the SSM [3].

Given the actual experimental evidence, we should justify a $hep$ flux four times larger than in the SSM. Bahcall and Krastev [8], discussing the possibility that the $hep$ flux be considerable larger than in the SSM, conclude that they cannot find a first-principle physical argument demonstrating that the astrophysical factor $S_{hep}(0)$ cannot exceed 20 or $40 \cdot 10^{-20} \, \text{keV barn}$, values which could explain the experimental value of the $hep$ flux. However, the latest detailed calculations [6, 7] exclude an astrophysical factor of this order of magnitude; the latest result [7] gives

$$S_{hep}(0) = (8.6 \pm 1.3) \cdot 10^{-20} \, \text{keV barn}.$$
Excluding that the cross section be wrong by a factor of four, the average rate can be four times larger only if the product of the thermal average of the cross section \( \langle \sigma v \rangle \) times the \( p \) and \( ^3He \) densities is four times larger. But both the temperature and the \( p \) density are accurately measured by helioseismology [9, 10] in the region where \( hep \) neutrino are produced. In addition increasing the temperature or the densities would affect much more the \( ^7Be, ^8B \)- and CNO-neutrino fluxes [11]. There remains the possibility of some improbable mechanism that increases the \( ^3He \) density, which is not measured by helioseismology [12], by a factor of four in the hep-neutrino production region \( (r/R_\odot > 0.12) \) but not in the \( ^7Be, \)

and \( ^8B \)-neutrino production regions \( (r/R_\odot < 0.12) \). Note that mixing would produce the opposite effect [12].

We propose that the large increase of \( hep \) flux which fits the present experimental data is a consequence of the enhancement the high-energy tail of the \( ^3He-p \) momentum distribution. The required \( hep \) flux of \( 36 \cdot 10^3 \) \( \text{cm}^{-2}\text{s}^{-1} \) is a signal that the thermal ion distribution of the solar plasma deviates slightly from the standard one in the region where the reaction \( ^3He + p \rightarrow ^3He + e^+ + \nu_e \) is active. In fact small deviations in the high-energy tail give dramatic effects on the fusion rates without affecting the bulk properties of the medium, which are the ones measured by helioseismology [13, 14].

Let us remind a few of the reasons that lead to deviations from the Maxwellian distribution [13, 14]. In plasmas with parameters such as those that can be found in part of the solar core, in the solar atmosphere or in the interior of giant planets [15], the Debye screening is approximately valid, but the essential many-body character of the interaction must be taken into account (nonlocality); the time necessary to build up again screening after hard collisions is comparable to the inverse of plasma frequency (memory effect or time nonlocality); since many collisions are necessary before particles lose memory of their initial state, the scattering process cannot be considered Markovian. The conditions are such that there is no clear separation between collective and individual degrees of freedom and the description of the system in the relevant energy range requires the use of additional scales, dynamically generated by the interaction, in addition to the temperature: the pure exponential \( \exp(-E/kT) \), which is determined by the single scale \( kT \), is insufficient. The Maxwellian regime ends below a given scale. It has also be shown [16] that a finite width of the quasi-particle is effectively equivalent to a distortion of the momentum distribution.

Small deviations from the Maxwellian distribution, as the ones that are relevant for
us, can be parameterized by the deformation parameter $\delta$ introduced many years ago by Clayton et al. [17]. This description could be related to the general framework for non-Maxwellian distributions that has been proposed by Tsallis and applied in many different contexts [18, 19]: the corresponding distribution depends on parameter $q$ that can be related to $\delta$ in the appropriate limit. The parameter $\delta$ can be related to the plasma parameter $\Gamma$ and the ion-ion correlation special parameter $\alpha$ [13, 14], and to the finite width of the quasi-particle [16].

In the next Section we derive the $hep$ flux. In the third Section we present the expression that relates the deformation parameter $\delta$ or $q$ to the plasma parameter and the ion-ion correlation special parameter $\alpha$; in addition we calculate the effect on the other neutrino fluxes, considering the LMA oscillation and the admissible deviations from Maxwellian distribution of the different reacting ions. In the last Section we summarize and comment our results.

II. THE $hep$ FLUX

The reaction $^3He + p \rightarrow ^4He + e^+ + \nu_e$ produces a neutrino with an endpoint energy of 18.81 MeV, the highest energy expected for solar neutrinos and the only one above 15 MeV. The rate of the $hep$ reaction is very slow and does not affect the solar structure.

The SSM $hep$ flux, which uses standard distribution in the plasma, is [8]:

$$\Phi_{hep}^{SSM} = 2.1 \cdot (1 \pm 0.03) \cdot \left( \frac{S_{hep}(0)}{2.3 \cdot 10^{-20} \text{keV barn}} \right) \cdot 10^3 \text{ cm}^{-2}\text{s}^{-1}.$$ 

Latest values of $S_{hep}(0)$ have ranged from $(2.3^{+0.9}_{-0.0}) \cdot 10^{-20}$ keV barn [20] to $(10.1 \pm 0.9) \cdot 10^{-20}$ [6]. Let us mention that Alberico et al. [21] have proposed a method to determine $S_{hep}(0)$ from a (challenging) laboratory experiment on electron scattering. The latest and more reliable determination [7]

$$S_{hep}(0) = (8.6 \pm 1.3) \cdot 10^{-20} \text{keV barn}$$

gives

$$\Phi_{hep}^{th} = (7.9 \pm 1.4) \cdot 10^3 \text{ cm}^{-2}\text{s}^{-1},$$

where the quoted error includes only the contribution due to $S_{hep}(0)$.

Super-Kamiokande best global fit (LMA) yields a $hep$ flux, $36 \cdot 10^3 \text{ cm}^{-2}\text{s}^{-1}$, about four times $\Phi_{hep}^{best}$, while their second best fit (only 1% of probability) yields $23 \cdot 10^3 \text{ cm}^{-2}\text{s}^{-1}$,
about three times $\Phi_{hep}^{\text{best}}$ [1]; therefore
\[ \Phi_{hep}^{\text{exp}} = (3 \div 4) \times \Phi_{hep}^{h} . \]

Fusion rates are proportional to the local densities of incoming particles and to the average cross section $\langle v \sigma(v) \rangle$ where $v$ is the relative velocity. Therefore, rates depend on the statistical distribution, i.e., on the local temperature if the distribution is Maxwellian, and also on at least an additional deformation parameter if the distribution deviates from the Maxwellian. For example, small deviations can be parameterized by the form proposed by Clayton [17]:
\[ e^{-E/kT} \rightarrow e^{-E/kT - \delta(E/kT)^2} \]
for $\delta > 0$; this distribution (Druyvenstein distribution) introduces a scale $kT/\delta$ in addition to $kT$: the second term in the exponential becomes important when $E \gtrsim kT/\delta$.

To first approximation, we disregard the changes of the other rates and write the change of the average $h_{He}$ cross section, and therefore of the rate, as [13, 14]:
\[ \langle v \sigma(v) \rangle_{1,3} = \langle v \sigma(v) \rangle_0 e^{-\gamma_{1,3} \delta_{1,3}} \]
where $\gamma_{1,3} = (E_0/kT)_{hep}^2$ with $E_0 = (E_G/(kT)^2/4)^{1/3}$; $E_G = 2\mu c^2(Z_1 Z_2 \sigma)^2$ is the Gamow energy and $\langle v \sigma(v) \rangle_0$ the Maxwellian rate. The indices $(1,3)$ indicate the reaction $^1\text{H} + ^3\text{He}$. For this reaction $\gamma_{1,3} = (9.07 \text{ keV}/1.01 \text{ keV})^2 \approx 81$. For small enhancement of the tail of the distribution the deformation parameter $\delta$ is negative and related to Tsallis entropic parameter $q$, $\delta = (1 - q)/2$ with $q > 1$. Note that even if the distribution of Eq. (1) is normalized only for $\delta > 0$, the resulting change of the rate in Eq. (2), which is derived as an asymptotic expansion in $\delta$, can be analytically continued to $\delta < 0$.

The required enhancement
\[ e^{-\gamma_{1,3} \delta_{1,3}} = 3 \div 4 \quad \text{implies} \quad -0.017 \lesssim \delta_{1,3} \lesssim -0.014 . \]

Let us compare this range of values of $\delta$ with the relation derived and discussed in Refs. [13, 14]
\[ |\delta_{i,j}| = 12 \alpha^{4}_{i,j} \Gamma_i \Gamma_j , \]
which gives $\delta_{i,j}$ in terms of $\Gamma_i = Z_i \sum_j Z_j \sigma^2(n_j)^{1/3}/kT$, the plasma (or ion-ion) coupling constants ($\Gamma_i \approx 0.14 Z_i$ in the Sun core), where $n_j$ is the number density of ion $j$, and
of $\alpha_{i,j}$, the ion-sphere-model parameters introduced by Ichimaru [22, 23], parameters that are related to the ion-ion correlation functions (0.4 $< \alpha < 0.9$). An alternative formula by Ichimaru [25], which uses $\Gamma_{i,j}^2 \equiv \left( Z_i Z_j e^2 \left( (n_i^{1/3} + n_j^{1/3}) / kT \right) \right)^2$ instead of $\Gamma_i \Gamma_j$, yields similar ion-sphere-model parameters.

III. THE OTHER FLUXES

In this Section we want to give a quantitative estimate of the range of $\delta$’s compatible with the experimental determination of the main neutrino fluxes, and compare it with the range of values suggested for the hep neutrino flux.

Previous calculations that assumed standard neutrinos have shown that $\delta$ of the order of a few times $10^{-3}$ were compatible with the neutrino experiments even if these deviations from the Maxwellian distribution were not sufficient to solve the solar neutrino problem [13, 14, 16]. Since we now have experimental evidence for oscillations and the best fit to the oscillation parameters (LMA) yields fluxes close to the one predicted in the SSM, we expect that the values of $\delta$ compatible with present data be even closer to zero. Therefore, the effect on the neutrino fluxes of small deviations from the Maxwellian distribution can be reliably obtained by using power laws that include the realignment of the solar model [14, 27].

We repeat the analysis of Ref. [14] taking into account the oscillation probability. We add the survival probability factors $P_s$ of $i$-th neutrino flux to the four equations of Castellani et al. [27]. The Gallium [24], Chlorine [26] and Luminosity equations yield:

\[
\begin{align*}
70.8 \pm 4.5 &= 73.185 \cdot R_{pp} P_{pp} + 34.196 \cdot R_{Be} P_{Be} + 9.0165 \cdot R_{CNO} P_{CNO} + 2.43 \cdot \Phi_{B}^{SNO} \\
2.56 \pm 0.23 &= 0.0226 \cdot R_{pp} P_{pp} + 1.1448 \cdot R_{Be} P_{Be} + 0.4239 \cdot R_{CNO} P_{CNO} + 1.11 \cdot \Phi_{B}^{SNO} \\
63.85 &= 0.980 \cdot 59.85 \cdot R_{pp} + 0.939 \cdot 4.77 \cdot R_{Be} + 0.937 \cdot 1.034 \cdot R_{CNO} + 0.498 \cdot 10^{-3} \cdot 5.05 \cdot R_B
\end{align*}
\]

where 5.05 is the produced $^8$B flux; the $^8$B flux in units of $10^6 \text{ cm}^{-2} \text{ s}^{-1}$

\[
\Phi_{B}^{SNO} = (5.05^{+1.01}_{-0.81}) \cdot R_B P_B = 1.76 \pm 0.05
\]

is measured in the experiment SNO. The SSM fluxes $\Phi$ are the ones calculated by Bahcall et al. [4]. We also use the survival probability $P_s$ extracted from the experiments by Berezinsky and Lissia [28]: $P_{pp} = 0.58$ at 0.265 MeV, $P_{Be} = P_{CNO} = 0.55$ at 0.814 MeV, and $P_B = 0.32$
at 6.71 MeV. The quantity $R_i$, defined in Ref. [13], represents the rate enhancement or depletion due to the nonmaxwellian tail. Eliminating $R_{pp}$ and using the relation between $R$ and $\delta$ [13, 14], the other $R_i$ (with conservative errors) are:

\[
R_B = e^{235\delta_{pp}-139.7\delta_{He}} = 0.87 \pm 0.09
\]

\[
R_{CNO} = e^{62.11\delta_{pp}+6.5\delta_{He}-407\delta_p} = 0.87 \pm 0.09
\]

\[
R_B = e^{62.11\delta_{pp}+149.5\delta_{He}-190\delta_p} = 1.10 \pm 0.20
\]

(4)

where for the purpose of this estimation we have introduced several $\delta$’s: $\delta_{pp}$ for protons in the $pp$ production region ($r/R_\odot \gtrsim 1$), which is also the region where the $hep$ neutrinos are produced, $\delta_p$ for protons in the inner core ($r/R_\odot \lesssim 1$), where all other neutrinos are produced, and $\delta_{He}$ for the distribution of the $He$ nuclei ($^3H$ and $^4H$). The result is that $\delta_{pp} \sim -10^{-2}$, while $|\delta_p| \sim |\delta_{He}| \lesssim 2 \cdot 10^{-3}$.

We note that neutrinos from the $pp$ and $hep$ reactions are produced in the same region ($r/R_\odot \gtrsim 1$), and they have similar values (limit) for $\delta$. The difference is that values of $\delta \sim 2\%$ have large effect on the $hep$ reaction [16], while they have a much smaller effect on the $pp$ reaction both because the Coulomb barrier is lower and because the adjustment of the solar structure to a change of the $pp$ rate, which is directly linked to the luminosity, is such that it minimizes the change of the rate, that ends up being between 0.5% and 3.0% larger than in the SSM (at the solar surface).

The other neutrino fluxes are all produced in the inner core ($r/R_\odot \lesssim 1$), where the plasma conditions are different, and they show much smaller deviation from Maxwellian distribution. We note that the value of $\alpha$ deduced from the $\delta$ of the proton component, \(\alpha\), a value between 0.54 and 0.72, is in agreement with the range of values allowed for a weakly coupled and weakly non ideal plasma (\(\Gamma \lesssim 1\)). In addition the values of the different parameters $\delta$ are within the constraints imposed by helioseismology, as it was checked in Ref. [29] without considering oscillations: oscillations makes the agreement even better. In detail protons superdiffusion in the inner core would yield a $^7$Be flux reduced by 4–22% with respect to SSM calculations; the same range of reduction is obtained for the CNO flux (these fluxes are at the solar surface, before oscillation).

Such values of $\delta$ could be measured or excluded by more precise measurements and solar model calculations.
IV. DISCUSSION AND CONCLUSION

Our work is based on the following points:

- the global best fit (LMA) to the neutrino experiments requires a hep-neutrino produced flux of $36 \times 10^3$ cm$^{-2}$ s$^{-1}$ [1], about four times the one predicted by the SSM [4]; also the second best fit (LOW) requires a flux three times the one in the SSM; if one consider the contribution to the signals on earth from hep neutrinos, this contribution, even after oscillation is larger than the SSM prediction;

- the latest precise evaluation [7] of the astrophysical factor, $S_{hep}(0) = (8.7 \pm 1.3) \cdot 10^{-20}$ keV barn, which is sligher smaller than the values used in the SSM [6], rules out that the cross section could be a factor of four larger;

- temperature and $p$ density are measured very precisely by helioseismology in the region of interest and could not vary significantly without dramatically affecting the $^8$B- and $^7$Be-neutrino fluxes;

- it is not known a physical mechanism that could increase the density of $^3$He in the hep-neutrino production region without increasing the same density in the $^8$B- and $^7$Be-neutrino production region (any mixing would produce the opposite effect) [12];

- there exist several mechanisms that deform the high-energy tail of the ion velocity distribution in plasmas [13, 14, 16, 30]; we do not exclude that other mechanisms could be found that also produce deformation of the distribution.

We have proposed that the experimental results are signals of plasma effects on the fusion rates.

We have verified that the slight enhancement of the high-energy tail of the distribution naturally produces the needed large hep-neutrino flux; the resulting deformation parameter is in agreement with the theory of weakly coupled and weakly non-ideal plasmas. This kind of modifications of the distribution do not affect the bulk properties and, in particular, the measured helioseismologic quantities. The other neutrino fluxes are not significantly modified.

Future more precise experimental determinations of the oscillation parameters, $\Delta m^2$ and mixing angle $\theta$, and of the high-energy neutrino spectrum (and in particular a direct mea-
surement of the $\alpha$ contribution) and improved theoretical calculations, or experimental
determination [21], of the astrophysical factor $S_{\text{h,exp}}(0)$ could corroborate (or bound) the
importance of plasma effects for some of the fusion rates in the Sun.

Acknowledgments

This work is partially supported by M.I.U.R. (Ministero dell’Istruzione, dell’Università e

[arXiv:hep-ex/0103033].

ex/0106015].

ex/0205075].

/010346].

ex/0204008].


/ph/9509116].


