PROPOSAL TO MEASURE SPIN EFFECT AND DIFFERENTIAL CROSS-SECTIONS
IN HADRON-HADRON ELASTIC SCATTERINGS AT SPS ENERGIES

CERN-IPN Orsay-Oxford Collaboration

J. Antille, N. Booth, D. Crabb, L. Dick, A. Gonidec,
K. Green, A. Gsponer, K. Kuroda, A. Michalowicz,
D. Perret-Gallix, G. Salmon and M. Werlen

RHF2 selection panel
Cost didn't justify physics
very negative
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1. **INTRODUCTION**

One of the most striking results in particle physics in recent years has been the mounting evidence for a kind of internal structure for hadrons. Beginning with the successes of unitary symmetry and its "quark" picture, one found additional support in the appearance of high transverse momentum secondaries in hadron collisions. The evidence from deep inelastic electron scattering and from high q^2 electroproduction strengthens this view. The theoretical views of structure follow many lines, but a bound system of spin (\(\frac{1}{2}\)) constituents (partons) seems to be a compelling picture. It seems clear, therefore, that a study of spin-dependent phenomena at the SPS could aid in distinguishing the various views of hadron structure. That is to say, to what extent are the "parton" spins correlated with the nucleon spin so that, if parton interactions are spin dependent, effects would be seen in hadron collisions? Are there phenomena that have analogy to the nucleon spin-orbit coupling in nuclei? Is there a circulation of hadronic matter that correlates with the spin? Yang and Chou\(^1\) point to rising cross-sections as a source of asymmetry in interaction of spinning hadronic matter that would depend on spin orientation. Bourrely et al.\(^2\) using a spin-dependent impact parameter picture, predict spin correlation effects.

The first answers to these broad questions can, of course, only come from spin measurements. It is crucial, however, to be able to perform the measurement of "depolarization tensors\(^*)\), since the so-called polarization parameter alone probably vanishes since it is a reflection of the interference of spin-flip amplitudes with the large diffractive non-flip amplitude. All of the conjectures of possible spin dependence in the interaction of hadronic matter are most transparent from the depolarization measurements. For these reasons a study of spin phenomena that includes the depolarization tensors (e.g. \(R + R'\) for \(\pi p + pp\)) in elastic scattering at high |t| is proposed below.

Now depolarization measurements, \(R\) for example, require scattering from a polarized target and a determination of the polarization of the recoil nucleon. (The possibility of a polarized beam of protons will, of course, enhance the number of tensor amplitudes that one can determine with no change in the physical system.) A system that satisfies the requirements for the spin measurements is a solenoid spectrometer. The system described below provides the possibility of determining all the relevant momenta and has the property of spin precession sufficient to optimize the \(R\) measurements.

The solenoid satisfies the basic requirements of the elastic scattering experiments in this proposal. This facility is well suited for other studies of

\(^*)\) We call "the depolarization tensors" in a general sense, not only the D parameter defined by Wolfenstein, but also A, R, etc.
quasi-two-body reactions\textsuperscript{3}). It is a vertex spectrometer and has many unique features that are of distinct advantage. Its near uniform field makes on-line analysis efficient and fast\textsuperscript{4}). Its axial symmetry is ideal for polarization measurements but also aids in the reduction of bias in studies of angular distributions in multi-body decays. It is a complement to a dipole magnet spectrometer which can be placed downstream of the solenoid to analyse the beam fragments of a quasi-two-body reaction\textsuperscript{3}). Furthermore, it can accommodate both planar and cylindrical wire chambers to advantage. All in all, the solenoid is a universal vertex spectrometer and would find uses in many directions.

In this connection it may be worth while to illustrate some other experimental studies that could profit from the solenoid facility. Hyperon beams will be available at the SPS so that studies of elastic scattering on polarized targets will be possible. Here again, depolarization measurements can be done, especially if a forward dipole magnet is available to analyse the hyperon polarization.

High-energy diffractive dissociation can be studied in some detail in the region of the target fragmentation. Finally, the possibility of helicity preservation in associated productions\textsuperscript{5}) at very high energies allows for the appearance of polarized hyperons recoiling from a polarized target. Although polarized hyperons and their decay is itself of interest, a new possibility may exist, that is the associated production of charmed baryons.

The recent discoveries of the \(\psi\) and \(J\) particles seem to indicate a new spectroscopy involving charm as a quantum number. In fact an ISR experiment (proposal ISRC/74-38) is designed to detect the leptonic decay of presumed charmed baryons and mesons. The expectation of associated production, in analogy with ordinary strange particle production, has been elucidated by M. Block (internal report R-604, Appendix to ISRC/75-45). This mechanism would provide a polarized charmed baryon as a target fragment of a polarized proton target. The charmed baryon would appear at low momentum in the laboratory, and the subsequent decay asymmetries of the hyperon to lepton should provide the information for the weak interaction parameters. Here again a large acceptance for target fragmentation would be a distinct advantage.

In this proposal, we suggest that a solenoid spectrometer can be a general facility useful for a large class of experiments needing

- high spatial acceptance
- precise vertex determination
- good momentum measurement for recoil and secondary particles, for momenta lower than 4 GeV/c (see also Appendix II).

Combined with a forward spectrometer, the solenoid can be a very powerful tool for many investigations. Nevertheless, we will restrict ourselves here to a study concerning spin in high-energy hadron-hadron elastic scattering.
2. **THE EXPERIMENTAL PROGRAM AND THE PHYSICAL INTERESTS**

Several theorists have tried to extrapolate known spin effects to higher energies. This is not easy, due partly to the fact that the rise in total cross-sections is not well understood. What is generally believed is that at high energies the polarization parameter \( P_0 \) will approach zero, particularly at small momentum transfers [see Figs. 1 and 2, from Neal's proposal\(^6\)]. At energies of 400 GeV there could still be an appreciable polarization at large momentum transfers, but the effects of spin are most likely to appear in the spin correlation and rotation parameters, such as \( R \). It is therefore important to measure \( R \) as well as \( \frac{d\sigma}{dt} \) and \( P_0 \).

If all three quantities, \( \frac{d\sigma}{dt} \), \( P_0 \), and \( R \), are measured in meson-nucleon scattering, then it is possible to do an amplitude analysis. Such an analysis has been performed only at 6 GeV/c and has given an important insight into the construction of models. The most difficult quantity to measure is \( R \), and we will discuss this in detail in this proposal.

On the basis of a spin-dependent diffractive model, Bourrely et al.\(^2\) have predicted that the \( R \) parameter is quite large, for example at \(-t = 1 \) GeV\(^2\), \( R = -30\% \). They also predict a strong variation of the \( D_{pp} \) parameter in \( pp \) elastic scattering near the dip of the cross-section. Measurement of spin parameters in this region will provide interesting information about the structure of the scattering amplitudes.

We propose to measure \( \frac{d\sigma}{dt} \), \( P_0 \), and some depolarization tensors for elastic scattering processes up to the highest energies available at the SPS. Since there are experiments planned at NAL to measure \( \frac{d\sigma}{dt} \) and \( P_0 \) in \( pp \) scattering over a wide range of \( s \) and \( t \), and \( \frac{d\sigma}{dt} \) and \( P_0 \) in \( \pi p \) scattering at small \( t \), we concentrate on measurements which we do not anticipate will be made by other experiments.

Specifically we propose:

- \( \frac{d\sigma}{dt} \) for \( K^+p \) and \( \bar{p}p \) scattering, and for \( pp \) and \( \pi^+p \) at large \(|t|\),
- \( P_0 \) for \( K^+p \) and \( \bar{p}p \) scattering, and for \( pp \) and \( \pi^+p \) at large \(|t|\),
- \( R \) for \( \pi^+p \), and a linear combination of \( A \) and \( A' \) for \( pp \) scattering.

The experimental set-up which we propose can accommodate all these measurements with very little modification. It is designed to optimize the more difficult \( R \) measurements, but it turns out to be optimal also for \( \frac{d\sigma}{dt} \) measurements. An easy extension will be measurements of spin effects in inclusive reactions.
3. GENERAL DESCRIPTION OF APPARATUS

3.1 Solenoid spectrometer

It is well known that for high-energy reactions, especially in elastic scattering, we need better information about the recoil particles in order to compensate for the lack of experimental precision on the fast forward particles. The only way is to use a spectrometer giving, in a large angular acceptance, precise momentum measurements.

A $2\pi$ homogeneous and strong magnetic field could be an ideal tool for this purpose. To fix the performance limit, two interesting quantities have to be kept in mind:

$$d \cdot H = \frac{200}{3} p_{xy},$$

$$\lambda \cdot H = \frac{200\pi}{3} p_z,$$

where $d$ is the diameter (in metres) of the projected trajectory in the $x$-$y$ plane perpendicular to the field ($z$-axis), $H$ the magnetic field in kG, $p_{xy}$ the particle momentum component in the $x$-$y$ plane, and $p_z$ the momentum component along the $z$-axis in GeV/c; $\lambda$ represents the pitch of the helicoidal trajectory. Other important factors are, of course, the space resolution of detectors, and the time resolution limiting counting rate.

If each particle trajectory is detected by more than three detectors placed in the homogeneous field and giving simultaneously three coordinates, we can reconstruct and separate, without ambiguity, each trajectory; a pair of two coordinates among three gives, for example, $p_{xy}$ and $\phi$, and the other pair gives the $p_z$ component and $\alpha$ (see Appendix I). The momentum precision is only limited by the precision of the coordinate measurements. The geometrical reconstruction can be made easier by the appropriate choice of coordinate axes.

The reconstruction method and calculation are greatly simplified when the field is axially symmetric, with the beam along the symmetry axis and the detectors in the coaxial symmetry configuration (see Appendix I for the reconstruction method in a cylindrical coordinate system).

Some characteristics of a solenoid spectrometer are summarized here:

i) This configuration is the best way to obtain the largest useful volume in a homogeneous field.

ii) Reconstruction of the recoil particle trajectory is relatively easy with minimum calculation time.

iii) There is a large azimuthal angular acceptance.
iv) It is easy to use with the beam parallel to the field, and in this case there is no azimuthal dependence of events (2π acceptance).

v) Forward particles with low \( p_{xy} \) component cross the solenoid along the axis without perturbation. Very low momentum particles are confined in a small cylindrical region around the beam axis and do not contribute to the background; for example, for \( p_T = 200 \text{ MeV}/c \), the trajectory is confined at 33 cm from the axis when the magnetic field is 40 kG; they diverge from the axis when coming out of the axial field region.

The specific qualities mentioned above are appreciable in a general spectrometer device.

Furthermore, if one uses spin precession in a magnetic field, the initial target spin orientation being along the axis, we can optimize the rotation parameter measurement.

Taking account of the advantages of an axial magnetic spectrometer, we will develop our proposal on the hypothesis of using a solenoid spectrometer facility.

As a basis of planning our experimental set-up, we propose to use a superconducting magnet of up to 100 kG·m and 2 m long (for comparison, BEBC uses a magnet of 160 kG·m, 5 m long). This gives a limit of the rotation parameter measurement at about \( |t| = 1.5 \text{ (GeV}/c)^2 \); cross-section measurements in elastic scattering can be performed up to \( |t| = 4 \text{ (GeV}/c)^2 \) with a momentum precision \( \Delta p/p \) less than 3% when \( z \) and \( \phi \) resolutions are 1 mm and 3 mrad, respectively.

Figure 3 gives with these conditions the momentum resolution function of the momentum transfer \( t \).

Figure 4 presents the trajectories in the \( r,z \) plane for different values of \( t \) in elastic scattering. We indicate also, in the vertical coordinate, the normalized radii for 40 kG of the existing magnets at CERN.

3.2 Cylindrical multiwire proportional chambers

The axial symmetry of the solenoid spectrometer makes possible a very simple detection system.

For recoil protons from elastic scattering up to \( |t| = 5 \text{ (GeV}/c) \) and \( p_{lab} = 300 \text{ GeV} \), the best system will be cylindrical multiwire proportional chambers (MWPC). With a structure such as that sketched in Fig. 5 we can read out the coupled cylindrical coordinates \( r-\phi \) and \( r-z \) simultaneously. As mentioned in Appendix I, one of the important advantages of axial symmetry is the fact that we can solve the problem of event reconstruction independently in these reference frames without ambiguity of decoupled coordinates due to the multiplicity. In practice, this detection system will require the minimum number of wires
for the same space resolution. The z-wires may be eliminated when we have a good resolution in the z-coordinate by means of the conventional method using pulse propagation time in the φ-wires. In this case, we can simplify again the read-out system by using a single coding device per chamber.

The total number of chambers will be \(\sim 6\); 3 before the second scattering, and 3 after, in case of the measurements of the depolarization parameters. For a space resolution of \(\pm 1\) mm in the \(r-\phi\) plane, the total number of wires will not exceed 15K. A more accurate measurement can be achieved by means of drift chambers giving a space resolution of few tenths of \(\text{mm}^2\). In this case, the momentum resolution will be improved by a factor of 5 to 10.

It is also worth while to mention that the cylindrical configuration will allow the combination of a large solid angle, high resolution experiment with a beam intensity of the order of \(10^7\) to \(10^8\) particles/burst.

3.3 Detection of forward particles

As emphasized in the preceding section, important kinematic constraints (momentum analysis) will be put on the recoil particles. However, in order to profit from the high space resolution given by the solenoid spectrometer for recoil particles, we expect to use position-sensitive scintillation counters for the detection of forward particles.

Owing to the recent technical developments in channel electron multipliers, we can expect a space resolution of 0.1 mm for incident and fast forward particles\(^8\). Even with a scintillation hodoscope developed by the CERN-Orsay-Oxford Collaboration\(^9\), it is not difficult to realize an angular resolution of 0.1 mrad. At 300 GeV/c this resolution corresponds to \(\Delta t \sim 0.05\) (GeV/c)\(^2\) at \(|t| = 0.7\), and \(\Delta \phi \sim 4.4^\circ\) at \(|t| = 0.4\) and 2.7\(^\circ\) at \(|t| = 1\).

3.4 Trigger system

In the case where the solenoid axis is along the beam direction, the main trigger system will consist of fast coincidences between two cylindrical 2π scintillation counters (divided into \(\phi\) angles) for recoil protons, and two ring-type forward scintillation counters placed at \(\sim 25\) m and \(\sim 50\) m downstream.

The latter type of counters can be expected to be position-sensitive as described above.

A simple way to reduce the inelastic background in the trigger is to impose the coplanarity constraint between the forward and the backward particles. The backward counters are disposed as shown in Fig. 6a (projection on a perpendicular plane to the beam direction). The outer counters are shifted with respect to the internal ones in azimuthal angle, such that for a given upper limit of the
momentum transfer $|t|$ all the trajectories passing through one of the internal counters hit the corresponding counter of the outer system.

For smaller momentum transfers there exists a loss of elastic events owing to the larger rotation of trajectories, but it will be largely compensated by the increase of the cross-section.

Supposing $|t|_{\text{max}} = 1 \text{ (GeV/c)}^2$, as in the case of the R parameter measurement, Fig. 6b shows the trigger efficiency for different coplanarity constraints. The upper two curves correspond to $\Delta \phi = \pm 7.5^\circ$ and $\pm 15^\circ$ at the inner counters $R_1$, but without the $\phi$-angle limit on the outer counters $R_2$.

A more severe constraint will be obtained, if necessary, with a coincidence matrix, $R_1(I) \ast [R_2(I) + R_2(I+1)]$. In this case the efficiency goes down by a factor of $\sim 6$ at $|t| = 0.4$, but we will still have a counting rate $\sim 6$ times larger than at $|t| = 1$ because of a factor of $\sim 40$ in the elastic cross-section.

Besides the coplanarity constraint, our set-up makes it possible to put, if necessary, a large solid-angle anticoincidence counter between the forward and backward detector system to kill the inelastic background associated with $\gamma$ and neutrals.

3.5 Particle identification

The identification of incident particles will be performed with the aid of a DISC Čerenkov counter.

For $\pi^+$ identification, however, the beam intensity might be limited to a few $\times 10^6$ per burst owing to the problems of beam stability (parallelism) as well as the intrinsic multiplicity of particle production at very high energies. Considering these new problems, we prefer, at the present time, to put a more important weight on pp and $\pi^- p$ reactions. The ratio $K^-/\pi^-$, for example, being expected to be 1.5\% at 150 GeV/c and 0.4\% at $\geq$ 300 GeV/c\(^{16}\), we can carry out at least these parts of our program without the problem of incident particle identification.

In order to reduce the background from the $\pi$ production in the target fragmentation region, which might be important, we expect to use a time-of-flight technique with the backward scintillation counters at least for the small momentum transfer region ($|t| \leq 1$). Figure 7 shows the time of flight of different particles with the same momentum for the paths of the target to $R_2$ and $R_1$ to $R_2$. Using a time-to-digital converter or Lissajous method\(^*)\ recently developed in our group, a time resolution of a few hundred picoseconds will be realized at the stage of data analysis. That is, knowing the precise impact position of particles on the backward counters, we can reduce the resolving time down to the limit of the photomultipliers by compensating for the difference of light path in the scintillator.

3.6 The data acquisition system

The system is built around three processors: the one in front for data acquisition and integer processing, the central one for floating-point calculation, and the back one to control the peripherals. Such a system has the following characteristics:

a) A group-oriented system with high flexibility and computing power.
b) A high-level software support such that anybody can develop, compile, and execute his own FORTRAN programs.
c) An integrated system level access to CAMAC to ease the physics interfacing.
d) Direct memory access for fast data-block transfer.
e) Batch and real-time capability.
f) Very high data "throughput" rates.
g) Permanent experiment monitoring and checking.
h) Link access to the CERN central computing facility for extra complex computation.

Such a system will benefit from the simplicity of the event reconstruction in the solenoidal field, and allow very rapid on-line analysis.

4. EXPERIMENTAL DETAILS

Using the apparatus described above, we present here possible experimental set-ups for the measurement of the various parameters for elastic hadron-hadron scattering at 150-300 GeV/c.

The solenoid spectrometer will be set in two different positions according to the parameter to be measured. Table 1 summarizes the suitable measurements in the two different configurations, one with the solenoid axis along the beam, and the other with it perpendicular to the beam.

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<th>Axial configuration</th>
<th>Perpendicular configuration</th>
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<tr>
<td>( \frac{d\sigma}{dt} )</td>
<td>pp, ( \pi^\pm ), ( K^\pm_p ), ( \pi^- )</td>
<td>pp, ( \pi^\pm ), ( K^\pm_p ), ( \pi^- )</td>
</tr>
<tr>
<td>( P_0 )</td>
<td>pp, ( \pi^\pm ), ( K^\pm_p )</td>
<td>pp, ( \pi^\pm ), ( K^\pm_p ), ( \pi^- )</td>
</tr>
<tr>
<td>D</td>
<td>( \pi^- )</td>
<td>pp *)</td>
</tr>
<tr>
<td>R</td>
<td>( \pi^- )</td>
<td>pp *)</td>
</tr>
<tr>
<td>A</td>
<td>( \pi^- )</td>
<td>( \pi^- ) *)</td>
</tr>
<tr>
<td>A + ( \pi^- )</td>
<td>pp</td>
<td>pp *)</td>
</tr>
<tr>
<td>R + ( bR )</td>
<td>pp</td>
<td>pp *)</td>
</tr>
</tbody>
</table>

*) Possible measurement but not easy.
4.1 Axial configuration

As seen in Table 1, the major part of our program will be performed in this configuration in order to profit from the advantages of axial symmetry.

The experimental set-up for the different parameters will be very similar, except for the polarimeter and the target; there will be a polarized target for the depolarization tensor and a liquid hydrogen target for $\frac{d\sigma}{dt}$ and $P_0$.

As a typical example of a set-up, we show in Fig. 8 a possible arrangement of apparatus for measurement of $R$ in $\pi p$ elastic scattering at 300 GeV/c for $0.5 \leq |t| \leq 1.0$ (GeV/c)$^2$. The main advantage of the set-up is, of course, a complete azimuthal angular acceptance (2$\pi$ geometry).

As a general feature of the axial configuration, one should note also an important advantage concerning inelastic background. The invariant cross-sections of inclusive reactions as a function of transverse momentum $p_T$ obey the well-established exponential law:

$$f \sim e^{-3.5p_T} \cdot C e^{-3p_T^2}$$

$C$ being adjusted such that $(p_T) \sim 0.35$ GeV/c. This means that a large portion of the inelastic events will be limited to a small cylindrical region around the beam, owing to the high longitudinal magnetic field. This feature allows us to make a momentum-sensitive trigger system as well as ensuring the good quality of the events hitting the polarimeter in the case of the second-rank spin parameter measurements. We estimate that random multiple tracks in the detector system should be limited to 4% of the events.

4.1.1 Measurement of depolarization tensors

Using a polarized proton target we propose to measure the $R$ parameter in $\pi^+ p$, and a linear combination of $A$ and $A'$ parameters in $pp$ elastic scattering. The spin direction of the polarized target will be oriented parallel (or antiparallel) to the beam direction. The position of the polarimeter is determined such that the $R$ component of recoil proton polarization in $\pi p$ scattering is a maximum at the secondary scattering; the calculation of spin precession is presented in detail in Appendix III.

As described in the preceding section, the main trigger of the detection system will consist of fast coincidences between two cylindrical scintillation counters $R_1$ and $R_2$ for recoil protons, and two ring-type forward scintillation counters $S_1$ and $S_2$ placed respectively at $\sim 25$ m and $\sim 50$ m downstream (see Fig. 8). The secondary scattering of the recoil proton is not recognized by the main trigger but, as described in Appendix I, the symmetrical feature of our system
will allow rapid pattern recognition of events to reduce the computer time for off-line analysis.

The trajectory of the recoil proton before and after secondary scattering is determined by a series of cylindrical MWPCs. Since our system is not symmetric at the secondary scattering -- that is, the path length of scattered protons depends considerably on the azimuthal angle -- we have checked the effect of multiple Coulomb scattering on the azimuthal distribution of the secondary scattering. As shown in Appendix IV, for an effective path length of ~10 cm of carbon for the incident proton, the bias of measured asymmetry is almost negligible with respect to the statistical error.

4.1.2 Estimate of experimental error in the measurement of $R$

Using the results presented in Fig. 23 and Eq. (AIII.6), we have roughly estimated the experimental error $\Delta R$ in the elastic scattering $\pi^- p$ at 300 GeV/c and $|t| = 1.0$ (GeV/c)$^2$.

a) Estimate of $(\Delta \varepsilon/c)$

**Counting rate of events from the first scattering:** As the azimuthal angular acceptance is $2\pi$ in our set-up, the number of events after the first scattering is given by

$$n_1 = N_{\text{beam}} \cdot N_{\text{target}} \cdot \left( \frac{d\sigma}{dt} \right) \Delta t .$$

$d\sigma/dt$: Supposing that the differential cross-section does not change so much from 200 to 300 GeV/c, we use the cross-section obtained at NAL for $\pi^- p$ at 200 GeV/c $^{10}$, i.e. $(d\sigma/dt) = 0.017$ mb/(GeV/c)$^2$ at $t = -1.0$ (see Fig. 9).

$N_{\text{target}}$: We suppose a propanediol target, 10 cm long, giving $N_{\text{target}} = 4.8 \times 10^{23}$/cm$^2$.

$N_{\text{beam}}$: We suppose a $\pi^-$ beam of an intensity of $1 \times 10^7$/burst, for example the H8 beam in the North Hall. For $\Delta t = 0.1$ (GeV/c)$^2$ we obtain $n_1 = 8.1$ events/burst.

**Counting rate after the secondary scattering:** Using the effective cross-section of the carbon analyser defined by Peterson $^{11}$, the number of events after the scattering on a carbon plate of 10 cm thickness, for example, is given by

$$n_2 = n_1 \cdot N_{\text{carbon}} \cdot (\sigma_{\text{eff}}) ,$$

where
\[ \sigma_{\text{eff}} = 30 \text{ mb for } \sim 500 \text{ MeV protons (see Fig. 10)}, \]
\[ N_{\text{carbon}} = 0.9 \times 10^{24}, \]
\[ n_2 = 0.22 \text{ events/burst}. \]

Now, supposing a machine cycle of 6 sec\(^{12}\) and a total efficiency of the detector system of \(\sim 50\%\), we obtain 52,000 events after a reasonable machine time of \(\sim 800 \text{ hours (\sim 5 weeks net)}\). Therefore we can reasonably expect a statistical error of \(\sim 0.4\%\) on \(\varepsilon\).

The magnitude of \(\varepsilon\) depends on \(P_C, |s| \sin \theta_R\), the \(\phi\)-dependence of the asymmetry at the secondary scattering, and also on the background from bound protons in the polarized target.

Supposing a background of roughly 20\% to 30\%, the value found in the low-energy Saclay experiment\(^{13}\) at \(t = -0.2 \text{ to } -0.5 \text{ (GeV/c)}^2\) (with an LMN target), we can estimate
\[ \varepsilon = f_\phi \cdot k \cdot P_C \cdot |s| \sin \theta_R \approx 0.64 \times 0.27 \times |s| \sin \theta_R, \]
where \(f_\phi\) is a factor due to the \(\phi\)-dependence of \(\varepsilon\), \(k\) is a dilution factor due to the background (1/1.3), and \(P_C\) is supposed to be \(\sim 35\%\)\(^{14}\).

Finally, including a systematic error of \(\sim 0.2 \sim 0.3\%\)^*) on \(\Delta \varepsilon\), we get
\[ \left( \frac{\Delta \varepsilon}{\varepsilon} \right) = 0.005 \frac{1}{0.17} \frac{1}{|s| \sin \theta_R} = 0.03 \frac{1}{f(R)}, \]
where \(f(R) = |s| \sin \theta_R\).

b) Estimate of \((\Delta P_C/P_C)\)

Table 2 shows the analysing power of carbon measured at 575 MeV by Coignet et al.\(^{15}\). Owing to the lack of the differential cross-section, this table does not give us the complete information on \(P_C\) corresponding to the effective cross-section, but we can reasonably expect
\[ (\Delta P_C/P_C) \approx 0.02 \]
if, eventually, one performs a calibration experiment at SC energies.

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*) Aschmann et al.\(^{15}\) have measured the asymmetry in inclusive reactions with an experimental error of less than \(\sim 0.3\%\).
Table 2

<table>
<thead>
<tr>
<th>$\theta_{\text{lab}}$</th>
<th>$\delta$</th>
<th>$\bar{E}$</th>
<th>$P_C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7°30'</td>
<td>0.83%</td>
<td>15.56 ± 0.65%</td>
<td>39.45 ± 0.82%</td>
</tr>
<tr>
<td>11°30'</td>
<td>0.63%</td>
<td>14.13 ± 0.46%</td>
<td>35.82 ± 0.90%</td>
</tr>
<tr>
<td>15°30'</td>
<td>0.33%</td>
<td>11.62 ± 0.30%</td>
<td>29.45 ± 0.45%</td>
</tr>
<tr>
<td>19°30'</td>
<td>0.32%</td>
<td>10.38 ± 0.37%</td>
<td>26.31 ± 0.77%</td>
</tr>
</tbody>
</table>

c) Calculation of $\Delta R$

Using $(\Delta \varepsilon / \varepsilon)$ and $(\Delta P_C / P_C)$ estimated above, we obtain from Eq. (AIII.6) of Appendix III:

$$\Delta R = \left( \frac{f}{f'} \right) \sqrt{(0.03/\varepsilon)^2 + (0.02)^2} .$$

With the aid of Fig. 23 of Appendix III we obtain $\Delta R$ for different values of $R$, as presented in Table 3.

Table 3

<table>
<thead>
<tr>
<th>$R$</th>
<th>-30%</th>
<th>-20%</th>
<th>-10%</th>
<th>0%</th>
<th>+10%</th>
<th>+20%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$</td>
<td>0.46</td>
<td>0.53</td>
<td>0.58</td>
<td>0.63</td>
<td>0.68</td>
<td>0.72</td>
</tr>
<tr>
<td>$f'/f$</td>
<td>1.41</td>
<td>1.14</td>
<td>0.95</td>
<td>0.79</td>
<td>0.67</td>
<td>0.53</td>
</tr>
<tr>
<td>$\Delta R$ a)</td>
<td>4.8%</td>
<td>5.3%</td>
<td>5.8%</td>
<td>6.5%</td>
<td>7.3%</td>
<td>8.7%</td>
</tr>
</tbody>
</table>

a) Note that the statistical error in $\varepsilon$ is still an important source of $\Delta R$, i.e. the precision depends very much on the beam intensity and the machine time.

In an extreme case, where the statistical error in $\Delta \varepsilon$ is negligible with respect to the systematic one -- for example, at 150 GeV/c with an intensity of $2 \times 10^8 \pi^-$/burst -- we obtain $\Delta R$ varying from 2.7 to 5.4% following the value of $R$ from -30% to +20%.

4.1.3 Measurement of $d\sigma/dt$

For $d\sigma/dt$ measurements at large $|t|$ our set-up will be able to accommodate a liquid hydrogen target of up to 50 cm length with a slight modification of the detection system as regards longitudinal size.
With four days data-taking for each energy, and assuming a relative error of 10\% on the differential cross-section, we obtain the upper limit for |t| values as shown in Table 4. The counting rate has been estimated for each reaction, assuming a hydrogen target 50 cm long, Δt = 0.1 (GeV/c)^2, and a beam intensity equivalent to that of the H2 or the HB beam in the North Area\(^1\).

**Table 4**

| P\(_{lab}\) (GeV/c) | \(|t|_{max} (GeV/c)^2\) |
|-----------------|-----------------|
|                 | pp              | π\(^+\) p | K\(^+\) p | π\(^-\) p | K\(^-\) p | π\(^-\) p |
| 150             | 2.5             | 3         | 2.5       | 2         | 3         | 1.5       |
| 300             | 2.5             | 2.5       | 2         | 2.5       | 1         | 1         |

At \(|t| < 2\) (GeV/c)^2 the precision Δ(\(d\sigma/dt\))/(d\(\sigma/dt\)) is better than 1\% for pp, π\(^+\)p, and π\(^-\)p reactions.

Taking account of the slowdown of the differential cross-section slope at high \(|t|\), it will probably be possible to measure, with sufficient accuracy, the cross-section up to \(|t| = 4\) (GeV/c)^2 at 100 GeV/c. In Fig. 11 we have a schematic picture of the experimental elastic cross-sections at high energy, shown by heavy lines, and their extrapolation to large \(|t|\) values. With this extrapolation, which is probably too pessimistic, the limits of \(|t|\) are shown, which could be measured in 10 days with the conditions detailed above.

4.1.4 The measurement of P\(_0\) with an axial field

With the axial field, and the same experimental arrangement as for the R-parameter measurement, the ideal situation would be to measure the asymmetry, \(c = P_0 P_{c'}\), by scattering hadrons on a frozen-spin target located near one end of the solenoid in a localized field perpendicular to the solenoid axis. Without this ideal facility, it is possible to measure P\(_0\) with the above axial field setup, by using a liquid-hydrogen target first scatter, and the carbon second scatter as polarization analyser. The asymmetry will in this case be \(c = P_0 P_{c'}\). Owing to the solenoid and analyser geometry required for the R determination, the measurement will be limited to \(|t| \leq 1.5\) (GeV/c)^2, giving recoil protons of 700 MeV kinetic energy. Up to this energy (Section 4.1.2) the carbon cross-section and analysing power are well known. The second scattering introduces a reduction of 1/30 in the counting rate, but this is compensated for by using a 50 cm hydrogen target rather than 10 cm of propanediol, with the added advantage of a much lower
background and no bound proton problems. In addition, there is 2π azimuthal acceptance, without the cos φ dependence of the spin effect in the first scatter.

The mean analysing power of carbon for second scattering angles > 5°, is about 35% at 600 MeV and more than 50% at 150 MeV \[|t| = 0.4 \text{(GeV/c)}^2\]. From the hydrogen target to the analyser position, the normal spin component P₀ rotates in the field by just 180° (Appendix III).

Counting rate and precision estimates are of the same order of magnitude as those given in Section 4.2.1 below.

4.2 Perpendicular configuration

4.2.1 The measurement of P₀ with a perpendicular field

When the magnetic field \[\text{[and the spin direction of the polarized proton target (PPT)]}\] is perpendicular to the beam direction we lose about 50% in the effective acceptance owing to the cos φ dependence of spin effect, but this configuration allows us to measure P₀ by a single scattering on a polarized target. Another advantage of this configuration is the fact that, owing to a large magnetic field (~100 kG·m) almost perpendicular to the forward particles, we can expect a momentum analysis of \(\Delta p/p = 1\%\) with a space resolution of 0.1 mrad.

The counting rates have been estimated for different reactions, supposing a propanediol target 10 cm long, \(\Delta t = 0.1 \text{(GeV/c)}^2\), and a beam intensity equivalent to that of the H2 or the H8 beam in the North Area\(^\text{16}\).

We used the differential cross-sections extrapolated from recent data obtained at NAL\(^\text{10}\). At \(|t| > 1 \text{(GeV/c)}^2\), counting rates are largely underestimated owing to the fact that the extrapolation is made with the \(d\sigma/dt\) slope at low \(|t|\). Under these conditions, assuming a statistical error of \(\Delta P₀ = 0.05\) and 10 days data-taking at each energy, Table 5 shows the upper limit attainable on \(|t|\) for polarization measurements.

| \(P_{\text{lab}}\) (GeV/c) | \(|t|_{\text{max}} \text{ (GeV/c)}^2\) |
|--------------------------|-----------------|
|                          | pp  | \(\pi^+p\) | \(K^+p\) | \(\pi^-p\) | \(K^-p\) | \(\bar{pp}\) |
| 150                      | 2   | 2.5        | 2        | 2.5        | 1.5        | 1.5        |
| 300                      | 2.5 | 2          | 1.5      | 2          | 1          | 0.5        |
Under the same conditions, we also obtain $\Delta P_0 \lessapprox 0.01$ for $|t| < 1$ for all reactions except $\bar{p}p$ and $K^- p$ at 300 GeV/c.

4.2.2 Possibility of measuring some depolarization tensors

As shown in Appendix III 2.2, the A component of the proton spin in $\pi p$ scattering quickly leaves the particle direction and stays for a long time in the transverse position (Fig. 25 of Appendix III). This suggests the possibility of measuring the A parameter without high precision on the position of the polarimeter. For a momentum transfer $|t| < 1 (\text{GeV/c})^2$, this sensitive region of A measurement is limited to a region $r < 1 \text{ m}$ and $|z| < 1 \text{ m}$, as shown in Fig. 26 of Appendix III. In pp scattering it will be possible to measure a linear combination of the R and R' parameters for smaller $\phi$ angles ($< 45^\circ$) and D for larger $\phi$ angles ($> 45^\circ$).
APPENDIX 1

EVENT RECONSTRUCTION

A great advantage of the axial spectrometer will be the possibility of solving the problem of the event recognition independently in the two reference frames, defined by cylindrical coordinates \((r, z)\) and \((r, \phi)\), where \(z\) is the direction of the magnetic field. As described in Section 3.1, the spark points in these frames will be defined uniquely by a simple cylindrical chamber without ambiguity of decoupled coordinates due to the multiplicity.

Among many algorithms of the pattern recognition, we have studied a method derived from the Koh representation of trajectories in axial \(\beta\)-spectrometers\(^{17,18}\).

1. ALGORITHMS

In the first reference frame \((r, z)\), the helicoidal trajectory is described by

\[
r = D \sin \alpha \cdot \sin \frac{z}{D \cos \alpha}, \tag{AI.1}
\]

where \(\alpha\) is the angle between the particle momentum \(\vec{p}\) and the \(z\)-axis, \(D\) is related to the momentum \(p\) by \(D = 2p/\hbar c\). The diameter and the pitch of the helix are given by \(d = D \sin \alpha\) and \(\lambda = \pi D \cos \alpha\), respectively. An example of the trajectory in this frame is presented in Fig. 12a.

In the second frame \((r, \phi)\), the trajectory is presented by a circle

\[
r = d \cos (\phi - \theta), \tag{AI.2}
\]

where \(\phi\) is the cylindrical coordinate angle, and \(\theta\) the angle for \(r = d\) as shown in Fig. 12b.

The method of Koh consists of introducing \(d\) and the pitch \(c = \lambda/\pi\) as new variables. Thus Eq. (AI.1) transforms into

\[
d = \frac{r}{\sin (z/c)}, \tag{AI.3}
\]

For a given pair of coordinates \((r, z)\), Eq. (AI.3) now presents a group of trajectories passing through this point (a circular slit with infinitesimal width in three-dimensional space). As shown in Fig. 13, when we define three pairs of coordinates on a trajectory \(r_1, z_1\), \(r_2, z_2\), and \(r_3, z_3\), for example, the intersection of the curves presented by Eq. (AI.3) determines uniquely the diameter \(d\) and the pitch \(\lambda\), and consequently the angle \(\alpha\). It is evident that any pairs of \(r\)-\(z\)
coordinates not belonging to the trajectory do not give the intersection on the same point.

As a natural extension we developed this method in the \( r-\phi \) plane. Considering \( d \) and \( \theta \) as new variables, Eq. (AI.2) transforms into

\[
d = \frac{r}{\cos (\phi - \theta)}.
\]

For a given pair of coordinates \( \phi \) and \( r \), this equation presents a straight line, tangent to a circle \( r \) at \( \theta = \phi \). That is, we find a simple fact: that the Koh representation in the \( r-\phi \) plane is equal to a problem of a elementary Euclid geometry -- to find the diameter of a circle from its segments. An example of the representation is given in Fig. 14 in the case of two trajectories.

Having solved the problem of the pattern recognition independently in \((r,z)\) and \((r,\phi)\) frames, we can combine \( \phi \)- and \( z \)-coordinates through the diameter \( d \) of different tracks.

As a simple test of the algorithm in the \( r-\phi \) plane, we generated \( n \) tracks plus \( m \) spurious sparks randomly distributed in space, and reconstructed tracks from \((n+m)\) mixed pairs of \( r-\phi \) coordinates. An example of results is shown in Fig. 15.

2. APPLICATION TO FOUR-TRACK EVENT RECOGNITION

We have studied the algorithm in the \( r-\phi \) plane using four-track events generated by a Monte Carlo program.

2.1 Simulation of events

In order to test our method under a realistic condition, we have made a rough simulation of four-track events assuming

\[\pi^- p \rightarrow \pi^+ \pi^- \pi^- p\]

at 16 GeV/c. The routine GENBOD from the CERN computer library generates \( n \)-particle events weighted according to Lorentz-invariant Fermi phase space.

The events are then selected by the following rough dynamic constraints:
- effective mass of the \( \pi^+ \pi^- \) system = 770 \pm 60 MeV (\( \rho^0 \) mass);
- momentum transfer to the \( \pi^+ \pi^- \) system \( \leq 0.3 \) (GeV/c)\(^2\) according to the forward \( \rho^0 \) production.

The events are traced in a uniform magnetic field of 40 kG and define 12 spark points on three planes perpendicular to the beam axis. The space resolution
of the detector is taken to be $\Delta x = \Delta y = \pm 1$ mm and $\Delta z = \pm 2$ mm with uniform distribution. The vertex distribution is also assumed to be uniform inside a target of 10 mm diameter placed on the solenoid axis.

2.2 Definition of the search parameter

Using three spark points $P_1$, $P_2$, and $P_3$, we calculate the intersections of the tangents to circles $r_1$, $r_2$, and $r_3$ at $P_1$, $P_2$, and $P_3$ as shown in Fig. 16. The search parameter is then defined by

$$R = \frac{P_1 Q_2}{P_1 Q_1}.$$

When we have $n$ tracks per event, we calculate this ratio for all possible combinations of spark points in the different planes (for example, 64 combinations in the case of four-track events).

2.3 Algorithm results

Figure 17 shows the search parameter distribution obtained with 500 four-track events. The distribution illustrated by dotted lines shows what the real track signal looks like in the region of $R \approx 1$. The background under the peak comes from the false combinations giving accidentally $R \approx 1$. Table 6 summarizes the percentage of the signal as well as the noise-to-real-track ratio for different cuts on $R$.

The noise-to-real-track ratio is defined as

$$N/S = \frac{\text{total number of combinations}}{(\text{real track combination})} - \frac{(\text{real track combination})}{(\text{real track combination})}.\]

Spurious sparks treated in the same manner will produce a similar distribution to that obtained with the false track combinations. Therefore, this ratio will be related implicitly to the rejection ratio for the physical background when the algorithm is used for the event selection.

In the last column of Table 6 we present also the computing time of the search parameters (including logical tests) on the CERN CDC 7600.

<table>
<thead>
<tr>
<th>Cut on R</th>
<th>Signal (%)</th>
<th>N/S</th>
<th>Computer time on CDC 7600</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 \pm 0.25$</td>
<td>85</td>
<td>1.56</td>
<td>[440 \mu s/\text{event}]</td>
</tr>
<tr>
<td>$1 \pm 0.36$</td>
<td>89</td>
<td>1.95</td>
<td>[]</td>
</tr>
</tbody>
</table>
The long tail of the signal distribution is due to the fast forward pions which give a very short segment of trajectory in the sensitive volume of the spectrometer.

When the signal is of the order of 90%, the probability of recognizing more than \( n (\leq 4) \) real tracks per event is:

\[
\begin{align*}
\sim 100\% & \quad \text{for } n \geq 2 \\
\sim 96\% & \quad \text{for } n \geq 3 \\
\sim 66\% & \quad \text{for } n = 4
\end{align*}
\]

The N/S ratio will be considerably reduced by checking the vertex of four tracks.

Since the computer time depends very much on the software system, it is difficult to estimate the corresponding time for small on-line computers. However, 440 \( \mu \)sec/event on CDC 7600 will not exceed 10 msec/event on most of the typical on-line computers.

The method is still under development in order to combine with the algorithm in \((r-z)\) plane, and will be tested for the following applications:

i) recognition of secondary scatterings, and

ii) elimination of spurious sparks in high intensity experiments.
APPENDIX II

GENERAL FEATURES OF EXPERIMENTS USING A SOLENOID

We will not at this time make other specific proposals for the use of the solenoid spectrometer, but point out some of its general features which make it a powerful instrument in several classes of experiments. In addition, we restrict the discussion to the case where the incident beam is along the axis of the solenoid.

1. MULTIPARTICLE VERTEX DETECTION

In a large class of interactions at high energies, a large fraction of the produced particles are emitted within a narrow cone in the forward direction. With information on these particles alone there is a large error in the z-coordinate of the vertex, and consequently in the production angles. The use of cylindrical chambers around the target permits the measurement of the less copious large-angle particles.

In a similar way, the apparatus is well-suited to finding secondary vertices which arise from secondary interactions or decays. In both aspects it appears to be in some cases competitive with streamer chambers and rapid cycling bubble chambers.

2. LARGE $p_T$ TRIGGER AND MEASUREMENT

Particles with $p_T < 15$ BR (8 in kg, R in m) are confined and do not reach the periphery of the solenoid, for example for $B = 20$ kg, $R = 2$ m, $p_T < 0.6$ GeV/c. Thus a trigger system based on $p_T$ may easily be devised, and with complete azimuthal acceptance. Moreover, the apparatus may be made completely insensitive to particles of low $p_T$ simply by choosing the minimum radius for the cylindrical proportional chambers. High beam intensity is consequently also possible in some cases. The solenoid directly measures $p_T$ using only the ($r$, $\phi$) information. The internal volume is sufficiently large to accommodate detectors for particle identification. Large $p_T$ muons could, of course, be identified outside.

3. BACKWARD SCATTERING PROCESSES

Backward scattering processes have the property that the kinematics of the backward-going particle are essentially independent of the incident energy. This is true both for elastic and quasi-elastic processes and for inclusive interactions in the target fragmentation region. The solenoid spectrometer has full acceptance for these particles and gives good angular and momentum resolutions.
4. COMPLETE DETECTION OF CHARGED SECONDARIES

The solenoidal spectrometer used in conjunction with a forward dipole spectrometer for the high-energy small-angle particles allows the possibility to measure all the charged secondaries.
APPENDIX III

SPIN ROTATION OF RECOIL PROTON IN
UNIFORM MAGNETIC FIELD

The spin motion in a magnetic field is described by a covariant equation:

\[ \dot{s} = g \mu_0 SF + \left( \frac{e}{m} - g \mu_0 \right) V(SFV), \]  

(AIII.1)

where \( S \) and \( V \) are spin and velocity four-vectors, and \( V \) is a four-tensor representing the electromagnetic field.

When the field is homogeneous (and \( \dot{E} = 0 \)) we can easily solve the equation to get analytical formulae describing the polarization vector as a function of time.

We define the polarization vector of the recoil protons by

\[ \vec{s} = |s| (\hat{z} \cos \theta_R + \hat{n} \sin \theta_R), \]

(AIII.2)

where \( \hat{z} \) is a unit vector parallel to the velocity \( \vec{v} \), and \( \theta_R \) is the angle between \( \hat{z} \) and \( \vec{s} \) (defined in the rest frame). The unit vector \( \hat{n} \) is defined such that \( \vec{s} \) is contained in a plane spanned by \( \hat{z} \) and \( \hat{n} \).

The polar angles of the two unit vectors are shown in Fig. 18a.

Supposing the magnetic field to be parallel to the z-axis, we obtain the following formula describing the spin direction at any time \( \tau = \tau / \gamma \):

\[ \theta_R = \cos^{-1} \left\{ \frac{A \cos \alpha - B \sin \left[ (\gamma^2 \sin^2 \alpha + \cos^2 \alpha)^{1/2} (\omega \tau + \theta) \right]}{\gamma^2 \sin^2 \alpha + \cos^2 \alpha} \right\}, \]

(AIII.3)

\[ B = \frac{A - \cos \alpha \cos \theta_R}{\gamma \sin \theta_R}, \]

(AIII.4)

\[ \phi = \phi_0 - \omega \tau, \]

(AIII.5)

\[ |\delta| = \cos^{-1} (-\cot \alpha \cot \beta), \]

(AIII.6)

with constants:

\[ A = \gamma \cos \beta_0 \sin \theta_{R_0} + \cos \alpha \cos \theta_{R_0}, \]

\[ B = \gamma \sin \alpha \left[ (\gamma^2 - 1) \sin^2 \alpha + (1 - A^2) \right]^{1/2}, \]

\[ \omega = \left( g \mu_0 - \frac{e}{m} \right) R = \left( R / 2 - 1 \right), \]
\[ \theta = \frac{1}{(\gamma^2 \sin^2 \alpha + \cos^2 \alpha)^{\frac{1}{2}}} \sin^{-1} \left[ \frac{A \cos \alpha - (\gamma^2 \sin^2 \alpha + \cos^2 \alpha) \cos \theta_R}{\gamma \sin \alpha \left[ (\gamma^2 - 1) \sin^2 \alpha + (1 - A^2) \right]^{\frac{1}{2}}} \right], \]

where \( \theta_R, \beta_0, \) and \( \phi_0 \) refer to the initial orientation of the proton spin. The sign of \( \delta \) can be determined by checking the direction of \( \hat{z} \times \hat{n} \) with respect to \( \hat{z} \).

If \( \alpha = \pi/2 \) and \( \beta_0 = \pi/2 \), that is, a circular motion in the x-y plane containing the spin vector, Eq. (AIII.1) reduces to the simple well-known formula

\[ \theta_R = \omega t / \gamma + \theta_R^0. \]

In any case, the spin precession is governed essentially by the term \( \omega t = \omega_{P_L} (g/2 - 1)(t/\gamma) \) of Eq. (AIII.3).

1. GENERAL VIEW OF SPIN MOTION

Before presenting the results of calculation of spin rotation in the specific cases, it is worth while to mention some general features in order to see the maximum possibility of a spin experiment using a solenoid. Figure 19 is an approximate sketch of the spin and particle motion in a uniform magnetic field. Irrespective of the initial direction, the rotational frequency of the spin vector is approximately \( (g/2 - 1) \) = 1.9 times faster than the Larmor frequency (neglecting the relativistic factor, \( \gamma^2 \sin^2 \alpha + \cos^2 \alpha \), due to the Lorentz transformation of the magnetic field) in the rest frame of the particle. For example, when the initial direction is perpendicular to the scattering plane (x-direction), this transverse component will be reproduced after a particle rotation of about 90°, suggesting the possibility of measuring the \( P_0 \) parameter using a non-polarized hydrogen target.

2. SPIN ROTATION OF RECOIL PROTON FROM Tp ELASTIC SCATTERING

The calculation of the spin rotation has been performed in two different configurations, one with the magnetic field and the target polarization parallel to the beam direction, and the other perpendicular to it.

The kinematic and the spin conditions have been assumed as follows:

i) Kinematic conditions of recoil protons from Tp elastic scattering at \( P_{lab} = 300 \) GeV/c, \( |t| = 0.5 \sim 1.0 \) (GeV/c)^2.

ii) We assumed \( P_0 \) and \( R \) as unknown parameters in regions \( +10\% \geq P_0 \geq -10\% \) and \( +30\% \geq R \geq -40\% \), but an important weight of the present calculation is centred around \( P_0 \approx 0\% \) and \( R \approx -20\% \).

iii) For each pair of \( P_0 \) and \( R \), the parameter \( A \) is calculated from \( P_0^2 + A^2 + R^2 = 1 \).
2.1 Magnetic field parallel to the beam direction

In this configuration the polarization vector of recoil protons from \( p \) elastic scattering before the precession is given by three polarization parameters, \( P_0, R, \) and \( A \) as shown in Fig. 18b, where the target polarization is taken as being antiparallel to the beam direction.

Knowing \( P_\perp, P_0, R \) and \( A \) we obtain the initial angles \( \beta_0 \) and \( \theta_0 \) from

\[
\cos \beta_0 = \frac{-AP_\perp \sin \alpha}{\sqrt{P_0^2 + (AP_\perp)^2}}
\]

(11.1)

\[
\cos \theta_0 = \frac{-RP_\perp}{\sqrt{P_0^2 + (AP_\perp)^2 + (RP_\perp)^2}}
\]

where \( \alpha \) coincides with the production angle of the recoil proton, and is the constant of motion when the magnetic field is uniform. The results of the calculation are given in Figs. 20 to 24.

2.1.1 Variation of \( \theta_R \) as a function of time

Figure 20 shows variations of \( \theta_R \) as a function of time calculated with \( H = 25 \text{ kG}, P_0 = 0.01, P_\perp = 0.8, R = -0.4 \text{ to } 0.2, \) and \( |t| = 0.8 \text{ (GeV/c)}^2 \).

As illustrated in Fig. 18c, when \( R \) is strongly negative, the initial spin direction before precession makes a large angle with the magnetic field, resulting in a large variation of \( \theta_R \) as shown in Fig. 20.

Since the asymmetry of secondary scattering is proportional to \( \sin \theta_R \), the most sensitive measurement of \( R \) will be realized when \( \theta_R \) is maximum.

2.1.2 Representation of \( \theta_R^{\text{max}} \) and \( \theta_R^{\text{min}} \) in space

Figure 21 shows the trajectories of recoil protons in a magnetic field of 25 and 40 kG (\( r \) is the distance from the \( z \)-axis) and indicates points on the trajectories where \( \theta_R \) is a maximum (●) and minimum (○). This figure allows us to determine the dimensions of the magnetic field and the position of the polarimeter for \( R \) measurements. The spin conditions are \( P_\perp = 0.8, P_2 = 0.01, \) and \( R = -0.2 \).

2.1.3 Orientation of \( \hat{n} \)

Figure 22 shows the projection of the unit vector \( \hat{n} \) on the \( x-y \) plane, results being obtained under the same conditions as above.

In principle, the orientation of \( \hat{n} \) will be given experimentally by the \( \phi \)-angle distribution of the secondary scattering. Therefore, the calculation of the \( \hat{n} \) vector orientation will serve only to check the experimental distributions.
2.1.4 \[ |s| \cdot \sin \theta_{R_{\text{max}}} \] as a function of R

Figure 23 gives very important information on the sensitivity of R measurements.

The asymmetry in the secondary scattering is given by

\[ \varepsilon = k P_C \frac{|s| \sin \theta_{R}}{R} = k P_C f(R), \]

where $P_C$ is the analysing power of the polarimeter, and $k$ is a dilution factor ($< 1.0$) due to the $\cos \phi$ dependence of asymmetry in the secondary scattering and the inelastic background.

Putting $f(R) = \varepsilon/k P_C$ and assuming $\Delta k = 0$, we get

\[
\left( \frac{\Delta f}{f'} \right) = \left( \frac{\Delta \varepsilon}{\varepsilon} \right)^2 + \left( \frac{\Delta P_C}{P_C} \right)^2 .
\]  \hspace{1cm} (AIII.9)

On the other hand, $\Delta f$ reflects on $\Delta R$ through the relation

\[ \Delta f = \left( \frac{\partial f}{\partial R} \right) \Delta R = f' \Delta R . \]

Substituting this into Eq. (AIII.9) we obtain

\[
\Delta R = \left( \frac{f}{f'} \right) \sqrt{\left( \frac{\Delta \varepsilon}{\varepsilon} \right)^2 + \left( \frac{\Delta P_C}{P_C} \right)^2} .
\]  \hspace{1cm} (AIII.10)

Figure 23 presents $f(R)$ calculated under the following conditions:

$H = 25 \text{ kG}$, $P_0 = 0.01$, $P_t = 0.8$, and $|t| = 0.5$, 0.6, 0.8, and 1.0 (GeV/c)$^2$.

The sensitivity ($f'/f$) decreases rapidly as $R$ increases (1.37 at $R = -0.3$ to 0.48 at $R = 0.2$ for $t = -0.8$), but does not change so much as a function of the momentum transfer $|t|$.

2.1.5 \[ |s| \sin \theta_{R} \] as a function of $\lambda = c\beta t$

In order to see the effect of the $P_0$ parameter on the spin motion, we calculated $|s| \sin \theta_{R}$ for different values of $P_0$. Figure 24 presents results for:

$H = 25 \text{ kG}$, $R = -0.2$, $P_0 = -0.1$, 0 + 0.1, and $t = -0.8$ (GeV/c)$^2$, as a function of path length $\lambda = c\beta t$.

As easily expected from Fig. 18c, the change of the $P_0$ parameter introduces only a phase shift of the spin rotation. The figure also shows that, once the polarimeter position is adjusted for a certain value of $P_0$, the variation of $|s| \sin \theta_{R}$ in a reasonable range of polarimeter ($\pm 10$ cm) is almost negligible.
2.2 Magnetic field perpendicular to the beam direction

When the magnetic field and the target polarization are perpendicular to the beam, the spin direction of the recoil proton from πp scattering is defined by three components:

\[
\begin{align*}
u &= \frac{R_P \cos \phi}{1 + P_0 P_t \sin \phi} \\
v &= \frac{A_P \cos \phi}{1 + P_0 P_t \sin \phi} \\
w &= \frac{P_0 + P_t \sin \phi}{1 + P_0 P_t \sin \phi}
\end{align*}
\]

(AIII.11)

Now, the R parameter appears in the transverse component \(u\) with \(\cos \phi\) dependence, while the A parameter appears in the longitudinal component \(v\).

2.2.1 \(|s| \sin \theta_R\) as a function of time

Assuming \(P_0 \sim 0\), \(R = -20\%\), \(P_t = 80\%\), and \(H = 40\) kG, we calculated \(|s| \sin \theta_R\) for different azimuthal angles \(\phi\). Some examples of the results are shown in Fig. 25 as a function of time.

The steep variation of the transverse component at time = 0 comes from the fact that the initial spin direction is very close to the particle direction. The broad peak corresponds to the time interval where the A component has a maximum in the transverse component.

2.2.2 Representation of \(|s| \sin \theta_R\) max in space

In Fig. 26 we plotted the particle position at 5 nsec after the scattering for different momentum transfers and azimuthal angles. Since the variation of \(|s| \sin \theta_R\) is very slow in the peak region, as shown in Fig. 25, this map allows us to determine the position of the polarimeter for the measurement of A.
1. **GENERAL CONDITIONS**

   We study the effect due to the multiple scattering in the carbon analyser, on the azimuthal distribution of scattered particles. We will restrict the results to one kinematical case, with the following conditions for the scattered proton on a polarized target:

   - momentum transfer $t = -0.5 \text{ (GeV/c)}^2$ at 300 GeV/c
   - production angle $\theta = 69.25^\circ$
   - momentum $p = 0.7556 \text{ GeV/c}$.

   In order to fix the reference for the azimuthal angle of the secondary scattered particle, we fix the initial conditions for the spin parameters to:

   $$P_\theta = 0.01, \ R = 0.20, \ A = 0.9797$$

   and a target polarization $P_t = 0.80$.

   We choose the geometrical position of the analyser such that it corresponds to a maximum of $\beta R$ (see Fig. 21) with a field of 40 kG.

2. **EVENTS GENERATION**

   The generation of scattered particles in the analyser was done, taking into account:

   i) the differential cross-section for scattering of protons from carbon (Ref. 11);

   ii) the asymmetry; for this purpose we suppose an azimuthal distribution with the following form:

   $$f(\phi) = A + B \sin \phi$$

   A and B are evaluated by taking into account the analysing power of carbon, the dilution factor due to inelastic background;

   iii) the multiple scattering on the particle before and after the secondary scattering. We assume a Gaussian law with a half width given by:

   $$\langle \phi \rangle = \frac{15 \times 10^{-3}}{\beta \times P} \sqrt{\frac{L}{27.9}}$$

   where L is the covered length in the carbon.
The initial $\phi$ distribution was generated from the "true" initial particle direction (with multiple scattering) at the secondary scattering time $t_d$.

Then we compute the true direction of the scattered particle (with multiple scattering) when it reaches the outside surface of the carbon analyser.

Using that result, we calculate the expected particle direction at $t_d$. And so we compute the reconstructed $\phi$ distribution with respect to a calculated direction (no multiple scattering) of the initial particle at $t_d$.

Figure 27 gives the initial and reconstructed $\phi$ distributions for 50,000 events. On the same figure we give also the result of a fit by a function $f(\phi) = \alpha + \beta \sin \phi$ of the reconstructed distribution.

We also fit the initial distribution, and we obtain the following results for the asymmetry:

$$\varepsilon_{\text{initial}} = 9.95\% \pm 0.4\%$$
$$\varepsilon_{\text{reconstructed}} = 9.85\% \pm 0.4\%$$

so we can conclude that we have almost no bias on the asymmetry due to the multiple scattering in the carbon.
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Figure captions

Fig. 1: Extrapolated polarization parameter in pp elastic scattering at a momentum transfer $t = -0.4 \ (\text{GeV}/c)^2$.

Fig. 2: Extrapolated polarization parameter in pp elastic scattering at $t = -1.6 \ (\text{GeV}/c)^2$.

Fig. 3: Momentum resolution $\Delta p/p$ as a function of $t$.

Fig. 4: Particle trajectory in the $r-z$ plane, for different $t$ values at 300 GeV/c.

Fig. 5: Sketch of the cylindrical wire chamber.

Fig. 6: a) Backward trigger counters.
    b) Trigger efficiency as a function of $t$.

Fig. 7: Time-of-flight for backward particles.

Fig. 8: Possible arrangement for the $R$ measurement.

Fig. 9: $\pi^-p$ cross-section at NAL (Ref. 10).

Fig. 10: Effective proton-carbon cross-section.

Fig. 11: Extrapolated cross-sections for $pp$, $\bar{p}p$, $\pi p$, $Kp$ from NAL results (Ref. 10).

Fig. 12: Projections of helicoidal trajectory:
    a) in the $r-z$ plane
    b) in the $r-\phi$ plane.

Fig. 13: Koh representation of one trajectory with three pairs of $(r,z)$ coordinates.

Fig. 14: Koh representation in the perpendicular plane for two trajectories and three cylindrical chambers.

Fig. 15: Reconstructed trajectories in the perpendicular plane.

Fig. 16: Search parameter definition.

Fig. 17: Search parameter distribution.
Fig. 18: a) Angles definition for spin precession.
b) Initial spin angles for $\pi p$ scattering.
c) Initial definition of $\theta_R$.

Fig. 19: General view of the spin rotation.

Fig. 20: $\theta_R$ as a function of time.

Fig. 21: $\theta_{R_{\text{max}}}$ in space.

Fig. 22: Projection of unit vector $\hat{n}$ on the x-y plane.

Fig. 23: $|s| \sin \theta_{R_{\text{max}}}$ as a function of the R parameter for different t values.

Fig. 24: $|s| \sin \theta_R$ as a function of the path for different values of the polarization parameter $P_0$.

Fig. 25: $|s| \sin \theta_R$ in the perpendicular configuration.

Fig. 26: $|s| \sin \theta_{R_{\text{max}}}$ in space, in the perpendicular configuration.

Fig. 27: a) Effect of multiple scattering on the azimuthal distribution of secondary scattered protons in carbon.
b) Fit of the asymmetry.
$t = -1.6 \ (\text{GeV/c})^2$

Fig. 2
Momentum Resolution 300 GeV\textit{c}

$\frac{\Delta P}{P} \%$

- $B = 20$ kG
- $B = 40$ kG

$Z$ resolution 1 mm
$\phi$ resolution 3 mrad

$-t \ (\text{GeV}\text{c})^2$

Fig. 3
Trajectories of recoil proton from π-p el. at 300 GeV/c

H = 40 kG

Fig. 4
Cylindrical wire chamber

Fig. 5
Fig. 6
Fig. 7
Fig. 8
PRELIMINARY

- $\times...200$ GeV/c
- $\bullet...100$ GeV/c

\[ \pi^- p \rightarrow \pi^- p \]

$\frac{d\sigma}{dt} [\text{mb}/(\text{GeV}/c)^2]$ vs. $-t (\text{GeV}/c)^2$

Fig. 9
Fig. 10
EXTRAPOLATED CROSS-SECTIONS
AND LIMITS OF MEASUREMENTS

Limit for 10% precision at

\( \triangle 300 \text{ GeV} \)
\( \bullet 150 \text{ GeV} \)
Fig. 12
Fig. 13
DEFINITION OF THE SEARCH PARAMETER

\[ R = \frac{R_1 Q_2}{P_1 Q_1} \]

Fig. 16
SEARCH PARAMETER DISTRIBUTION

(500 4-track events)

- All possible combinations
- Only good combinations

Fig. 17
VARIATION OF $\theta_R$ AT $t = -0.8 \text{(GeV/c)}^2$

$\theta_R$ vs. TIME (ns)

Fig. 20
$t = -0.5 \ (\text{GeV}c)^2$

$H = 25 \ \text{kG}$

- $\theta_{R_{\text{max}}}$
- $\theta_{R_{\text{min}}}$

$H = 40 \ \text{kG}$

$t = -0.5, -0.6, -0.8, -1.0 \ (\text{GeV}c)^2$
PROJECTION OF UNIT VECTOR $\vec{n}$ ON x-y PLANE

$H = 25$ kG

- $\Theta_R_{\text{max.}}$
- $\Theta_R_{\text{min.}}$

Fig. 22
ASYMMETRY/\textit{P}_{\text{r}}$ IN SECONDARY SCATTERING

AS A FUNCTION OF \textit{R}

\textit{t} = -0.5 (GeV/c)$^2$

\textit{P}_{\text{i}} = 0.8

\textit{P}_{\text{r}} = 0.01

Fig. 23
$|s| \sin \theta_R$ AS A FUNCTION OF TIME

Magnetic field perpendicular to the beam direction

$H = 40 \, \text{kG}$

$\varphi = 15^\circ$

$\varphi = 0^\circ$

$t = -0.8 \, (\text{GeV/c})^2$

$P_t = 0.8$

$P = 0 \quad R = -0.2$

Fig. 25
REPRESENTATION OF $l_1 l_2 \sin \theta_R$ max IN SPACE

Magnetic field perpendicular to the beam direction

$H = 40 \text{ kG}$, $P_0 = 0$, $R = 0.2$, $P_1 = 0.8$

Fig. 26
AZIMUTHAL DISTRIBUTION OF SECONDARY SCATTERED PARTICLE FOR A CARBON THICKNESS OF 4 cm

initial distribution
reconstructed distribution

--- fit by $f(\phi) = A + B \sin \phi$
for reconstructed distribution
Sadler P.15. inclusive at $x = 0$ if large $p_T$ of target fragmentation and elastic scattering.

Need big magnet for $E^+$ fields.

[i.e. vertical for $pp \rightarrow$ horizontal for $pp$] used in conjunction with E-chondroscopes.

[Doesn't include elastic interactions] $p$-magnet (?)

Trigger for large $p_T$ $(E^+ \text{ and } E^-)$ complicated trigger. (dangerous?)

> 3 GeV/c: 10⁶ c/pd ⨉ $2$ pd/pd at 6 GeV/c. $\delta$ becomes steep at $200$. (detectors)

14% of events on free protons.

$\delta$ subtraction $\rightarrow$ symmetric detection.

In OP $\approx 0.1$ attainable?

Note (proposed) make 4,5 m long 3.5 m SF.

for $x = 0$

Inclusive:

- other cases: simpler.

given big magnet. 1½ years to start work. time to prepare for experiment.

° Insight into polarization related to proton model.

- Puo. (Dik). $\pi^0, K^0, p^0, D$ (hl). ch.

\[
\begin{align*}
\text{P} & \rightarrow \text{P}, \text{P}^0, \text{P}^+ \\
\text{in place} & \geq 1 \\
\text{PP} & \rightarrow \text{P}\text{P}^+ \text{P}^0 \\
\text{PP} & \rightarrow \text{P}^0 \text{P}^+ \text{P}^0 \\
\text{PP} & \rightarrow \text{P}^0 \text{P}^+ \text{P}^0 \\
\text{PP} & \rightarrow \text{P}^0 \text{P}^+ \text{P}^0
\end{align*}
\]

\[\text{No competition on new top.} \]

؟ $bkg.$

؟؟. Do we have to decide today? -

Insights into polarization related to proton model.