Limiting fragmentation from the
Color Glass Condensate

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Abstract
We show how the limiting fragmentation phenomenon can arise from the Color Glass Condensate model of high energy QCD. We consider the very forward rapidity region in relativistic heavy ion collisions and argue that in this region, nucleus-nucleus collisions are similar to proton-nucleus collisions (up to shadowing corrections). We then use the known results for proton-nucleus cross sections to show that it leads to the phenomenon of limiting fragmentation in the very forward region of heavy ion collisions as observed at RHIC.
The Relativistic Heavy Ion Collider (RHIC) has opened up a new frontier in high energy nucleus-nucleus collisions. Many exciting and new phenomena have been observed which have challenged theoretical models and predictions. Suppression of high $p_t$ hadrons in mid rapidity, increase of baryon to pion ratio with $p_t$ and a large, constant anisotropy at high $p_t$ are yet to be explained satisfactorily. In the fragmentation region (very forward rapidity), the PHOBOS experiment [1] has observed the so called limiting fragmentation phenomenon [2], shown in Fig. (1) for two different energies, which clearly shows most central (0 – 6%) charged particle multiplicities are independent of the center of mass energy $^{1}$. In this note, we show that the Color Glass Condensate model of high energy nuclei can lead to a semi-quantitative understanding of this phenomenon.

![Figure 1: Limiting fragmentation observed at RHIC [1].](image)

It has been suggested that at high energies, due to high gluon density effects, a hadron or nucleus is a Color Glass Condensate and can be described by semi classical methods [3]. This approach has been applied to heavy ion collisions at RHIC with some success [4]. Proton (deutron) nucleus collisions at RHIC, scheduled to begin shortly, will greatly clarify the role and significance of the gluon saturation at RHIC energies [5]. Here we show that limiting fragmentation observed at RHIC can serve as yet another

$^{1}$The error bars shown for $\sqrt{s} = 200$ GeV data are the average of positive and negative uncertainties published by PHOBOS [1].
indication of the importance of high gluon density effects and the Color Glass Condensate at RHIC energies.

Unlike the mid rapidity region, the fragmentation region (very forward rapidities) in a high energy heavy ion collision, is expected to be quite similar to high energy proton nucleus collisions, up to shadowing corrections\(^2\). This is because the Quark Gluon Plasma is expected to be formed only in mid rapidity and will not affect particle production in the forward rapidity region. Also, in the fragmentation region, one can treat the target nucleus as a dilute system of quarks and gluons while the projectile nucleus must be treated as a Color Glass Condensate due to its large number of gluons. This is formally the same as a proton nucleus system treated in [5] where one considers scattering of quarks and gluons [6] coming from the proton on the dense nucleus. For definiteness, here we focus on quark nucleus scattering where the cross section is given by

\[
\frac{d\sigma^{qA\rightarrow qX}}{d^2q_t\,dq^-\,d^2b_t} = \frac{1}{(2\pi)^2} \delta(q^- - p^-) \int d^2r_t e^{iq\cdot r_t} \times \left[ \frac{1}{N_c} \int d^2R_t \text{Tr}_c \left( 2 - U(R_t + \frac{r_t}{2}) - U^\dagger(R_t - \frac{r_t}{2}) \right) \right] \rho_{\text{dipole}}(r_t) \tag{1}
\]

with a similar equation for gluon scattering. This is the multiple scattering generalization of quark gluon scattering in pQCD and is finite as \( q_t \to 0 \) due to higher twist effects. To relate this to nucleus nucleus scattering in the very forward rapidity region, we convolute this cross section with the quark and gluon distributions in the target nucleus

\[
\frac{d\sigma^{AA\rightarrow qX}}{d^2q_t\,dq^-\,d^2b_t} = \int dx_q f_{q/A}(x_q) \frac{d\sigma^{qA\rightarrow qX}}{d^2q_t\,dq^-\,d^2b_t} \tag{2}
\]

Furthermore, since we are interested in total number of produced particles per unit rapidity, we will integrate over the transverse momentum \( q_t \) of the scattered quark. We emphasize the fact that this integral is finite and can be done exactly. It gives

\[
\frac{d\sigma^{AA}}{dq^-\,d^2b_t} = \int dx_q f_{q/A}(x_q) \frac{d\sigma^{qA\rightarrow qX}}{dq^-\,d^2b_t} \tag{3}
\]

\(^2\)By shadowing here, we mean any modification of the nuclear parton distributions, be it anti-shadowing, EMC effect etc.
Using $p^- = x_q \sqrt{s/2}$, $q^- = k^-/z$ and $k^- = \frac{1}{\sqrt{2}} m^h e^{\eta_h}$, taking advantage of $\delta(p^- - q^-)$ in (1) to do the $x_q$ integration and including scattering of gluons from the nucleus, we get

$$\frac{d\sigma^{AA}}{d\eta_p \, d^2 b_t} \sim \left[ x_q f_{q/A}(x_q) + x_g G_A(x_g) \right]$$

(4)

where $x_q = x_g = \frac{m^h}{z m_P} e^{\eta_h - y_{beam}}$ and $f_{q/A}$ and $G_A$ are the quark and gluon distributions of the target nucleus. Experimentally, the maximum $(\eta_h - y_{beam})$ observed at RHIC is about +2 units of rapidity [1]. One can use this fact and that $x_q = x_g = 1$ to show that $\frac{m^h}{z m_P} \approx \exp(-2)$. We therefore have

$$x_q = x_g \approx e^{-2 + \eta_h - y_{beam}}$$

(5)

and all dependences on $m^h$ and $z$ drop out (this implicitly assumes that hadron multiplicities are dominated by low $p_t$ pions and that $z$ does not vary much with $p_t$ for low $p_t$ pions).

In Fig. (2) we plot $d\sigma/d\eta_p \, d^2 b_t$ from eq. (4) shifted for normalization and by the beam rapidity and compare with the most central $(0 - 6\%)$ data [1] from RHIC at $\sqrt{s} = 200$. We have used GRV98 [7] parton distributions and the EKS98 parameterization of nuclear shadowing [8].

Figure 2: Limiting fragmentation from (4) compared to data from RHIC.
There are a few caveats to our results. We can not predict the overall normalizations, only the slope and have to normalize our results to the data at one reference point taken to be the target beam rapidity. It is well known that leading order (in $\alpha_s$) calculations of cross sections suffer from large scale dependence. Also, the scale dependence of nuclear parton distributions, and in particular the gluon distribution, is very poorly known due to the limited $Q^2$ coverage of fixed target experiments. The current parameterizations of nuclear gluon distributions are at best an educated guess. Unfortunately, our results are quite sensitive to the change of scale $Q^2$ (the scale dependence of distribution functions is not written out explicitly in eq. 4). We therefore fix this scale by requiring that eq. (4) give a reasonable fit to the RHIC limiting fragmentation data at $\sqrt{s} \sim 20$ GeV for a couple of units of rapidity. It turns out that $Q = 2.3$ GeV works well. We then use the same scale $Q$ in (4) to predict the multiplicities at higher energies of $\sqrt{s} = 130$ GeV and $\sqrt{s} = 200$ GeV. We also show the case when $Q^2 = Q_s^2(y)$ as suggested in [9]. Here $y = \log 1/x$ and $Q_s^2(y) \equiv Q_{s0}^2 \exp(\lambda y)$ with $Q_{s0}^2 = 2.0 GeV^2$ at mid rapidity and $\lambda = 0.3$ [4]. The choice of $Q_{s0}^2 = 3.0 GeV^2$ leads to a much better agreement but is disfavored by RHIC data. Alternatively and if one insists on keeping $Q_{s0}^2 = 2.0 GeV^2$ at mid rapidity, the choice of $\lambda = 0.45$ leads to a good agreement with the data but this value of $\lambda = 0.45$ is too large to fit the HERA data (also, choosing a $x$ dependent scale in parton distributions would seem to violate the sum rules reflecting various conservation laws. Nevertheless, having this as the factorization scale might be theoretically tempting since otherwise there is really no hard scale left after we integrate over all hadrons transverse momenta). We will not pursue this further since we are not doing a detailed quantitative study here.

There are two principle reasons why our approach should break down as we get closer to the mid rapidity region. First, as one goes further away from the target nucleus, high gluon density effects in the target nucleus become important. This will show as the growth in the saturation scale of the target nucleus (which is of order $\Lambda_{QCD}$ right at the target nucleus rapidity). To estimate this, we use $Q_s^2(\Delta \eta) = \Lambda_{QCD}^2 \exp(\Delta \eta)$. As one goes about three units of rapidity away from the target nucleus, its saturation scale becomes appreciable ($\sim 1 GeV$) and one can not describe it as a dilute system of partons anymore [10].

3During completion of this note, we learned of [9] which however focuses on a different problem.
Another reason why this approach should break down as one gets closer to mid rapidity is that the classical fields of both nuclei will become strong and the system will be very different from a proton nucleus collision. Also, in the mid rapidity region one will have to include the media effects due to the deconfined matter presumably produced in heavy ion collisions at RHIC. The media effects are presently not well understood and are beyond the scope of this work.

To summarize, the underlying physics of limiting fragmentation in the Color Glass Condensate model is that since most particles are produced with transverse momenta which are below the saturation scale of the projectile nucleus (in the target nucleus reference frame), their cross sections are transverse momentum independent (the black disk limit). Thus the rise of the particle multiplicities in the very forward rapidity region (near the target nucleus) is due to the blackness of the projectile nucleus and the growth of the target nucleus parton distributions with rapidity.

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References

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