The mechanism of core-collapse supernovae and the ejection of heavy elements

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We present here the first results of two-dimensional hydrodynamical simulations of the neutrino-heating phase in the collapsed core of a 15 \( M_\odot \) star, where the neutrino transport is treated with a variable Eddington factor method for solving the Boltzmann transport equation, and the neutrino interactions include nucleon-nucleon bremsstrahlung, nucleon recoils and correlations, and weak-magnetism effects as well as direct interactions between neutrinos of different flavors. With the given input physics (neutrino reactions and nuclear equation of state), our best simulations do not develop strong convection in the neutrino-heating layer behind the shock and do not yield explosions. With about 30\% higher neutrino-energy deposition behind the shock, however, an explosion occurs on a timescale of 150 ms after core bounce. It leaves behind a neutron star with an initial baryonic mass of 1.4 \( M_\odot \) and ejects the \( N = 50 \) isotopes of Sr, Y and Zr in amounts consistent with Galactic abundances.

1. INTRODUCTION

Wilson and collaborators ([1]–[4]) have routinely obtained delayed supernova explosions by the neutrino-heating mechanism [5] for more than ten years in one-dimensional simulations. In order to get a sufficiently large neutrino-energy deposition for reviving the stalled shock, they assumed that neutron-finger convective instabilities boost the neutrino luminosities from the hot neutron star. Moreover, Mayle et. al. [3] used a special nuclear equation of state with a high abundance of pions in the nuclear matter to reproduce the explosion energy of SN 1987A. Both assumptions are not generally accepted. Therefore investigations of the neutrino-driven mechanism should be continued with making less controversial postulates about the involved physics.

 Forced by observations of large-scale anisotropies and mixing in SN 1987A and directed by the fact that the neutrino-heating layer is convectively unstable [6], supernova modelers began to perform multi-dimensional simulations of the post-bounce supernova evolution and discovered that violent motions set in on a timescale relevant for the delayed mechanism ([7]–[11]). In fact, increasing the postshock pressure by rising, outward pushing neutrino-heated matter and allowing for simultaneous accretion (thus maintaining high accretion luminosities) and shock expansion (thus enlarging the layer of neutrino-energy deposition), this postshock convection turned out to have a very helpful influence on the explosion: Models which failed to blow up in spherical symmetry were found to succeed
in two dimensions.

Making use of this effect and employing an advanced version of the Herant et. al. code [8], Fryer has started to explore a variety of progenitors to determine whether neutron stars or black holes are left behind as compact remnants [12]. Fryer and Heger expanded the study to rotating stars [13], and most recently Fryer and Warren have finished the first full supernova simulations in three dimensions, finding good agreement with results that were obtained in the two-dimensional case [14].

Though these simulations are undoubtedly pioneering first steps, they suffer from a major uncertainty: The transport of neutrinos in the supernova core is described in a much simplified way, using grey flux-limited diffusion and ignoring Doppler-shift and gravitational redshift effects on the neutrino spectrum. This approximation is too crude to allow for serious conclusions on the explosion mechanism. The importance of an accurate numerical description of the neutrino transport has repeatedly been pointed out by Bruenn and by Mezzacappa and collaborators (e.g., [15]–[17]).

For this reason, a new generation of supernova codes has been developed in the past years with the aim to replace the widely used (multi-group) flux-limited diffusion treatments (e.g., [18]–[22]) by a direct solution of the Boltzmann transport equation ([23]–[26],[15, 16]). Progress was also made for including the effects of general relativity (GR) in a rigorous way in neutrino-hydrodynamics codes in spherical symmetry ([27]–[29]). Although improved flux limiters for the neutrino transport [30] or two-moment closure schemes [31, and references therein] have been suggested and may work better than previously used flux limiting prescriptions (for an apparently successful new development in this direction, see the results by Bruenn shown in Ref. [29]), any such approximation has ultimately to be compared with Boltzmann solvers applied in dynamical supernova calculations.

In this conference contribution we shall summarize our efforts to combine a Boltzmann solver for the neutrino transport with a multi-dimensional hydrodynamics code, using a state-of-the-art description of neutrino-matter interactions and an approximation of GR which is (hopefully) sufficiently accurate to model supernova explosions and neutron star formation. First results of two-dimensional simulations with this new code named MuDBaTH (Multi-Dimensional Boltzmann Transport and Hydrodynamics) will also be presented.

2. A NEW RADIATION-HYDRODYNAMICS CODE FOR NEUTRINOS

For the integration of the equations of hydrodynamics we employ the Newtonian finite-volume code PROMETHEUS [32], which was supplemented by additional problem specific features by Keil [33]. PROMETHEUS is a direct Eulerian, time-explicit implementation of the Piecewise Parabolic Method (PPM) of Colella and Woodward [34].

The neutrino transport is done with the Boltzmann solver scheme that is described in much detail in Ref. [35]. The integro-differential character of the Boltzmann equation is tamed by applying a variable Eddington factor closure to the neutrino energy and momentum equations (and the simultaneously integrated first and second order moment equations for neutrino number). The variable Eddington factor is determined from the solution of the Boltzmann equation and the system of Boltzmann equation and its
moment equations is iterated until convergence is achieved. Employing this scheme in multi-dimensional simulations in spherical coordinates, we solve the moment equations on the different angular bins of the numerical grid but calculate the variable Eddington factor only once on an angularly averaged stellar background. This approximation is good only for situations without significant global deformations. Since the iteration of the Boltzmann equation has to be done only once per time step, appreciable amounts of computer time can be saved [35]. We point out here that it turned out to be necessary to go an important step beyond this simple “ray-by-ray” approach. Physical constraints, namely the conservation of lepton number and entropy within adiabatically moving fluid elements, and numerical requirements, i.e., the stability of regions which should not develop convection according to a mechanical stability analysis, make it necessary to take into account the coupling of neighbouring rays at least by lateral advection terms and neutrino pressure gradients.

General relativistic effects are treated only approximately in our code [35]. The current version contains a modification of the gravitational potential by including correction terms due to pressure and energy of the stellar medium and neutrinos, which are deduced from a comparison of the Newtonian and relativistic equations of motion. The neutrino transport contains gravitational redshift and time dilation, but ignores the distinction between coordinate radius and proper radius. This simplification is necessary for coupling the transport code to our basically Newtonian hydrodynamics. Although a fully relativistic treatment would be preferable, tests showed that these approximations seem to work reasonably well as long as there are only moderate deviations (\(\sim 10-20\%\)) of the metric coefficients from unity and the infall velocities do not reach more than 10–20\% of the speed of light.

As for the neutrino-matter interactions, we discriminate between two different sets of input physics. On the one hand we have calculated models with conventional (“standard”) neutrino opacities, i.e., a description of the neutrino interactions which follows closely the one used by Bruenn and Mezzacappa and collaborators [19, 36, 37]. It assumes nucleons to be uncorrelated, infinitely massive scattering targets for neutrinos. In these reference runs we have usually also added neutrino pair creation and annihilation by nucleon-nucleon bremsstrahlung [38]. Details of our implementation of these neutrino processes can be found in Ref. [35].

A second set of models was computed with an improved description of neutrino-matter interactions. Besides including nucleon thermal motions and recoil, which means a detailed treatment of the reaction kinematics and allows for an accurate evaluation of nucleon phase-space blocking effects, we take into account nucleon-nucleon correlations [following Refs. 39, 40], the reduction of the nucleon effective mass, and the possible quenching of the axial-vector coupling in nuclear matter [41]. In addition, we have implemented weak-magnetism corrections as described in Ref. [42]. The sample of neutrino processes was enlarged by also including scatterings of muon and tau neutrinos and antineutrinos off electron neutrinos and antineutrinos and pair annihilation reactions between neutrinos of different flavors (i.e., \(\nu_{\mu,\tau} + \bar{\nu}_{\mu,\tau} \rightarrow \nu_e + \bar{\nu}_e\); [43]).

Our current supernova models are calculated with the nuclear equation of state of Lattimer and Swesty [44]. For the density regime below \(6 \times 10^7 \text{ g/cm}^3\) we switch to an equation of state that considers electrons, positrons and photons, and nucleons and nuclei.
with an approximative treatment for composition changes due to nuclear burning and
shifts of nuclear statistical equilibrium [35].

3. RESULTS: NEW 1D AND 2D SUPERNOVA SIMULATIONS

We have performed a number of core-collapse simulations in spherical symmetry and
then followed the post-bounce evolution in one and two dimensions. All described cal-
culations were started from a 15 $M_\odot$ progenitor star, Model s15s7b2, provided to us by
S. Woosley. Adopting the naming, we label our models by s15N for Newtonian runs and
s15G for runs with approximate treatment of general relativity, followed by letters “so”
when “standard neutrino opacities” were used and by “io” in case of our state-of-the-art
improvement of the description of neutrino-matter interactions. The model names have
a suffix that discriminates between 1D (“_1d”) and 2D simulations (“_2d”).

We have varied yet another aspect in our simulations. Some of the models were com-
puted with a version of the transport code where the velocity dependent (Doppler shift
and aberration) terms in the neutrino momentum equation (and the corresponding terms
in the Boltzmann equation for the antisymmetric average of the specific intensity; see
Ref. [35]) were omitted. These terms are formally of order $v/c$ and should be small for
low velocities. This simplification of the neutrino transport, our so-called “Case A”, was
used in the Newtonian models and in those models with approximate GR which have
names ending with the letter “a”. Models with neutrino transport including all velocity
dependent terms also in the neutrino momentum equation (“Case B”) can be identified
by the ending “b” of their names. Table 1 provides an overview of the computed models.

Table 1
Input physics for our set of computed models. See text for details.

<table>
<thead>
<tr>
<th>Model</th>
<th>Dim.</th>
<th>Gravity</th>
<th>$\nu$ Reactions</th>
<th>Transport</th>
<th>Wedge $^\dagger$</th>
</tr>
</thead>
<tbody>
<tr>
<td>s15Nso_1d</td>
<td>1D</td>
<td>Newtonian</td>
<td>standard</td>
<td>Case A</td>
<td>±27°</td>
</tr>
<tr>
<td>s15Nso_2d</td>
<td>2D</td>
<td>Newtonian</td>
<td>standard</td>
<td>Case A</td>
<td>±27°</td>
</tr>
<tr>
<td>s15Gso_1d.b</td>
<td>1D</td>
<td>approx. GR</td>
<td>standard</td>
<td>Case B</td>
<td>±43.2°</td>
</tr>
<tr>
<td>s15Gio_1d.b*</td>
<td>1D</td>
<td>approx. GR</td>
<td>standard$^\dagger$</td>
<td>Case B</td>
<td></td>
</tr>
<tr>
<td>s15Gio_1d.a</td>
<td>1D</td>
<td>approx. GR</td>
<td>improved</td>
<td>Case A</td>
<td>±43.2°</td>
</tr>
<tr>
<td>s15Gio_2d.a</td>
<td>2D</td>
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<td>improved</td>
<td>Case B</td>
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</tr>
</tbody>
</table>

$^\dagger$ Angular wedge of the spherical coordinate grid around the equatorial plane.

$^\dagger$ Calculation without neutrino-pair creation by nucleon-nucleon bremsstrahlung.

The shock trajectories of all models are displayed in Fig. 1. No explosions were ob-
tained with the most complete implementation of the transport equations, Case B (see
lines labeled with H1, H2, S1, B1, corresponding to Models s15Gio_1d.b, s15Gio_2d.b,
s15Gso_1d.b and s15Gso_1d.b*, respectively). The shock trajectories of this sample of
models form a cluster that is clearly separated from the models computed with the transport version of Case A, which generally show a larger shock radius and therefore more optimistic conditions for explosions. Both classes of calculations differ mainly in the (co-moving frame) neutrino energy densities around and outside of the neutrinosphere. In Case A these energy densities are larger, corresponding to somewhat smaller neutrino losses from the cooling layer and somewhat higher neutrino energy deposition in the heating layer behind the supernova shock. Although the differences are moderate (typically 10–30%, depending on the quantity, radial position and time) during the first 80 ms after bounce, the accumulating effect clearly damps the shock expansion and leads to a dramatic shock recession after this initial phase in all models of Case B. This demonstrates the sensitivity of the long-time evolution of the collapsing stellar core to smaller “details” of the neutrino transport.

A sufficiently large shock radius for a sufficiently long time is also crucial for the growth of convective instabilities in the neutrino-heating layer. This is visible from the two-dimensional Models s15Gio\_2d.b and s15Gio\_2d.a. The former model fails to explode because the convective activity behind the shock is suppressed when the shock retreats, despite of a high energy transfer rate to the postshock matter by neutrinos. In contrast, convective overturn behind the shock becomes strong and is crucial for getting an explosion in Model s15Gio\_2d.a. This is obvious from a comparison with the corresponding one-dimensional simulation, Model s15Gio\_1d.a. It is interesting that the Newtonian 2D run, Model s15NSo\_2d (with standard opacities) does marginally not explode, whereas Model s15Gio\_2d.a with relativistic corrections (and state-of-the-art neutrino reactions) succeeds\(^1\). The influence of the neutrino opacities can be directly seen by comparing Models s15Gso\_1d.b and s15Gio\_1d.b. It is not dramatic but sufficient to justify the inclusion of all the improvements described in Sect. 2. Neutrino-pair creation by bremsstrahlung

\(^1\)At the time of the conference, we had just these two 2D runs and announced the possible success of Model s15Gio\_2d.a. A later 2D computation with the full neutrino moment equations (Model s15Gio\_2d.b for Case B), however, then turned out to produce a dud.
makes a minor difference during the considered phases of evolution (Model s15Gso_1d.b vs. Model s15Gso_1d.b).

The successfully exploding 2D model, Model s15Gio_2d.a, has very interesting properties. Figure 3 shows a snapshot at 248 ms post bounce, giving the distribution of entropy per baryon ($s$ in units of Boltzmann’s constant) and electron fraction. The shock is located at a radius of more than 1700 km at 300 ms after bounce and is expanding with about 10000 km/s. The explosion energy of this model might become rather low. It is only $\sim 4 \times 10^{50}$ erg at that time, but still increasing. The explosion starts so late (at $\sim 150$ ms post bounce) that the proto-neutron star has accreted enough matter to have attained an initial baryonic mass of 1.4 $M_\odot$. Therefore our simulation does not exhibit the problem of other successful models which produced rather small ($\sim 1.1 M_\odot$) neutron stars. Also another problem of published explosion models [e.g., 8, 45, 46, 12] has disappeared: The ejecta mass with $Y_e \lesssim 0.47$ is less than $10^{-4} M_\odot$ (Fig. 2), thus fulfilling a constraint pointed out by Hoffman et. al. [47] for supernovae if they should not overproduce the $N = 50$ (closed neutron shell) nuclei, in particular $^{88}$Sr, $^{89}$Y and $^{90}$Zr, relative to the Galactic abundances. Of course, we will have to follow the explosion for a longer time to make final statements about explosion energy and ejecta composition, and also the neutron star mass may grow by later fallback, especially when the explosion energy remains low.

4. SUMMARY AND CONCLUSIONS

We have presented a set of core collapse and supernova models, which were computed in spherical symmetry and in two dimensions, using a new Boltzmann solver for the neutrino transport and improved neutrino opacities compared to the standard treatment in current supernova codes. We did not find explosions, neither without nor with postshock convection, in our most complete simulations.
Omitting the velocity-dependent terms from the neutrino momentum equation, however, leads to sufficiently large changes in the neutrino quantities and neutrino energy loss or deposition (10–30% between neutrinosphere and shock) that very strong convective overturn in the neutrino-heating region develops and drives a successful explosion. This demonstrates that the models are rather close to the threshold for an explosion, causing an enormous sensitivity of the post-bounce evolution to an accurate treatment of the neutrino transport and hydrodynamics. It also confirms the importance of postshock convection, because the 1D model corresponding to our exploding 2D case does not explode. When the neutrino heating is too weak, and the shock recedes to a small radius quickly after it has reached its maximum radius, however, convective activity behind the shock is suppressed and has no crucial influence on the evolution.

We also found that Ledoux convection below the neutrinosphere, although setting in shortly after bounce and persistent until the end of our simulations, occurs so deep inside the neutron star that its effects on the $\nu_e$ and $\bar{\nu}_e$ luminosities and on the supernova dynamics are insignificant.

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References