\[ (\mathcal{A} + \mathcal{B} + Tc_{L}) = \Omega \]

\[ \mathcal{T}(\omega) = \mathcal{T}(\bar{\omega}) \]

The electroweak operator is defined in terms of the generators of the weak and electromagnetic currents, and \( T_{L} \) and \( T_{R} \) are the left-handed and right-handed components of the gauge fields. We will use the notation \( \mathcal{T}(\omega) \) for the transformation of \( \omega \) under the gauge group, and \( \mathcal{T}(\bar{\omega}) \) for the transformation of the adjoint representation of \( \mathcal{T}(\omega) \).

In this section, we will briefly review the most important features of the model. Details can be found in ref. 2.

II. THE MODEL

In this section, we will define the left-handed basis for the neutrino mass matrices and a parameter in the model, which will be used in the calculations and a parameter in the model, which will be used in the calculations.

The model is based on the assumption that the neutrino masses are generated by a seesaw mechanism, and that the right-handed neutrinos are the lightest.

We will use the notation \( \mathcal{A}(1)_{L} \times \mathcal{B}(1)_{L} \times T_{L} \) for the model, and \( \mathcal{A}(1)_{L} \times \mathcal{B}(1)_{L} \times T_{L} \) for the model.

In the introduction, we will discuss the neutrino masses and a parameter in the model, and the neutrino masses and a parameter in the model.

The neutrino masses and a parameter in the model are given in the first section.

In the introduction, we will discuss the neutrino masses and a parameter in the model, and the neutrino masses and a parameter in the model.
A general choice of the Higgs sector includes a Higgs field $\Phi$ in the mixed representation $(1/2, 1/2, 0)$ and two Higgs doublets

$$\chi_L = \begin{pmatrix} \chi_L^+ \\ \chi_L^{-} \end{pmatrix}, \quad \chi_R = \begin{pmatrix} \chi_R^+ \\ \chi_R^{-} \end{pmatrix},$$

with transformation properties

$$(1/2, 0, 1)_{\chi_L}, \quad (0, 1/2, 1)_{\chi_R}. \quad (4)$$

The breakdown of $SU(2)_L \times SU(2)_R \times U(1)_Y$ down to $U(1)_Y$ is realized through a non trivial pattern of vacuum expectation values for the Higgs fields, namely,

$$< \chi_L > = \begin{pmatrix} 0 \\ v_L \end{pmatrix}, \quad < \chi_R > = \begin{pmatrix} 0 \\ v_R \end{pmatrix}, \quad < \Phi > = \begin{pmatrix} k & 0 \\ 0 & k' \end{pmatrix}. \quad (5)$$

Higgs doublets are responsible for the gauge boson and fermion masses. To the Higgs sector we add two new Higgs singlets, one coupled to Dirac mass terms - $S_D$ - and the other coupled to Majorana mass terms - $S_M$.

At present there are several indications in favor of nonzero neutrino mass and mixing between families coming from solar and atmospheric neutrino data [9]. Neutrinos are predicted to be Majorana particles in many extensions of the standard model containing neutrinos with nonzero masses. Here we will do so and allow Majorana mass terms within the Yukawa sector of the lagrangian. For the first family we have

$$\mathcal{L}_M = f \left[ \bar{e}_L \gamma^\mu \gamma^5 \alpha_R e_R + \bar{\nu}_L \gamma^\mu \gamma^5 \beta_R \nu_R + \bar{\nu}_R \gamma^\mu \gamma^5 \alpha_L \nu_L \right] +$$

$$+ f' \left[ \bar{e}_L \gamma^\mu \gamma^5 \beta_R \nu_R + \bar{\nu}_L \gamma^\mu \gamma^5 \alpha_L \nu_L \right] + f'' \left[ \bar{e}_L \gamma^\mu \gamma^5 \nu_R + \bar{\nu}_L \gamma^\mu \gamma^5 \nu_L \right] +$$

$$+ g's_M \left[ \bar{\nu}_L \gamma^\mu \gamma^5 \nu_R + \bar{\nu}_R \gamma^\mu \gamma^5 \nu_L \right] + g''s_D \bar{\nu}_R \nu_L + g'''s_D \bar{\nu}_L \nu_R.$$

The generalization to three families is straightforward. Notice that the inclusion of Majorana terms spoils the invariance with respect to any global gauge transformation so that there is no conserved leptonic charge (see for example Ref. [10]).

Fermions masses arise after spontaneous symmetry breaking of the gauge structure $SU(2)_L \times SU(2)_R \times U(1)_Y$ down to $SU(2)_L \times U(1)_Y$. For the charged and neutral sectors the mass lagrangians are, respectively

$$\mathcal{L}_{M,c} = f \left[ \bar{e}_L \gamma^\mu \gamma^5 \alpha_R e_R + \bar{\nu}_L \gamma^\mu \gamma^5 \beta_R \nu_R + \bar{\nu}_R \gamma^\mu \gamma^5 \alpha_L \nu_L \right] +$$

$$+ g's_M \left[ \bar{\nu}_L \gamma^\mu \gamma^5 \nu_R + \bar{\nu}_R \gamma^\mu \gamma^5 \nu_L \right] + H.C., \quad (7)$$

and,

$$\mathcal{L}_{M,n} = f \left[ \bar{\nu}_L \gamma^\mu \gamma^5 \nu_R + \bar{\nu}_R \gamma^\mu \gamma^5 \nu_L \right] + f' \left[ \bar{e}_L \gamma^\mu \gamma^5 \nu_R + \bar{\nu}_L \gamma^\mu \gamma^5 \nu_L \right] +$$

$$+ g''s_D \bar{\nu}_R \nu_L + H.C. \quad (8)$$

One of the main points of the mirror left-right model is the presence of the term $g''s_D \bar{\nu}_R \nu_L$ in the charged lepton mass matrix. This term will imply a see-saw mass relation for the charged sector. We have then a natural mechanism to explain small charged lepton masses in a large unified mass scale.

In this model where CP violation is not taken into consideration and therefore the couplings $f, f', g, g', g''$ and $h'$ are $3 \times 3$ real matrices.

In matrix form, taking $k = k' = 0$, the charged sector reads

$$\mathcal{L}_{M,c} = \bar{\psi} M_c \psi,$$

$$= \left( \bar{\nu}_L, \bar{\nu}_R, \bar{\nu}_L, \bar{\nu}_R \right) \begin{pmatrix} 0 & 0 & f_{\nu L} & 0 \\ 0 & 0 & f_{\nu R} & 0 \\ f_{\nu L} & 0 & g''s_D & 0 \\ f_{\nu R} & 0 & 0 & g''s_D \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R \\ \nu_L \\ \nu_R \end{pmatrix}. \quad (9)$$

On the other hand, for the neutral sector it is convenient to introduce the self-conjugated fields defined, for each family, as

$$\chi_\mu = \nu_L + \nu_L^c, \quad (10)$$

$$w_N = N_R + N_L^c, \quad \chi_N = N_L + N_L^c$$

$$\nu_\mu = \nu_R + \nu_R^c.$$
In terms of the new fields, equations (8) may be rewritten as
\[
\mathcal{L}_{\text{eff}} = \bar{\xi} M_n \xi
\]
\[
\begin{pmatrix}
0 & 0 & f_v v_L & f_v L/2 \\
0 & 0 & f_v R' /2 & f_v R \\
0 & 0 & g_v^{SM} & g_v^{SD} /2 \\
\end{pmatrix}
\begin{pmatrix}
\chi v \\
v_N \\
g_{v}^{SM} \\
g_{v}^{SD} \\
\end{pmatrix}
\]

The mass matrices show the following block structure with different mass scales
\[
M = \begin{pmatrix}
0 & M_{LR} \\
M_{LR}^T & M_S
\end{pmatrix}
\]
(12)

where \(M_{LR}\) and \(M_S\) are \(n \times n\) matrices verifying \(\det(M_{LR}) \ll \det(M_S)\).

In view of the see saw structure \((\det(M_{LR}) \ll \det(M_S))\), mass matrices can be driven to a block diagonal form by expanding in power series of \(M_{LR} M_S^{-1}\) [11]. This results in a \(2n \times 2n\) light fermion mass matrix and \(2n \times 2n\) heavy one given by
\[
M^{(light)} \simeq M_{LR} M_S^{-1} M_{LR}, \quad M^{(heavy)} = M_S
\]
(13)

respectively.

III. CHARGED FERMION Masses

In order to obtain the fermion masses we need explicit textures for the coupling matrices in Eqs. (9) and (11). Mixing between families in the charged sector is phenomenologically disfavored and thus the coupling matrices \(f\) and \(g\) can be chosen diagonal. Taking \(f = \text{diag}(1, 1, 1)\) and \(g = \text{diag}(\lambda_1, \lambda_2, \lambda_3)\) we obtain for each family a light charged fermion, with mass eigenvalue \(m_i = v_L v_R / \lambda_i s_D\), and a heavy one with eigenvalue \(M_i = \lambda_i s_D\).

Flavor left handed and right handed fields \(\psi\) are connected to the physical fields \(\eta\) by means of an orthogonal transformation, that is
\[
\psi_j = \sum_{k=1}^{4n} V_{jk} \eta_k
\]
(14)

where \(\eta\) is the column matrix formed by the mass fields and \(n\) the number of families into consideration.

Explicitly, the mixing matrix \(V\) in the one family case is
\[
V = \begin{pmatrix}
1 & 1 & 0 & 0 \\
1 & -1 & 0 & 0 \\
0 & 0 & 1 & 1 \\
0 & 0 & 1 & -1
\end{pmatrix}
\]
(15)

From this matrix we can recover the Dirac structure of charged leptons by suitable rotations. The generalization to the three family case is easily done.

The three parameters \(\lambda_i\) in \(g\) allow us to recover the standard charged fermion spectrum in a simple way.

For the light fermions we have
\[
m_i = \frac{v_L v_R}{\lambda_i s_D}
\]
(16)

Fixing the vacuum parameter \(v_L\) equal to the Fermi scale \(v_{\text{Fermi}}\), we obtain the following constraints
\[
v_R / \lambda_1 s_D \simeq 10^{-6}, \quad v_R / \lambda_2 s_D \simeq 10^{-3}, \quad v_R / \lambda_3 s_D \simeq 10^{-2}.
\]
(17)

Consequently, for \(v_R \simeq 10^3 - 10^4\) GeV we have the following spectrum for the heavy sector
\[
M_1 = \lambda_1 s_D = 10^5 - 10^{10}\text{ GeV}, \quad M_2 = \lambda_2 s_D = 10^5 - 10^7\text{ GeV}, \quad M_3 = \lambda_3 s_D = 10^5 - 10^6\text{ GeV}.
\]
(18)

Taking the vacuum expectation value of the Higgs scalar \(s_D\) at the mass scale \(10^{10}\) GeV, then the coupling parameters \(\lambda_i\) are fixed to
\[
\lambda_1 = 1, \quad \lambda_2 = 10^{-3}, \quad \lambda_3 = 10^{-4}.
\]
(19)
IV. NEUTRAL FERMION MASSES

The mass lagrangian corresponding to the neutral sector contains Dirac and Majorana mass terms built up from the inclusion of right-handed neutrino fields and their mirror partners. In this framework, the description on the phenomenological neutrino mass matrix will differ from the most familiar schemes on three neutrino mixing found in the literature [12]. In equation (11), the Dirac mass terms arise from the off-diagonal submatrices of the blocks \( M_{LR} \) and \( M_{S} \), while the Majorana terms arise from the diagonal ones. As we mention before, the difference between \( M_{LR} \) and \( M_{S} \) mass scales ensures the see-saw mechanism for the neutral sector. We still have to choose suitable candidates for the textures of the coupling matrices. There are many possibilities that can be compatible with the present experimental status on neutrino masses and oscillations.

For the Dirac mass terms we chose diagonal couplings. The simplest choice is to take \( f \) and \( g' \) equal to the unity.

An important point in the left-right symmetric model comes from the Majorana mass terms. Since Majorana fields are completely neutral and therefore, are all physically equivalent, it is a natural requirement that all Yukawa couplings are to be taken equal. This corresponds to taking democratic textures for the couplings \( f', g' \) and \( g'' \), that is

\[
f' = g' = g'' = \rho \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix},
\]

where \( \rho \) may be set equal to 1 for simplicity. The analytic expression for \( M^{(\text{light})} \) is now

\[
M^{(\text{light})} = \frac{v_L^2}{s_M} \begin{pmatrix} \frac{1}{2} + \frac{1}{2} w^2 & \frac{1}{2} + \frac{1}{2} w^2 & \frac{1}{2} + \frac{1}{2} w^2 \\ \frac{1}{2} + \frac{1}{2} w^2 & \frac{1}{2} + \frac{1}{2} w^2 & \frac{1}{2} + \frac{1}{2} w^2 \\ \frac{1}{2} + \frac{1}{2} w^2 & \frac{1}{2} + \frac{1}{2} w^2 & \frac{1}{2} + \frac{1}{2} w^2 \end{pmatrix}
\]

where \( w \) is defined as \( w \equiv v_L / v_R \).

The interaction fields \( \xi \equiv (\chi_{\nu}, wN)^t \) are related to the physical ones \( \eta \equiv (\nu, N)^t \) by means of the orthogonal transformation \( \xi = U \eta \).

\[
\begin{pmatrix} \chi_{\nu i} \\ w_{Ni} \end{pmatrix} = U \begin{pmatrix} \nu_i \\ N_i \end{pmatrix} \quad i = 1, 2, 3
\]

The \( 2n \times 2n \) orthogonal matrix \( U \) is determined by requiring

\[
U^a M^{(\text{light})} U = \text{diag} (m_1, m_2, \ldots, m_6)
\]

where \( m_k \) are the eigenvalues of \( M^{(\text{light})} \) and correspond to the spectrum of the light sector. Explicitly, we found

\[
U = \begin{pmatrix}
\frac{\sqrt{12}}{4\pi} + O(w^2) & 0 & 0 & \sqrt{\frac{1}{12}} + O(w^2) & -\sqrt{\frac{1}{2}} & 0 \\
\frac{\sqrt{12}}{4\pi} + O(w^2) & 0 & 0 & \sqrt{\frac{1}{12}} + O(w^2) & \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\
\frac{\sqrt{12}}{4\pi} + O(w^2) & 0 & 0 & \sqrt{\frac{1}{12}} + O(w^2) & \frac{1}{\sqrt{3}} & -\frac{\sqrt{2}}{\sqrt{3}} \\
-\sqrt{\frac{2}{3}} & -\sqrt{\frac{1}{3}} & 0 & \sqrt{\frac{2}{3}} + O(w^2) & 0 & 0 \\
-\sqrt{\frac{2}{3}} & -\sqrt{\frac{1}{3}} & 0 & \sqrt{\frac{2}{3}} + O(w^2) & 0 & 0 \\
-\sqrt{\frac{2}{3}} & -\sqrt{\frac{1}{3}} & 0 & \sqrt{\frac{2}{3}} + O(w^2) & 0 & 0
\end{pmatrix}
\]

The neutrino fields are labeled \( \nu \) or \( N \) according to their characteristic mass scales \( v_L^2 / s_M \) or \( v_R^2 / s_M \), respectively. The spectrum of light Majorana neutrino masses is

\[
m_{\nu_1} = \frac{1}{4} \frac{v_L^2}{s_M}, \quad m_{\nu_2} = \frac{1}{4} \frac{v_L^2}{s_M}, \quad m_{\nu_3} \approx \frac{125}{48} \frac{v_L^2}{s_M}, \quad m_{\nu_1} \approx \frac{37}{4} \frac{v_R^2}{s_M}.
\]

\[
m_{N_1} = \frac{1}{4} \frac{v_R^2}{s_M}, \quad m_{N_2} = \frac{1}{4} \frac{v_R^2}{s_M}, \quad m_{N_3} \approx \frac{37}{4} \frac{v_R^2}{s_M}.
\]
It is interesting to notice that the model leads naturally to a hierarchical mass spectrum, with different square mass scales. As we will see in the next section, this feature is essential if the model is to account for the mass pattern coming from neutrino oscillation data.

From Eq. 25 we can also redefine the six Majorana fields in terms of two Dirac and two Majorana neutrino fields

The main theoretical constraints on neutrino masses come from cosmological considerations related to typical bounds on the universe mass density and its lifetime. Specifically, the cosmological bound follows from avoiding the overabundance of relic neutrinos. For neutrinos below $\lesssim 1 \text{ MeV}$ the limit on masses for Majorana type neutrinos is [14]

$$\sum_{\nu} m_{\nu} \lesssim 100\Omega_{\nu}h^2 \text{ eV} \simeq 30 \text{ eV} \quad (26)$$

where $\Omega_{\nu}$ is the neutrino contribution to the cosmological density parameter, $\Omega$, defined as the ratio of the total matter density to the critical energy density of the universe and the factor $h^2$ measures the uncertainty in the determination of the present value Hubble parameter $h$. The factor $\Omega_{\nu}h^2$ is known to be smaller than $1$.

In Eq. (26) the matter component represented by the factor $\Omega_{\nu}h^2$ was chosen smaller than 0.3, according to reference [15], in order to obtain an age of the Universe $t \geq 12$ Gyeans.

From (25), the sum of neutrino masses satisfy

$$\sum_{i} m_{\nu_i} \lesssim 10^{3} \frac{m_{R}^{2}}{s_{M}} \quad (27)$$

so that the cosmological criterium (26) is verified if

$$\frac{m_{R}^{2}}{s_{M}} \lesssim 10\Omega_{\nu}h^2 \text{ eV.} \quad (28)$$

This constrains the breaking scale $s_{M}$ to be $s_{M} \gtrsim 10^{15} \text{ GeV}$ when $v_{R}$ is fixed at $v_{R} \simeq 10^{3} \text{ GeV}.$

V. OSCILLATIONS OF NEUTRINOS

The oscillations in neutrino beams are one of the most fundamental consequences of neutrino mixing. Experimental results concerning a two-generation transition are quoted in terms of $\Delta m^{2} = m_{3}^{2} - m_{1}^{2}$, and the mixing angle. We will see in this section that the model presented previously yields satisfactory results for the democratic texture of the Majorana terms coupling matrices when we fix $\frac{v_{R}^{2}}{s_{M}} \simeq 10^{-2} \text{ eV}$.

Taking into account the orthogonality of the mixing matrix, the probability of transition $\nu_{\alpha} \rightarrow \nu_{\beta}$ between two generations $\alpha$ and $\beta$ is

$$P(\nu_{\alpha} \rightarrow \nu_{\beta}) = |\delta_{\alpha\beta} + \sum_{i=1}^{2} U_{\alpha i}U_{\beta i} \exp(-i\Delta m^{2}_{i1}L/2E - 1)|^2 \quad (29)$$

where $L \simeq l$ is the distance between neutrino source and neutrino detector and $E$ is the neutrino energy.

Notice that as a general feature of the transition probability, neutrino oscillations can be observed whenever the condition $\Delta m^{2}_{11}L/E \sim 1$ is satisfied.

Specifically, considering the model in question supplemented by the democratic texture input, we obtain for the transition $\nu_{e} \rightarrow \nu_{\mu}$

$$P(\nu_{e} \rightarrow \nu_{\mu}) = |U_{e3}U_{\mu 4} \exp(-i\Delta m^{2}_{31} - 1) + U_{e5}U_{\mu 5} \exp(-i\Delta m^{2}_{51} - 1)|^2 \quad (30)$$

where the explicit values of the matrix elements $U_{\alpha i}$ are given in Eq. (24).

In first approximation we neglect $|U_{e3}U_{\mu 4}| = 1/111$ in front of $|U_{e5}U_{\mu 5}| = 1/3$, yielding to the simpler expression

$$P(\nu_{e} \rightarrow \nu_{\mu}) = |U_{e5}U_{\mu 5} \exp(-i\Delta m^{2}_{51} - 1)|^2 \quad (31)$$

$$= \frac{1}{2} |U_{e5}|^2 |U_{\mu 5}|^2 \left(1 - \cos \Delta m^{2}_{51} \frac{L}{2E}\right)$$
and therefore the amplitude of the probability mixing and the relevant scale of mass are

\[ 4|U_{e5}|^2 |U_{\mu 5}|^2 = \frac{4}{9}, \quad \Delta m^2 = \Delta m^2_{51} \simeq \frac{1}{16} \left( \frac{v_R^4}{s_M} \right) \]  

(32)

Recent solar neutrino oscillations results (SNO) strongly favor the large mixing angle Mikheyev-Smirnov-Wolfenstein (MSW) solar solution at the scale

\[ \Delta m^2_{31} \simeq 10^{-5} eV^2 \]  

(33)

Replacing this value in Eq. (32) and choosing \( v_R \approx 10^3 \) GeV, we found that in the left-mirror model the singlet breaking scale should be fixed \( s_M \approx 10^{17} \) GeV in order to recover the solar neutrino experimental results.

We now turn our attention to the \( \nu_\mu \to \nu_\tau \) transition. In this case, Eq. (29) leads to the following result

\[ P(\nu_\mu \to \nu_\tau) \simeq \frac{1}{4} |U_{\mu 5}|^2 |U_{\tau 5}|^2 \left( 1 - \cos \Delta m^2_{51} \frac{L}{2E} \right) + \frac{1}{2} |U_{\mu 6}|^2 |U_{\tau 6}|^2 \left( 1 - \cos \Delta m^2_{16} \frac{L}{2E} \right) + \frac{1}{2} U_{\mu 6}^* U_{\tau 6} \left( 1 - \cos \Delta m^2_{16} \frac{L}{2E} \right) \]  

(34)

Now the oscillations are also characterized by a new scale of masses, namely \( \Delta m^2_{16} \) that didn’t appear in the transitions \( \nu_e \to \nu_\mu, \nu_\tau \). As a rough approximation we consider just the dominant term (\( \sim U_{\mu 6} U_{\tau 6} \)) which implies large mixing at the scale \( \Delta m^2_{16} \), that is

\[ 4|U_{\mu 6} U_{\tau 6}|^2 = 1, \quad \Delta m^2_{16} \simeq \left( \frac{37}{4} \right) \left( \frac{v_R^2}{s_M} \right) \]  

(35)

Using the estimate value for \( s_M \), we found

\[ \Delta m^2_{16} \simeq 10^{-3} eV^2. \]  

(36)

The recent data on atmospheric neutrino by Super-Kamiokande show that the origin of the zenith angle dependence of the neutrino flux is due to oscillations between \( \nu_\mu \) and \( \nu_\tau \). The data is consistent with maximal \( \nu_\mu \) and \( \nu_\tau \) mixing at a square mass difference scale \( \Delta m^2_{atm} \simeq 10^{-3} eV^2 \). Indeed, the preferable values of mass and mixing parameters are

\[ \sin^2 2\theta_{atm} = 1.0, \quad \Delta m^2_{atm} = 3.5 \times 10^{-3} eV^2. \]  

(37)

VI. PHENOMENOLOGY

In order to analyze some phenomenological consequences of the model we’ll work out the interaction Lagrangian. We will see that the standard model results are safely recovered at the Fermi scale and that the connection between the left and right sectors appears at the breaking scale of the new gauge group \( SU(2)_R \) where non negligible effects, involving a new neutral current, are predicted.

As done elsewhere [13], grouping all fermions of a given electric charge and a given helicity \( (h = L, R) \) in a vector column \( \psi_h = (\psi_O, \psi_E)^T \) of \( n \) ordinary \( (O) \) and \( m \) exotic \( (E) \) gauge eigenstates, the interaction Lagrangian for the neutral current is simply written as

\[ \mathcal{L}^{nc} = \sum_h \bar{\psi}_h \gamma^\mu \left( g_L T^3_L g_R T^3_R g \frac{Y}{2} \right) \psi_h \left( W^3_L \right)_L, \]  

(38)

or, in terms of the physical neutral vector bosons \( (Z, Z', A) \)

\[ \mathcal{L}^{nc} = \sum_h \bar{\psi}_h \gamma^\mu R' \left( g_L T^3_L g_R T^3_R g \frac{Y}{2} \right) \psi_h R \left( \begin{array}{c} Z \\ Z' \\ A \end{array} \right). \]  

(39)
$R$ is a $3 \times 3$ matrix representation of the orthogonal transformation which connects the weak $(W_{L\mu}^\ast, W_{R\nu}^\ast, B_{\mu})$ and mass eigenstates basis $(Z_\mu, Z'_\nu, A_\mu)$. In its standard form,

$$
R = \begin{pmatrix}
    c_{\theta_W}c_\alpha & c_{\theta_W}s_\alpha & s_\theta_W \\
    -s_\alpha c_\beta - c_\alpha s_\theta_W s_\beta & c_\beta c_\alpha - s_\alpha s_\theta_W s_\beta & s_\beta c_\theta_W \\
    s_\alpha c_\beta - c_\alpha s_\theta_W c_\beta & -s_\beta c_\alpha - s_\alpha s_\theta_W c_\beta & c_\beta c_\theta_W
\end{pmatrix}
$$

(40)

where $\theta_W$, $\alpha$, and $\beta$ are the mixing angles between the $Z - A$, $Z - Z'$ and $Z' - A$ bosons.

By direct calculation from the neutral bosons mass matrix one can obtain an analytic expression for $R$ in powers of $w = v_L/v_R$

$$
R = \begin{pmatrix}
    \frac{-s_\alpha (g_2^2 g_3^2)^{1/2}}{\Delta^{1/2}} + O(w^4) & \frac{g_3 g_2^2}{\Delta^{1/2} (g_2^2 g_3^2)^{1/2}} w^2 & \frac{g_2^2}{\Delta^{1/2} (g_2^2 g_3^2)^{1/2}} w^2 \\
    \frac{g_3^2}{\Delta^{1/2} (g_2^2 g_3^2)^{1/2}} w^2 & \frac{-s_\alpha (g_2^2 g_3^2)^{1/2}}{\Delta^{1/2}} - \frac{g_3 g_2^2}{\Delta^{1/2} (g_2^2 g_3^2)^{1/2}} w^2 & \frac{g_2^2}{\Delta^{1/2} (g_2^2 g_3^2)^{1/2}} w^2 \\
    \frac{g_2^2}{\Delta^{1/2} (g_2^2 g_3^2)^{1/2}} w^2 & \frac{g_2^2}{\Delta^{1/2} (g_2^2 g_3^2)^{1/2}} w^2 & \frac{g_2^2}{\Delta^{1/2} (g_2^2 g_3^2)^{1/2}} w^2
\end{pmatrix}
$$

(41)

with $\Delta = g_3^2 g_2^2 + g_1^2 g_3^2 + g_1^2 g_2^2$.

In the limit $w = 0$, which corresponds to no mixing between $Z - Z'$ (or $\alpha = 0$), one recovers the standard model case.

The following identities arise by comparing Eqs. (40) and (41),

$$
\sin^2 \theta_W = \frac{g_3^2 g_2^2}{g_2^2 g_1^2 L + g_2^2 g_1^2 R + g_1^2 g_2^2}, \quad \sin^2 \beta = \frac{g_3^2}{g_2^2 + g_1^2}.
$$

(42)

Expressed in terms of the rotation angles, the neutral currents in (38) coupled to the massive vector bosons $Z$ and $Z'$ are respectively,

$$
J_\mu = \frac{g_L}{\cos \theta_W} \gamma_\mu \left[ (1 - w^2 \sin^2 \beta) T_{3L} - w^2 \sin^2 \beta T_{3R} \right]
$$

$$
- \quad Q \sin^2 \theta_W \left( 1 - w^2 \sin^2 \beta \theta_W \right) \left[ 1 + w^2 \frac{\sin^2 \beta \cos^2 \beta}{\sin^2 \theta_W} \right] T_{3L} + \frac{T_{3R}}{\sin^2 \beta}
$$

(43)

$$
J'_\mu = g_L \tan \theta_W \tan \beta \left[ \left( 1 + w^2 \frac{\sin^2 \beta \cos^2 \beta}{\sin^2 \theta_W} \right) T_{3L} + \frac{1}{\sin^2 \beta} T_{3R} \right]
$$

(44)

The corrections to the standard model neutrino NC coming from the extended group symmetry are

$$
\mathcal{L}^{\nu, N} = - J_{\mu, N}^{\nu, N} Z^\mu - J_{\mu, N}^{\nu, N} Z'\mu
$$

(45)

$$
= - \frac{g_L}{2 \cos \theta_W} \left[ (1 - w^2 \sin^2 \beta) \nu_L \gamma_\mu \nu_L - w^2 \sin^2 \beta \tilde{N} \gamma_\mu \tilde{N} \right] Z_{\mu}
$$

$$
- \quad \frac{1}{2} g_L \tan \theta_W \tan \beta \left[ \left( 1 + w^2 \frac{\sin^2 \beta \cos^2 \beta}{\sin^2 \theta_W} \right) \nu_L \gamma_\mu \nu_L + \frac{1}{\sin^2 \beta} \tilde{N} \gamma_\mu \tilde{N} \right] Z'_{\mu}
$$

(46)

or, in terms of the Majorana fields defined in (10),

$$
\mathcal{L}^{\nu, N} = - \frac{g_L}{2 \cos \theta_W} \left[ (1 - w^2 \sin^2 \beta) \nu_L \gamma_\mu \nu_L - w^2 \sin^2 \beta \tilde{N} \gamma_\mu \tilde{N} \right] Z^\mu
$$

$$
- \quad \frac{1}{2} g_L \tan \theta_W \tan \beta \left[ \left( 1 + w^2 \frac{\sin^2 \beta \cos^2 \beta}{\sin^2 \theta_W} \right) \nu_L \gamma_\mu \nu_L + \frac{1}{\sin^2 \beta} \tilde{N} \gamma_\mu \tilde{N} \right] Z'_{\mu}
$$

(47)

$$
\mathcal{L}'^{\nu, N} = - \frac{g_L}{2 \cos \theta_W} \left[ (1 - w^2 \sin^2 \beta) \nu_L \gamma_\mu \nu_L + \frac{1}{2} \nu_L \gamma_\mu \nu_L \right] Z^\mu
$$

$$
- \quad \frac{1}{2} g_L \tan \theta_W \tan \beta \left[ \left( 1 + w^2 \frac{\sin^2 \beta \cos^2 \beta}{\sin^2 \theta_W} \right) \nu_L \gamma_\mu \nu_L + \frac{1}{2} \nu_L \gamma_\mu \nu_L \right] Z'_{\mu}
$$

(48)
In order to express the neutral currents in terms of mass eigenstates one has to use the transformation relation (22) into the interaction lagrangian (45). This yields

\begin{equation}
\mathcal{L} = -J^{\nu,N}_{\mu} Z^\mu - J^{\mu,N}_{\nu} Z^\nu
\end{equation}

\begin{align*}
&= -\frac{g_L}{2 \cos \theta_W} \left( 1 - w^2 \sin^2 \beta \right) \sum_{i=1}^{2n} \sum_{j,k} \left( \begin{array}{c}
U_{ij} U_{ik} \eta_j \gamma_\mu \frac{1 - \gamma_5}{2} \\
\end{array} \right) \eta_k + \\
&- \frac{1}{2} g_L \tan \theta_W \tan \beta \left( 1 + w^2 \sin^2 \beta \cos^2 \beta \right) \sum_{i=1}^{2n} \sum_{j,k} \left( \begin{array}{c}
U_{ij} U_{ik} \eta_j \gamma_\mu \frac{1 - \gamma_5}{2} \\
\end{array} \right) \eta_k + \\
&+ \frac{1}{\sin^2 \beta} \sum_{i=4}^{2n} \sum_{j,k} \left( \begin{array}{c}
U_{ij} U_{ik} \eta_j \gamma_\mu \frac{1 + \gamma_5}{2} \\
\end{array} \right) \eta_k \end{align*}

As a consequence of the neutral gauge boson mixing in (40), mirror neutrinos couple to the \( Z \) boson and \( \tilde{Z} \) may contribute to \( Z \) decay \( \Gamma_Z \). Correspondence with the experimental results may be achieved by constraining the angle \( \alpha \), or equivalently, the factor \( w^2 \), which parametrize the \( Z - \tilde{Z} \) mixing. This is indeed the case for \( \epsilon_R > 30 \epsilon_L \) [2]. It should be noticed that the non standard \( E_L - Z \) coupling contain a term that is not suppressed by a \( w^2 \) factor, namely,

\begin{equation}
\mathcal{L}_{E_L} = -g_L \tan \theta_W \tan \beta \eta E_L \eta \end{equation}

However, this contribution is excluded at energies lying in the electroweak scale due to the large charged fermion masses in the heavy sector (see Sec 3). Therefore, the standard model results are recovered in the limit \( w^2 < < 1 \).

The neutral current coupled to the massive vector boson \( Z' \) contains non suppressed couplings which involves either standard or exotic neutrinos and are important to test the model at the \( SU(2)_R \) breaking scale. These contributions are

\begin{equation}
\mathcal{L}_{Z'} = -\frac{1}{2} g_L \tan \theta_W \tan \beta \left( 1 + w^2 \sin^2 \beta \cos^2 \beta \right) \sum_{i=1}^{2n} \sum_{j,k} \left( \begin{array}{c}
U_{ij} U_{ik} \eta_j \gamma_\mu \frac{1 - \gamma_5}{2} \\
\end{array} \right) \eta_k + \\
+ \frac{1}{\sin^2 \beta} \sum_{i=4}^{2n} \sum_{j,k} \left( \begin{array}{c}
U_{ij} U_{ik} \eta_j \gamma_\mu \frac{1 + \gamma_5}{2} \\
\end{array} \right) \eta_k \end{equation}

The new \( Z' \) gauge boson can be produced at the Large Hadron Collider with masses in the 1-4 TeV region [2]. The implications of a new \( Z' \) to the high precision electroweak data was studied by Erler and Langacker [10].

VII. CONCLUSION

The recent experimental reports on neutrino oscillations, suggesting non zero masses for neutrinos, are certainly the strongest indication for physics beyond the standard model. Enlarging the fermion spectrum by introducing mirror matter is a simple way to implement non zero neutrino masses in extended theories. In the present paper we saw that a consistent spectrum of neutrino masses and oscillation pattern can arise in such a scenario, which is motivated by an underlying left-right symmetric structure in the gauge group. We show that the well known \( SU(2)_L \times SU(2)_R \times U(1)_Y \) theory, spontaneously broken into the standard \( SU(2)_L \times U(1) \) at the mass scale \( v_F \lesssim 10^9 \) GeV, and supplemented by two Higgs singlets with vacuum parameters at the scales \( s_D \sim 10^{18} \) GeV and \( s_M \sim 10^{17} \) GeV reproduce the observed charged and neutral fermion masses.

The new physics predicted by the model is consistent with the theoretical arguments and experimental results available on neutrino physics. The connection between the known leptons and their mirror states can be experimentally tested by a new neutral gauge boson present at the 7 TeV mass scale. New Majorana neutrinos, such as those considered here, may be experimentally tested at the large hadron collider at CERN [17].
Acknowledgments

This work was partially supported by the Centro Latino Americano de Fisica (CLAF) and the following Brazilian agencies: CNPq, FUJB, FAPERJ and FINEP.