We discuss different QCD approaches to calculate the form factor $F^{\gamma^*\gamma\pi}(Q^2)$ of the $\gamma^*\gamma \rightarrow \pi^0$ transition giving preference to the light-cone QCD sum rules (LCSR) approach as being the most adequate. In this context we revise the previous analysis of the CLEO experimental data on $F^{\gamma^*\gamma\pi}(Q^2)$ by Schmedding and Yakovlev. Special attention is paid to the sensitivity of the results to the (strong radiative) $\alpha_s$-corrections and to the value of the twist-four coupling $\delta^2$. We present a full analysis of the CLEO data at the NLO level of LCSRs, focusing particular attention to the extraction of the relevant parameters to determine the pion distribution amplitude, i.e., the Gegenbauer coefficients $a_2$, $a_4$. Our analysis confirms our previous results and also the main findings of Schmedding and Yakovlev: both the asymptotic, as well as the Chernyak-Zhitnitsky pion distribution amplitudes are completely excluded by the CLEO data. A novelty of our approach is to use the CLEO data as a means of determining the value of the QCD vacuum non-locality parameter $\lambda_2^q = \langle \bar{q}D^2q\rangle/\langle \bar{q}q\rangle = 0.4 \text{ GeV}^2$, which specifies the average virtuality of the vacuum quarks.


Keywords: Transition form factor, Pion distribution amplitude, QCD sum rules, Factorization, Renormalization group evolution
This data has been processed by Schmedding and Yakovlev (SY) [?] using light-cone QCD sum rules (LCSR), taking also into account the perturbative QCD contributions in the next-to-leading order (NLO) approximation. In this way SY obtained useful constraints on the shape of the pion distribution amplitude (DA) in terms of confidence regions for the Gegenbauer coefficients $a_2$ and $a_4$, the latter being the projection coefficients of the pion DA on the corresponding eigenfunctions. Note that SY have extended to the NLO the LCSR approach suggested before by Khodjamirian [?] for the leading order (LO) light-cone sum rule method.

The present analysis gives further support to the claim, expressed by the above mentioned authors, that LCSRs provide the most appropriate basis in describing the form factor of the $\gamma^*\gamma \to \pi^0$ transition. This is intimately connected with peculiarities of real-photon processes in QCD [? , ?]. But the method of the CLEO data processing, adopted in [?], seems to be not quite complete from our point of view. We think that an optimal analysis should take into account the correct ERBL evolution of the pion DA to the scale $Q_{\text{exp}}^2$ of the process (the latter not to be fixed at some average point, $\mu_{\text{SY}}=2.4$ GeV, as done in [?]) and to re-estimate the contribution $\delta^2$ from the next twist term. The influence of both these effects appears to be important and it is examined here in detail. Furthermore, we are not satisfied with the error estimation performed in the SY analysis, for reasons to be explained later, and prefer therefore to use a more traditional treatment to determine the sensitivity to the input parameter $\delta^2$ and the construction of the 1-$\sigma$ and 2-$\sigma$ error contours.

Our main goal in the present work will be to obtain new constraints on the $(a_2, a_4)$ DA parameters from the CLEO data, taking into account all the remarks mentioned above, and then to compare them with the constraints following from QCD SRs with nonlocal condensates (NLC). We will not repeat here the derivation of the main results of LCSRs, as well as those related to NLC SRs, but we will refer the interested reader to [? , ?] and correspondingly to [? , ?] and references therein. But for the sake of convenience we included a technical exposition of our approach in comprehensive appendices. The paper is organized as follows. In Sec. 2 we review different QCD approaches to calculate the transition form factor $F_{\gamma^*\gamma\pi}(Q^2)$, having recourse to QCD “factorization theorems” [?], encompassing both perturbation theory and LCSRs. The analysis of the CLEO data is discussed in Sec. ?? in conjunction with the SY approach in comparison with other approaches/approximations. In Sec. ?? we present a complete NLO analysis of the CLEO data with a short discussion of the BLM setting procedure. Sec. ?? includes a comparison of the QCD SR pion DA models with the results obtained in Sec. ?? from the CLEO data processing. In Sec. ?? we summarize our conclusions. The paper ends with five appendices: in Appendix A we re-estimate the value of the twist-four scale $\delta^2$. In Appendix B the old Chernyak–Zhitnitsky (CZ) result for $a_2$ is discussed, paying attention to evolution effects. In Appendices C and D the two-loop results [? , ?] for the purely perturbative part of the form-factor calculations and the ERBL evolution of the pion DA are outlined. Finally, in Appendix E, all needed calculation details for the NLO LCSR are presented.

2 Transition form factor $F_{\gamma^*\gamma\pi}(Q^2)$ with LCSR

2.1 Factorization of the $F_{\gamma^*\gamma\pi}$ form factor. Standard results

The form factor of the process $\gamma^*(q_1)\gamma^*(q_2) \to \pi^0(p)$ is defined by the matrix element

$$
F_{\gamma^*\gamma\pi}(Q^2) = \frac{\langle \pi^0(p) | J_{\mu} | \gamma^*(q_1)\gamma^*(q_2) \rangle}{\sqrt{2} \alpha_s \frac{\mu^2}{Q^2} \frac{1}{Q^2}}
$$

where

2