a complex function. The introduction of the imaginary part of the self-energy accounts for taking into account the
effects of the quasi-particle spectral function, which have been studied elsewhere[9, 10]. The complex nature of the
potential is due to finite time propagation and decay of processes shown in Fig. 1. In the present investigation
$\mathcal{V}$ will be assumed to be a real function what implies the gap function to be real also.

The self-energy $\Sigma^{(1)}(\omega)$ in neutron matter has been calculated within the above described approximation using the
Gogny force D1 [2] at each coupling vertex shown in Fig. 1(c),(d) and (e). This corresponds to implicitly dressing those vertices by a G-matrix and replacing
the latter with its phenomenological counterpart given by the Gogny force. The second-order contributions $\Sigma^{(2)}$ and
$\Sigma^{(3)}$ (graph b) and c in Fig. 1, respectively) are reported as a function of the energy for three values of the neutron matter density, which are in the range where
the pairing is expected to be largest. $\Sigma^{(1)}$ exhibits a pronounced maximum in the vicinity of the Fermi energy
due to the high probability amplitude for particle-hole excitations near $\varepsilon_F$. It is in very good agreement with the results obtained from Brueckner-Hartree-Fock (BHF)
calculation with G-matrix [2].

From the point of view of the hole-line expansion one would expect $\Sigma_{hh}$ to give the largest contribution of the
ring series in the low-density limit. The particle-particle contribution is much less sizeable and has been also considered, since the Gogny force has no energy dependence like the G-matrix.

The second-order potential is given by the one-bubble exchange term (plotted in Fig. 1b), which is the first
one of the ring diagram series. Physically it represents the screening to the pairing due to the medium polarization. Again we take the Gogny force for all vertices in
Fig. 1(a,b). Our prediction for $\Sigma^{(2)}(\omega)$ at three typical densities is reported in Fig. 3. We plot the symmetric part

$$
(\omega, \omega')^{(2)}_{hh} = \frac{1}{2(2\pi)^3} \varepsilon_{(\omega, \omega')}^{(2)} \left[ \varepsilon_{(\omega, \omega')}^{(2)} + \varepsilon_{(-\omega, -\omega')}^{(2)} \right],
$$

which is the only one relevant for the pairing gap. The strength of $\mathcal{V}$ is concentrated around the Fermi energy
($\omega = 0$) with a peak value at $k = k' = k_F$ and a width increasing with the density. Its $\omega$ dependence is shaped by the
polarization part, i.e., Lindhard functions[11] which at $\omega = 0$ (static limit) is repulsive at any momentum

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{The diagrams of NN interaction and self-energy discussed in the text. The exchange terms are understood.}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{Rearrangement contributions to the self-energy. On the right panel the HF mean field is plotted vs momentum k.}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig3.png}
\caption{Screening potential vs energy}
\end{figure}
and density, but it becomes attractive for $|\omega| \gg \varepsilon_F$. One therefore expects a reduction of the gap due to screening.

The gap equation, Eq. (1), is to be coupled to the closure equation for the Green's function

$$\rho = \sum_k \int \frac{d\omega}{2\pi i} \frac{\omega \mu^+ G_k(\omega)}{\bar{\mu}^2 - \varepsilon_k[\omega]} \quad (3)$$

fixing the chemical potential in the superfluid phase.

The anomalous propagator

$$F_k(\omega) = \frac{\Delta(\omega)}{[\bar{\mu}^2 - \varepsilon_k[\omega]] [\bar{\mu}^2 + \varepsilon_k(-\omega)]}, \quad (4)$$

has two poles, symmetric with respect to the imaginary axis of the complex $\omega$-plane[5], which are the roots of the equation

$$\pm \omega_k = \varepsilon_k(\pm \omega_k). \quad (5)$$

The quasiparticle energy $\varepsilon_k$ is given by

$$\varepsilon_k(\omega) = M_k^\pm(\omega) + \sqrt{\varepsilon_k^2 + M_k^\pm(\omega)^2} + \Delta_k^2(\omega), \quad (6)$$

where

$$M_k^\pm(\omega) = \frac{1}{2} [M_k(\omega) \pm M_k(-\omega)]. \quad (7)$$

The two poles are located close to the real axis on opposite sides of the imaginary axis. Leaving aside a general integration of the gap equation, we adopt the pole approximation relying on replacing the full propagator by its pole part:

$$F_k(\omega) = \frac{Z_k \Delta_k(\omega)}{\varepsilon_k(\omega) + \varepsilon_k(-\omega) \frac{1}{\omega - \omega_k + i\eta} - \frac{1}{\omega + \omega_k - i\eta}}. \quad (8)$$

In general the residue $Z_k$ at the poles is defined as

$$Z_k = \left[ 1 - \frac{\partial \varepsilon_k(\omega)}{\partial \omega} \right]_{\omega = \omega_k}^{-1}, \quad (9)$$

calculated at the Fermi surface. In the calculations we took the limit of $Z_k$ for $\Delta \to 0$, which corresponds to the quasiparticle strength.

Inserting Eq. (8) for the anomalous propagator into the gap equation, after $\omega$ integration we obtain

$$\Delta_k(\omega) = -\frac{1}{2} \int d^3k \frac{d\varepsilon_k}{d\omega} (\omega - \omega_{k'}) \frac{Z_k \Delta_k(\omega_{k'})}{\varepsilon_k(\omega_{k'}) + \varepsilon_k(-\omega_{k'})}, \quad (10)$$

Notice that the reason why only the $\omega$-even part of the interaction contributes to the integral can be traced to time reversal invariance of the superfluid ground state for which the anomalous propagator as well as the gap function are even functions of $\omega$.

The remarkable advantage of this approximation is that the gap depends only parametrically on $\omega$ and its energy dependence is only related to the energy dependence of the interaction. The on-shell gap $\Delta_k(\omega_k)$ fulfills the equation

$$\Delta_k(\omega_k) = -\frac{1}{2} \int d^3k' \frac{d\varepsilon_k}{d\omega} (\omega_{k'} - \omega_{k'}) \frac{Z_k \Delta_k(\omega_{k'})}{\varepsilon_k(\omega_{k'}) + \varepsilon_k(-\omega_{k'})} + \varepsilon_k(-\omega_k) \quad (11)$$

equivalent to the gap equation in the static limit.

The approximate version of the gap equation, Eq. (10), has been solved using as input the self-energy and pairing potential discussed in the previous section. The focus was on the $^1S_0$ neutron-neutron pairing for neutron matter, which is by far the most important component of pairing as to possible implications in nuclear systems. The energy gap at the Fermi surface $(k = k_F$ and $\omega = 0$) as a function of $k_F$ is reported in Fig. 4. The domain of existence of the superfluid state is mainly at low densities with a peak value of about 1.4 MeV at $k_F = 0.6$ fm$^{-1}$. In the same figure we also report the result in the BCS limit [for a review see [1]] (neither self-energy effects nor screening) and the result with self-energy effects but without screening [2]. From the comparison of the three predictions one sees that the main suppression of $\Delta$ is due to the strong g.s. correlations which lead to a Z-factor much less than unit. However screening of the pairing interaction produces an additional suppression. It has to be noticed that the screening potential also shifts the peak value of the gap to lower density, where the suppression is less sizeable. This implies that the medium effects are not expected to reduce dramatically the pairing at the nuclear surface and the feature of pairing as surface effect is still confirmed by our fully consistent calculation of pairing in neutron matter.

Screening effects on the pairing interaction have also been studied in different contexts. One of the earliest calculations has been performed in the framework of the second-order correlated basis perturbation theory [4], where a pairing suppression by a factor 4 is predicted.
In that work the momentum dependence is substantially neglected but amounts to overestimate the suppression. Close to the present approach is the polarization potential \cite{12, 13} calculated from the induced interaction theory \cite{14}, which gives substantially the same gap when treated within the Landau parameter approximation \cite{15}. Finally, we should mention a parallel study on finite nuclei \cite{16}, where the polarization potential is given by the coupling to surface vibrations. While the self-energy plays the same role as in neutron matter, the phonon exchange produces an enhancement of the pairing at variance with the dominant repulsive effect of the spin fluctuations in neutron matter.

In conclusion, this work constitutes a continuation of a previous one \cite{2} where we investigated self-energy effects on the pairing gap in infinite neutron matter. Here we treated consistently the additional inclusion of vertex corrections. On the same footing we considered the lowest order particle hole polarization bubble both in the self-energy and in the screening of the pairing force. Instead of the G-matrix we used the phenomenological Gogny force at all coupling vertices. We verified that this replacement has only very little influence on the numerical results. The screened pairing interaction is in principle energy dependent but in the quasiparticle approximation used here, this dependence on energy becomes only a parametrical one which greatly facilitates the numerical task of solving the gap equation. The outcome of the inclusion of vertex corrections is that the gap as a function of \( k_F \) maintains approximately its bell-shaped form, but with respect to the self-energy corrections only, a further substantial reduction of the gap is induced by screening the bare NN interaction in the gap equation. Therefore, with respect to the lowest order approach, i.e., without any polarization effects, that is with bare interaction and \( k \)-mass only, the gap at its maximum is now reduced by about 50\%. This strong reduction is a common feature of all previous calculations and in this sense our investigation is a confirmation of what has been found by other authors earlier, even though the approaches differ in detail. Nonetheless this strong reduction of the pairing due to the polarization remains intriguing. It is very likely that the same situation prevails in nuclear matter and then an estimate via the local density approximation \cite{7} would lead to a by far too low value of the gap in finite nuclei. One may contemplate the validity of the local density approximation in the case of pairing but it should not be off by important factors \cite{8, 17}. This remark even holds true for the inclusion of polarization effects. Whether the resummation of the bubbles (RPA) and therefore the inclusion of surface phonons in the case of finite nuclei \cite{16} can resolve this important question remains to be seen in future studies.


