Two-Loop Results for $M_W$ in the Standard Model and the MSSM

A. Freitas, a S. Heinemeyer b and G. Weiglein c

a Fermilab, Batavia, IL 60510-0500, USA

b Institut für Theoretische Elementarteilchenphysik, LMU München, D–80333 Munich, Germany
c Institute for Particle Physics Phenomenology, University of Durham, Durham DH1 3LE, UK

Recent higher-order results for the prediction of the W-boson mass, $M_W$, within the Standard Model are reviewed and an estimate of the remaining theoretical uncertainties of the electroweak precision observables is given. An updated version of a simple numerical parameterisation of the result for $M_W$ is presented. Furthermore, leading electroweak two-loop contributions to the precision observables within the MSSM are discussed.

1. INTRODUCTION

The comparison of electroweak precision measurements with the theoretical predictions allows to test the electroweak theory at the quantum level. In this way indirect constraints on unknown parameters of the theory can be obtained, in particular constraints on the Higgs-boson mass, $M_H$, within the Standard Model (SM) and constraints on the parameters of the Higgs and scalar top and bottom sector within the Minimal Supersymmetric extension of the SM (MSSM).

Fig. 1 shows the result of a global fit to all data within the SM [1,2]. The theoretical predictions are affected by two kinds of uncertainties: the uncertainties from unknown higher-order corrections, indicated by a “blue band” in Fig. 1, and uncertainties from experimental errors of the input parameters, indicated in Fig. 1 by two fit curves corresponding to two different values of $\Delta \alpha_{\text{had}}$, the hadronic contribution to the shift in the fine structure constant (the experimental error of the top-quark mass, $m_t$, is directly included in the fit). The upper plot in Fig. 1 shows the result based on the most recent data (summer 2002 [1]), and the currently best estimate of the theoretical uncertainties from unknown higher-order corrections, while the lower plot shows the fit result based on the previous estimate of the theoretical uncertainties and the winter 2001 data [2]. The comparison in Fig. 1

*Talk presented by G. Weiglein.
shows that the present estimate of the theoretical uncertainties from unknown higher-order corrections yields a larger value than the previous estimate. This was triggered by the recently obtained result for the complete fermionic two-loop corrections to the W-boson mass, \( M_W \), in the SM \([3]\), leading to an improved estimate of the remaining theoretical uncertainties in the prediction for the leptonic effective weak mixing angle, \( \sin^2 \theta_{\text{eff}} \) \([4]\).

In the following section the present status of the prediction for \( M_W \) in the SM is reviewed and an estimate of the remaining theoretical uncertainties of the electroweak precision observables is given. Sect. 3 summarises the impact of new electroweak two-loop contributions on the precision observables within the MSSM \([5]\).

2. PREDICTION FOR \( M_W \) IN THE SM

The prediction for \( M_W \) is obtained from relating the result for the muon lifetime within the SM (and analogously for the MSSM) to the definition of the Fermi constant, \( G_\mu \) (by convention, the QED corrections within the Fermi Model, which are known up to two-loop order \([6]\), are split off in the defining equation for \( G_\mu \)). This leads to the relation

\[
\Delta r = \Delta r_0 + \Delta r_\text{sub} + \Delta r_\text{tb} + \Delta r_\text{lf} + \Delta r_\text{bos}.
\]

Complete results for the fermion-loop contributions at two-loop order were first obtained for the Higgs-mass dependence of \( M_W \) in Ref. \([12]\). The full result for the fermion-loop contributions at two-loop order was derived in Refs. \([3,4]\).

Figure 2. Relative importance of different two-loop contributions to \( \Delta r \) with one closed fermion loop as a function of the Higgs-boson mass, see text.

Figure 2 shows the relative importance of the different contributions with one closed fermion loop to \( \Delta r \) at two-loop order, whose sum is denoted by \( \Delta r_0 + \Delta r_\text{sub} + \Delta r_\text{tb} + \Delta r_\text{lf} + \Delta r_\text{bos} \). It can be seen that both corrections with a top-/bottom-loop, \( \Delta r_\text{sub} + \Delta r_\text{tb} \), and with a light-fermion loop, \( \Delta r_\text{lf} \), yield important contributions. It should be noted, however, that the light-fermion contribution contains the numerically relatively large term \( 2\Delta \alpha \Delta r_\text{lf}^\text{bos} \), which can easily be separated from the genuine two-loop contribution of the light fermions. In order to investigate the numerical relevance of the latter, in Fig. 2 also the difference \( \Delta r_\text{sub,1-loop} = \Delta r_\text{sub} + \Delta r_\text{tb} - 2\Delta \alpha \Delta r_\text{lf}^\text{bos} \) is shown. While these genuine light-fermion two-loop contributions do not exceed the top-/bottom contributions for any value of the Higgs-boson mass below 1 TeV, they nevertheless amount up to \( 3.3 \times 10^{-4} \), which corresponds to a shift in \( M_W \) of more than 5 MeV.

Recently also the Higgs-mass dependence of the purely bosonic two-loop corrections became available \([4]\). Finally, the full result for the purely bosonic two-loop corrections has been obtained in Ref. \([13]\), completing in this way the calculation of muon decay at the two-loop level. The numerical effect of the purely bosonic two-loop corrections turned out to be relatively small, giving rise to a shift in \( M_W \) of less than \( \pm 1 \text{ MeV} \) for \( M_H \leq 1 \text{ TeV} \). From the higher-order contributions to \( \Delta r \) (for a discussion, see e.g. Ref. \([4]\)) only the top-bottom contributions at \( \mathcal{O}(\alpha^4) \) \([14]\) were found to be non-negligible in view of the present experimental accuracies.

Below a simple parameterisation of the result for \( M_W \) is given, being based on taking into account the following contributions to \( \Delta r \):

\[
\Delta r = \Delta r_0 + \Delta r_\text{sub} + \Delta r_\text{tb} + \Delta r_\text{lf} + \Delta r_\text{bos}.
\]

where \( \Delta r_0 \) is the one-loop result, \( \Delta r_\text{sub} \) and
\( \Delta r^{(\alpha \alpha^2)} \) are the two-loop and three-loop QCD corrections, \( \Delta r(N_f \alpha^2) \) and \( \Delta r(N_f^2 \alpha^4) \) are the electroweak two-loop contributions with one and two fermion loops, respectively, and \( \Delta r^{(\alpha^2; \text{bos})} \) is the purely bosonic two-loop contribution according to the expression given in Ref. [15]. The numerically rather small electroweak higher-order corrections have been neglected here. The parameterisation of the result for \( M_W \) reads

\[
M_W = M_W^0 - d_1 dH - d_2 dH^2 + d_3 dH^4
- d_4 d\alpha + d_5 dt - d_6 dt^2 - d_7 dH dt
- d_8 d\alpha_s + d_9 dZ,
\]

where the dependence on the variables \( M_H, m_t, \alpha, \alpha_s \) and \( M_Z \) is expressed by \( dH = \ln \left( \frac{M_H}{100 \, \text{GeV}} \right), dt = \left( \frac{m_t}{174.3 \, \text{GeV}} \right)^2 - 1, \)
\( d\alpha = \Delta \alpha/0.05924 - 1, d\alpha_s = \alpha_s(M_Z)/0.119 - 1, \)
and \( dZ = M_Z/(91.1875 \, \text{GeV}) - 1. \)
The coefficients \( d_1, \ldots, d_9 \) have the following values (in GeV):

\[
\begin{align*}
M_W &= -0.012, -0.010, -0.008, -0.006, -0.004, -0.002, 0.000, 0.002, m_h \quad \text{[GeV]} \\
\delta M_W &= 0, 0, 0, 0, 0, 0, 0, 0, m_h \quad \text{[GeV]} \\
\tan \beta &= 3, 40 \\
M_A &= 100 \, \text{GeV}, 300 \, \text{GeV} \\
M_A &= 100 \, \text{GeV}, 300 \, \text{GeV} \quad \text{(MSSM - SM)}
\end{align*}
\]

The remaining theoretical uncertainties of the electroweak precision observables from unknown higher-order corrections, taking into account all known contributions, can be estimated with the methods described in Refs. [4,16] as:

3. LEADING ELECTROWEAK 2-LOOP CORRECTIONS IN THE MSSM

The situation concerning theoretical uncertainties of the electroweak precision observables \( M_W \) and \( \sin^2 \theta_{\text{eff}} \) from unknown higher-order corrections within the MSSM is significantly worse than in the SM. Comparing the available results for higher-order corrections in both models, the uncertainties from unknown higher-order corrections within the MSSM can be estimated to be at least a factor of 2 larger than the ones in the SM as given in eq. (??).

The leading higher-order corrections from quark and squark loops enter via the quantity \( \Delta \rho, \)

In Fig. 3 the numerical effect of the \( \mathcal{O}(\alpha_t^2) \) corrections on \( M_W \) is analysed. In addition to the

Figure 3. Contribution of the \( \mathcal{O}(\alpha_t^2) \) MSSM corrections to \( M_W \) as a function of \( m_h \) (upper plot) and \( \tan \beta \) (lower plot).

MSSM \( \mathcal{O}(\alpha_t^2) \) correction to \( \delta M_W \) also the ‘effective’ change from the SM result (where the value of the SM Higgs boson mass has been set to \( m_h \)) to the new MSSM result is shown. The parameters in Fig. 3 are chosen according to the \( m_{h_{\text{max}}} \) benchmark scenario [21], i.e. \( M_{\text{SUSY}} = 1 \, \text{TeV}, X_t = 2 M_{\text{SUSY}}, \) where \( m_t X_t \) is the off-diagonal entry in the \( t \) mass matrix. The other parameters are \( \mu = 200 \, \text{GeV}, A_b = A_t. \) The Higgs-boson mass \( m_h \) is obtained in the upper plot from varying \( M_A \) from 50 GeV to 1000 GeV, while keeping \( \tan \beta \) fixed at \( \tan \beta = 3, 40. \) In the lower plot, \( \tan \beta \) is varied from 2 to 40, \( M_A \) is kept fixed at \( M_A = 100, 300 \, \text{GeV}. \)
$m_h$ from the other MSSM parameters contains corrections up to two-loop order, as implemented in the program FeynHiggs [22].

The effect of the $O(\alpha_t^2)$ MSSM contributions on $\delta M_W$ amounts up to $-12\text{ MeV}$. For large $\tan \beta$ it saturates at about $-10\text{ MeV}$. The ‘effective’ change in $M_W$ in comparison with the corresponding SM result with the same value of the Higgs-boson mass is significantly smaller. It amounts up to $-3\text{ MeV}$ and goes to zero for large $M_A$ as expected from the decoupling behaviour. For a small $CP$-odd Higgs boson mass, $M_A =100\text{ GeV}$, a shift of $-2\text{ MeV}$ in $M_W$ remains also in the limit of large $\tan \beta$, since the two Higgs doublet sector does not decouple from the MSSM. For large $M_A$, $M_A =300\text{ GeV}$, for nearly all $\tan \beta$ values the effective change in $M_W$ is small.

The absolute contribution for $\delta \sin^2 \theta_{\text{eff}}$ (which is not shown here) is around $+6 \times 10^{-5}$. The effective change ranges between $+3 \times 10^{-5}$ for small $\tan \beta$ and small $M_A$ and approximately zero for large $\tan \beta$ and large $M_A$.

Acknowledgements: G.W. thanks the organisers of “RADCOR 2002 – Loops & Legs 2002” for the invitation and the pleasant atmosphere at the meeting. This work has been supported by the European Community’s Human Potential Programme under contract HPRN-CT-2000-00149 Physics at Colliders.

REFERENCES

15. M. Awramik, M. Czakon, hep-ph/0211041,
these proceedings.


