RECENT RESULTS ON QUARKONIUM PRODUCTION AND DECAY

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I summarize the current status of the comparison between experiment and the predictions of the NRQCD factorization approach to quarkonium decay and production. I also present the results of some recent calculations and theoretical developments in the NRQCD factorization approach.

1. Some Successes of the NRQCD Factorization Approach

Since its formulation, the NRQCD factorization approach has enjoyed a number of successes in predicting inclusive decay rates and production cross sections for quarkonium states.

The initial success of the NRQCD factorization method was in the computation of infrared-finite predictions for the inclusive decay rates of $P$-wave quarkonium states. A subsequent global fit to the $P$-wave charmonium data has yielded a $\chi^2$ per degree of freedom of 15.0/10 and values of the nonperturbative NRQCD matrix elements that are in good agreement with lattice determinations and with estimates based on the NRQCD velocity-scaling rules.

In the area of quarkonium production, the first success of the NRQCD factorization approach was in explaining the Tevatron data for the inclusive production of $J/\psi$, $\chi_c$, $\psi(2S)$, $\Upsilon$, and $\Upsilon(2S)$ states. Previous calculations in the color-singlet model had yielded results that were smaller than the data by more than an order of magnitude. In the NRQCD predictions, the unknown NRQCD matrix elements were determined from fits to the data. Nevertheless, the comparison with the Tevatron data gives a nontrivial confirmation of the NRQCD factorization approach because the extracted matrix elements satisfy the velocity-scaling rules, and the shape of the data as a function of $p_T$ is consistent with NRQCD factorization, but not with
A recent success of the NRQCD factorization approach is in predicting the cross section for $\gamma\gamma \rightarrow J/\psi + X$ at LEP.\textsuperscript{6} In this case, the prediction makes use of NRQCD matrix elements extracted from the Tevatron data, and hence, provides a test of the predicted universality of the matrix elements. The principal theoretical uncertainties arise from uncertainties in the renormalization and factorization scales and from estimates of the color-octet matrix elements. The data from the Delphi experiment are consistent with the NRQCD factorization prediction and clearly disfavor the prediction of the color-singlet model.

Another prediction that makes use of the NRQCD matrix elements extracted from the Tevatron data is that for quarkonium production in deep-inelastic scattering at HERA.\textsuperscript{7} Again, the theoretical uncertainties arise mainly from uncertainties in the renormalization and factorization scales and in the color-octet matrix elements. Data from the H1 experiment plotted as functions of $p_T$ or $Q^2$ favor the prediction of NRQCD factorization over that of the color-singlet model. Neither prediction is entirely consistent with the data as a function of $z$. It should be noted that the most recent calculation\textsuperscript{7} disagrees with a number of previous theoretical results, and that these disagreements have not yet been resolved fully.

\section*{2. Some Problematic Comparisons with Experiment}

There are some processes for which the comparisons between the experimental data and the NRQCD factorization predictions are less satisfactory.

The production of transversely polarized quarkonium at the Tevatron is potentially a “smoking gun” for the color-octet production mechanism, which is an integral part of the NRQCD factorization formalism. For quarkonium production at large $p_T$ ($p_T \gtrsim 4m_c$ for $J/\psi$), gluon fragmentation into quarkonium via the color-octet mechanism is the dominant process. At large $p_T$, the fragmenting gluon is nearly on mass shell, and, so, is transversely polarized. NRQCD predicts\textsuperscript{8} that the polarization is largely transferred to the $J/\psi$, although it is diluted somewhat by nonfragmentation processes, radiative corrections, and feeddown from higher quarkonium states.\textsuperscript{9,10,11} The data from the CDF experiment show no evidence for the predicted increase of transverse polarization with increasing $p_T$. However, the error bars are large, and only the highest-$p_T$ data point is actually inconsistent with the prediction. There are also large theoretical uncertainties, primarily from corrections of higher order in $\alpha$ and $v$.\textsuperscript{9,10,11}
including $v^2$ effects in the polarization transfer.

Another problematic process is inelastic quarkonium photoproduction at HERA. The HERA data\textsuperscript{12,13} are just barely compatible with the prediction.\textsuperscript{14,15,16,17} The color-octet mechanism leads to a prediction of increasing rate with increasing energy fraction $z$. This is not observed. However, theoretical uncertainties arising from corrections of higher order in $\alpha_s$ and $v$, uncertainties in $m_c$, and the breakdown of the $v$ expansion near $z = 1$ have not been addressed or have been addressed incompletely.\textsuperscript{18,19,20} It should be noted that the experimental data differential in $p_T$ are compatible with color-singlet production alone,\textsuperscript{18} even at large $p_T$.

A recent striking result from the Belle collaboration\textsuperscript{21} concerns production of a double $c\bar{c}$ pair in $e^+e^-$ collisions: $\sigma(e^+e^- \to J/\psi c\bar{c})/\sigma(e^+e^- \to J/\psi X) = 0.59^{+0.15}_{-0.13} \pm 0.12$. Perturbative QCD plus the color-singlet model leads to the prediction\textsuperscript{22} $\sigma(e^+e^- \to J/\psi c\bar{c})/\sigma(e^+e^- \to J/\psi X) \approx 0.1$. This seems to be a major discrepancy between theory and experiment.

3. General Difficulties in the Theory

There are several difficulties that arise when one attempts to make accurate theoretical predictions for quarkonium production and decay processes.

Foremost among these is the fact that the color-octet NRQCD matrix elements are poorly determined. For some processes, only linear combinations of color-octet matrix elements can be determined from the data with reasonable accuracy. Different linear combinations are fixed by different processes—a situation that makes it difficult to test the universality of the color-octet matrix elements.

A further difficulty in making accurate theoretical predictions is the fact that corrections to the short-distance coefficients of next-to-leading-order (NLO) in $\alpha_s$ are often large. For example, for the process $J/\psi \to \gamma\gamma\gamma$, the NLO correction is $-12.62\alpha_s/\pi$, relative to the leading-order coefficient. A related problem is that the dependence of the calculated short-distance coefficients on the renormalization scale is often large. These issues raise doubts about the convergence of the perturbation expansion for the short-distance coefficients. For some key processes, such as $J/\psi$ production in hadronic collisions, the NLO corrections have not yet been computed. For certain processes, resummations of large logarithms of $p_T^2/M^2_\psi$, $M^2_\psi/M^2_\sigma$, and $z^2$ have been carried out.\textsuperscript{23,24} In some instances, resummations of logarithms of $1-x$ and $x$ may also be required.

Another significant source of inaccuracy in theoretical predictions is
the existence of large relativistic corrections. For example, for the process $J/\psi \rightarrow \gamma\gamma\gamma$, the order-$v^2$ correction is $-5.32v^2$. Since $v^2 \approx 0.3$ for charmonium and $v^2 \approx 0.1$ for bottomonium, one can question whether the $v$ expansion converges. Furthermore, for many processes, the order-$v^2$ corrections have not yet been computed.

In the remainder of this paper, I describe some recent progress in confronting these obstacles to accurate theoretical predictions.

4. Relativistic Corrections to Gluon Fragmentation into S-wave Quarkonium and to S-wave Quarkonium Decay

Motivated by the importance of the fragmentation process for $S$-wave quarkonium production at large $p_T$, J. Lee and I calculated the order-$v^2$ corrections to that process. Our preliminary results are that the correction to the color-singlet process is approximately 74% for the $J/\psi$, and the correction to the color-octet process is approximately -54% for the $J/\psi$. The correction to the color-singlet process reduces the predicted value of the transverse polarization parameter $\alpha$ by about 10% at large $p_T$. The corrected prediction is still far above the highest-$p_T$ CDF data point. The correction to the color-octet process directly affects the value of the $3S_1$ color-octet matrix element that is obtained by fitting to the Tevatron data.

A. Petrelli and I have computed the coefficients of color-singlet operators of order $v^4$ for $S$-wave quarkonium decays. This is the first decay or production calculation at next-to-next-to-leading order in $v$. The series for $1S_0$ decay into two photons and for the color-singlet part of $1S_0$ decay into light hadrons are $1 - 1.33v^2 + 1.51v^4 + \cdots$, the series for $3S_1$ decay into $e^+e^-$ is $1 - 1.33v^2 + 1.61v^4 + \cdots$, and the series for the color-singlet part of $3S_1$ decay into light hadrons is $1 - 5.32v^2 + 7.62v^4 + \cdots$. All of these series seem to be converging in order $v^4$, even the one for $3S_1$ decay into light hadrons, which receives a large contribution in order $v^2$. It is known that color-octet contributions in the first non-trivial order ($v^3$ and $v^4$) are large. The hope is that these, too, will receive small corrections in the next order in $v$.

5. Lattice Computation of Bottomonium Decay Matrix Elements

D.K. Sinclair, S. Kim, and I have recently computed matrix elements for bottomonium decay on the lattice, using two dynamical light quarks. The essential steps in this computation are to measure matrix elements in
a lattice simulation, to use perturbation theory (at the cutoff scale \(m_b\)) to relate the lattice and continuum matrix elements, and to extrapolate to three light-quark flavors and to physical light-quark masses. For the matrix element of the color-singlet \(S\)-wave operator in the \(\Upsilon\) state, we obtain \(4.10(1)(9)(41) \text{ GeV}^3\), which compares well with the phenomenological value \(3.86(14) \text{ GeV}^3\) extracted from the measured rate for \(\Upsilon \rightarrow e^+e^-\). We also obtained values for matrix elements of the color-singlet \(P\)-wave operator and the color-octet \(S\)-wave operator in the \(\chi_b\) state, which can be used to make predictions for the decays of \(\chi_b\) states. The matrix element of the order-\(v^2\) color-singlet \(S\)-wave operator in the \(\Upsilon\) state is poorly determined, owing to large lattice-to-continuum corrections. The values of all of these matrix elements are strongly affected by the inclusion of light dynamical quarks in the calculation.

6. Resummation of QCD Corrections to Quarkonium Decay Rates

In this Section I describe work carried out in collaboration with Y.-Q. Chen. Expressions for the decay rates of the \(\eta_c\) into light hadrons (two gluons in perturbation theory) and two photons are both known through NLO in \(\alpha_s\) and \(v\). Each expression depends on two NRQCD matrix elements. However, if we take the ratio of the rates, the dependence on the matrix elements cancels:

\[
R^{\text{NLO}}(\mu) = \frac{\Gamma(\eta_c \rightarrow gg)}{\Gamma(\eta_c \rightarrow \gamma\gamma)} = R_0 \left\{ 1 + \left[ \left( \frac{199}{6} - \frac{13\pi^2}{8} \right) - \frac{8}{9} n_f \right] \frac{\alpha_s(\mu)}{\pi} \right. \\
+ 2\beta_0 \alpha_s \ln \frac{\mu^2}{4m_c^2} + O(\alpha_s^2) + O(v^3) \right\},
\]

where \(R_0 = 9\alpha_s^2(\mu)/(8\alpha_s^2m_c^2)\), and \(\beta_0 = (33 - 2n_f)/(6\pi)\).

One might hope that Eq. (1) would yield an accurate prediction for the ratio. Unfortunately, the term in Eq. (1) proportional to \(\alpha_s\) is approximately \(1.1R_0\), casting doubt on the convergence of the perturbation expansion. Ignoring this difficulty and setting \(\mu = 2m_c\), we obtain \(R^{\text{NLO}}(2m_c) = 2.1 \times 10^4\). The experimental value is \(R^{\text{Exp}} = (3.3 \pm 1.3) \times 10^3\). The agreement is reasonable, however the \(\mu\) dependence of the theoretical result is strong. For example, using BLM scale setting\(^{28}\) we obtain \(\mu_{\text{BLM}} \approx 0.52m_c\). This leads to an NLO term that is 0.5 times the leading term, and, so, the convergence appears to be better. However, the agreement with experiment is poor: \(R^{\text{NLO}}(\mu_{\text{BLM}}) = 9.9 \times 10^3\).
It is instructive to re-write Eq. (1) in terms of $\beta_0$:

$$R^{\text{NLO}}(\mu) = R_0 \left\{ 1 + \left[ \left( \frac{37}{2} \cdot \frac{13\pi^2}{8} \right) + \pi\beta_0 \left( \frac{16}{3} + 2 \ln \left( \frac{\mu^2}{4m_c^2} \right) \right) \right] \frac{\alpha_s(\mu)}{\pi} \right\}.$$  \hspace{1cm} (2)

The terms proportional to $\alpha_s$ that do not contain a factor $\beta_0$ have a value $2.5R_0$, while the terms proportional to $\beta_0$ have a value $12R_0$ for $\mu^2 = 4m_c^2$. The fact that the terms proportional to $\beta_0$ dominate in the NLO correction suggests that we should resum the contributions proportional to $(\alpha_s/\beta_0)^n$ to all orders in $\alpha_s$. To do this, we make use of the method of naive non-Abelianization (NNA). That is, we sum the fermion-loop vacuum-polarization contributions and then take gluon loops into account by replacing the fermion-loop contribution to $\beta_0$ with the full $\beta_0$.

In order to carry this out, one must compute all cuts through the two final-state complete gluon propagators, where each propagator consists of a chain of free gluon propagators and vacuum-polarization bubbles. The result is

$$R^{\text{Bub}} = \sum_{n,m} \left\{ \int_0^1 \frac{dx}{2\pi x} \int_0^1 \frac{dy}{2\pi y} f(x, y) I_R^{(n)}(x) I_R^{(m)}(y) \theta(1 - \sqrt{x} - \sqrt{y}) + 2G_V^{(n)} \int_0^1 \frac{dx}{2\pi x} f(x, 0) I_R^{(m)}(x) + f(0, 0) G_V^{(n)} G_V^{(m)} \right\},$$ \hspace{1cm} (3)

where $x = k^2/(4m_c^2)$, $y = l^2/(4m_c^2)$, $k$ and $l$ are the gluon momenta, $f(x, y)$ is the phase-space and heavy-quark factor ($f(0, 0) = 1$), $I_R^{(n)}(x) = -2\text{Im}[-i\Pi(x)]^n$ is the sum of quark cuts of the complete gluon propagator in $n$th order, $G_V^{(n)} = [-i\Pi(0)]^n$ is the sum of gluon cuts of the complete gluon propagator in $n$th order, and $\Pi_{\mu\nu}(x) = (k^2 g_{\mu\nu} - k_\mu k_{\nu})\Pi(x)$ is the one-loop vacuum polarization. In Eq. (3), the first, second, and third terms correspond to the $qq$, $qg$, and $gg$ cuts, respectively.

The individual terms in Eq. (3) are infrared divergent, but we can write $R^{\text{Bub}} = G_1^2 + G_2$, where

$$G_1 = \sum_n \int_0^1 \frac{dx}{2\pi x} f(x, 0) I_R^{(n)}(x) + \sum_n G_V^{(n)},$$ \hspace{1cm} (4)

and

$$G_2 \equiv \sum_{n,m} \left\{ \int_0^1 \frac{dx}{2\pi x} \int_0^1 \frac{dy}{2\pi y} f(x, y) I_R(x)^{(n)} I_R^{(m)}(y) \theta(1 - \sqrt{x} - \sqrt{y}) - \int_0^1 \frac{dx}{2\pi x} f(x, 0) I_R^{(n)}(x) \int_0^1 \frac{dy}{2\pi y} f(0, y) I_R^{(m)}(y) \right\}.$$ \hspace{1cm} (5)
$G_1$ is infrared finite because of the KLN theorem, and $G_2$ is infrared finite because the integrands in its two terms become identical when $x$ or $y$ vanishes.

Let us first consider the properties of a single vacuum-polarization-bubble chain, as manifested in $G_1$. A calculation of $G_1$ yields

$$G_1(\mu) = \frac{1}{\pi \alpha_s(\mu) \beta_0} \arctan \frac{\pi \alpha_s(\mu) \beta_0}{1 - \alpha_s(\mu) \beta_0 d} - \frac{1}{\pi} \sum_{n=1}^{\infty} \int_0^1 dx \text{Im} \left[ \alpha_s(\mu) \beta_0 (d - \ln x + i\pi) \right]^n,$$

where $d$ is a renormalization-scheme-dependent constant. Now, integration of $\ln^n x$ down to $x = 0$ produces factorial growth: $\int_0^1 dx x^m \ln^n x = (-1)^n n! / (m + 1)^{n+1}$. Therefore, the second term of $G_1$ may contain contributions that grow as $n!$. This is an indication that, owing to the failure of asymptotic freedom at small $x$, the perturbation series may diverge. Factorial growth in the perturbation series is characteristic of the presence of a renormalon—a singularity in the Borel transform of the decay rate—that signals the importance of nonperturbative effects.

However, when the imaginary part of Eq. (6) is computed, a seemingly miraculous cancellation of the factorial growth occurs. The singularity in the Borel transform also vanishes in the imaginary part. In order to understand why this happens, let us re-write the second term of $G_1$ as a contour integral:

$$\sum_n \int_0^1 dx \frac{\pi}{\pi} \text{Im} \left[ \alpha_s \beta_0 (-\ln x + d + i\pi) \right]^n = \sum_n \int_C \frac{dx}{2\pi} [\alpha_s \beta_0 (-\ln x + d)]^n,$$

where the contour runs from $-1$ to 0 to 1 around the cut in $\ln x$. We can deform the contour out of the region of small $x$ into a circle of radius unity. Now the entire contour lies in a region in which perturbation theory applies. No factorial growth occurs. The series is bounded by a geometric series and, hence, is convergent.

Suppose that, for the second term of $G_1$, we carry out the perturbation summation before performing the integration. Then, summing the resulting geometric series, we obtain

$$\int_0^1 dx \frac{\pi}{\pi} \text{Im} \left[ \frac{1}{1 - \alpha_s \beta_0 (-\ln x + d + i\pi)} \right].$$

This differs from the expression (7) by a small, but nonzero, amount: $[1/(\alpha_s \beta_0)] \exp \{ -1/[\alpha_s(\mu) \beta_0 + d] \} \approx e^d \Lambda^2_{\text{QCD}} / (\alpha_s \beta_0 \mu^2)$, which has the typical form of a nonperturbative contribution. Both orders of operations lead...
to convergent expressions. However, \( \log x \) becomes unbounded at small \( x \), and, so, the convergence is not uniform, and different orders of limits can produce different results. If we carry out the summation before performing the integration, as in the expression (8), there is a Landau pole at \( x_0 = \exp[-1/(\alpha_s \beta_0) + d] \). The residue at this pole is the difference between the expressions with different orders of operations.

This raises a question as to which order of limits, if either, is correct. In the unintegrated expression, for \( x \) sufficiently small, we are outside the radius of convergence of the perturbation series, and the perturbation sum is unreliable. In the integrated expression, at any finite order in \( \alpha_s \), we can deform the \( x \) contour to get out of the region of small \( x \). Then, the perturbation expansion converges, and we can take the \( n \to \infty \) limit. Hence, this latter order of limits is the only one of the two that leads to a perturbation expansion that is not obviously unreliable.

Now let us consider the properties of two vacuum-polarization-bubble chains, as manifested in \( G_2 \) [Eq. (5)]. For \( x \sim 1 \), kinematics force \( y \) to be in the nonperturbative region near 0. But, \( x \sim 1 \) implies that \( \sqrt{k^2} \) is large compared with \( l \) or with the three-momentum of the heavy quark. Then, in NRQCD, the propagator carrying \( k \) can be shrunk to a point. The result is a one-loop correction to the color-octet operator \( O_8(3S_1) \). We subtract this nonperturbative piece from the perturbative calculation and absorb it into the quarkonium matrix element of \( O_8(3S_1) \). After these subtractions, the \( x \) and \( y \) contours can be deformed out of the nonperturbative region. Note that the contour-deformation argument is crucial. Without it, there would be contributions from the nonperturbative region in which both \( x \) and \( y \) are small. Such contributions would not have the form of a contribution to an NRQCD operator matrix element and would invalidate the NRQCD factorization formula.

Combining NNA resummation with the complete NLO calculation, we obtain \( R^{NNA} = (3.01 \pm 0.30 \pm 0.34) \times 10^3 \), for \( \alpha_s(2m_c) = 0.247 \pm 0.012 \). This is about 50% larger than \( R^{NLO}(2m_c) \) and is in good agreement with experiment. The first uncertainty arises from the uncertainty in \( \alpha_s(2m_c) \). The second uncertainty comes from a velocity-scaling estimate of the unknown matrix element of \( O_8(3S_1) \), which we simply treat as an error of order \( v^3 \). In the resummed expression, the uncertainty from the choice of \( \mu \) is reduced to about 11% because the bubble sum is a renormalization-group invariant (at leading order in the \( \beta \) function), and, hence, only the residual non-bubble-sum contribution depends on \( \mu \). As is the case with any resummation, the result depends on the choice of resummation scheme. We can

\[ R^{NNA} = (3.01 \pm 0.30 \pm 0.34) \times 10^3, \quad \text{for} \quad \alpha_s(2m_c) = 0.247 \pm 0.012. \]
check that our choice is reasonable by comparing with a different resummation scheme, namely, the use of the background-field-gauge gluon vacuum polarization instead of naive non-Abelianization. This scheme yields a very similar result: \( R^{BFG} = (3.26 \pm 0.31 \pm 0.47) \times 10^3 \).

One may ask why the BLM method, which is supposed to account for vacuum-polarization corrections, yields a different result than NNA resummation. The BLM method approximately resums the vacuum-polarization correction through a choice of scale. A change of scale generates a geometric series: 

\[
\alpha_s(\mu') \approx \alpha_S(\mu)[1 + 2\alpha_s(\mu)\beta_0 \ln(\mu/\mu') + \ldots].
\]

However, in the NNA resummation, the geometric series in the integrand may not yield a geometric series after integration over the gluon virtualities. For example, 

\[
G_1(2m_c) = 1 + 1.91\alpha_s(2m_c) + 2.47\alpha_s^2(2m_c) + 0.97\alpha_s^3(2m_c) - 4.49\alpha_s^4(2m_c) - 11.76\alpha_s^5(2m_c) + \cdots,
\]

which is nothing like a geometric series.

7. Summary and Discussion

The NRQCD factorization approach has had significant successes in predicting the rates for a number of processes. Among these are inclusive P-wave quarkonium decays, quarkonium production at the Tevatron, \( \gamma \gamma \rightarrow J/\psi + X \) at LEP, and quarkonium production in deep-inelastic scattering at HERA. Other processes are, so far, more problematic. The predictions of NRQCD factorization are unconfirmed for quarkonium polarization at the Tevatron, inelastic quarkonium photoproduction at HERA, and double \( c\bar{c} \) production at Belle. For the former two processes, owing to large error bars, there is no real discrepancy at present.

More precise theoretical predictions are hampered by uncertainties in the NRQCD matrix elements and large corrections in NLO in \( \alpha_s \) and \( v \), which have cast some doubt on the convergence of the series in \( \alpha_s \) and \( v \). Lattice calculations can help to pin down the decay matrix elements. It is not yet known how to formulate the calculation of production matrix elements on the lattice. Calculations of higher order in \( v \) are still lacking for many processes, but there is now hope that the \( v \) expansion settles down in order \( v^4 \). In general, resummation of large corrections to the \( \alpha_s \) series are needed. Standard methods exist for dealing with logarithmic corrections. The bubble resummation method is a promising technique for dealing with nonlogarithmic corrections associated with vacuum-polarization contributions. However, other types of large nonlogarithmic corrections are not yet understood. A possible clue to their analysis may lie in the fact that large order-\( \alpha_s \) corrections seem to be correlated with large order-\( v^2 \) corrections.
It is clear that there are still many interesting and challenging problems in heavy-quarkonium physics that remain to be solved.

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**References**