Quark-Hadron Duality in Photoabsorption Sum Rules and Two Photon Decays of Meson Resonances

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Abstract
The idea of quark-hadron duality is developed and applied to integral sum rules for the photoexcitation of meson resonances. Some applications of the presented approach in the light and heavy quark sectors are made, and the role of the scalar diquark cluster degrees of freedom in the radiative formation of light scalar mesons is discussed.

1 Introduction
In the present report, we continue consideration of aspects of the quark hadron duality, i.e. the equivalence of two complete sets of state vectors, saturating certain integral sum rules, one of the sets being the solution of the bound state problem with colour-confining interaction, while the other describes free partons. Further exploration of relations between exclusive and inclusive production of hadrons in processes induced by virtual and real photons is worthwhile as means of more extensive tests of approximations accompanying the description of the transition between hadron and quark-gluon degrees of freedom in QCD. The choice of sum rules satisfying the assumed duality condition is suggested by correspondence with the well-known results in the nonrelativistic theory of interaction of the radiation with matter. The message is that sum rules connected with the dipole moment fluctuation seem to be singled in both nonrelativistic [1] and relativistic regions [2, 3, 4]. Following this idea, which has first been tested in the models of quantum field theory, we present, within the relativistic constituent quark model approach, the relations between two-photon decay widths of the lowest meson resonances following from the derived sum rules for polarized gamma-gamma cross sections.

2 Quark-hadron duality for the bremsstrahlung -weighted sum rules. The case of meson resonance photoexcitation

We first remind that applying the adopted duality principle of two complete sets of final state vectors (one - with the confined $q\bar{q}$-states representing the sum over all resonances, the other- with the ”gedanken” free $q\bar{q}$ -pair) to the Cabibbo-Radicati sum rule [5] for the pion, we get [4]

$$<r^2>_{\pi^\pm} = \frac{3}{4\pi^2\alpha} \int_0^\infty \frac{d\nu}{\nu} (\sigma(\gamma^-\pi^\pm) - \sigma(\gamma^+\pi^\pm)) = \frac{3}{4\pi^2F^2_\pi}$$  (1)
Further useful relations were obtained [4] from the assumed approximate equality of the derivatives
\[ F'_{\gamma^*\pi^0} (q^2) \big|_{q^2 \to 0} \simeq F'_{\gamma^*\pi^\pm\pi^\mp} (q^2) \big|_{q^2 \to 0} \]
of form factors normalized to unity at zero momentum transfers
\[ m_q \simeq \sqrt{\frac{2}{3}} \pi F_{\pi} (q = u, d) \]  \hspace{1cm} (2)
\[ \frac{g_{\pi^0qq}}{4\pi} \simeq \frac{\pi}{6} \]  \hspace{1cm} (3)
where the numerical value of the pseudoscalar \( \pi qq \)-coupling constant follows from an analogue of the Goldberger-Treiman relation for quarks.

The successful local approximation for the \( \pi q \bar{q} \) -vertex in the calculation of \( \langle r^2 \rangle_\pi \) means that it should be approximately valid in computation of the radiative decays \( M^* \to \gamma \pi \) of the lowest meson resonances contributing to the sum rule (1). Therefore, a similar mechanism of the local annihilation of constituent quarks in the reactions \( q \bar{q} \to e^+e^- \) and \( q \bar{q} \to \gamma \pi \) enables one to obtain, e.g., for the \( \omega \)-meson [4]
\[ \frac{\Gamma(\omega \to e^+e^-)}{\Gamma(\omega \to \pi^0\gamma)} \simeq \frac{\sigma((q\bar{q})_\omega \to e^+e^-)}{\sigma((q\bar{q})_\omega \to \pi^0\gamma)} \big|_{s=m_\omega^2} \simeq \frac{18}{18} \left( \frac{g_{\pi^0qq}^2}{4\pi} \right)^{-1} = \frac{\alpha}{3\pi} \]  \hspace{1cm} (4)
where \( (q\bar{q})_\omega = (1/\sqrt{2})(u\bar{u} + d\bar{d}) \) and numerically the ratio of the widths turns out to be within the experimental uncertainties of the data. The seemingly paradoxical relevance of the local approximation combined with the application of the Goldberger-Treiman relation to the pion-quark vertex as compared to an appearing more general ”softened” description in which this vertex is described by a Bethe-Salpeter type equation, may be interpreted as an exemplification, in the considered context, of the twofold picture of the pion. As is well-known, the pion presents itself in twofold ways: on the one hand, its anomalously small mass suggests that it should be identified with the pseudo-Goldstone mode of dynamical breaking of chiral symmetry in QCD; on the other, it should be described equally well from the QCD Lagrangian in terms of current quarks, or in terms of the QCD motivated relativistic models of constituent quarks with strong attractive forces acting in the \( J^P = 0^- \) channel. The complementarity of these pictures may also reflect a kind of duality between the effective hadronic description based on symmetries, and the microscopic description in terms of partons. This duality can be alternatively exploited in calculations of hadron properties. While the high momentum transfer reactions reveal characteristics of the quark and gluon distributions inside the pion, at low energies its role as a (pseudo)Goldstone boson mode became essential in describing the long-range structure and strength of its interaction with hadrons and effective hadronic constituent degrees of freedom, e.g., constituent quarks. However, the inclusion of hadronic corrections to \( \langle r^2 \rangle_\pi \) in the form of the one-pion-exchange graph contribution to the Cabibbo-Radicati sum rule, which provides the chiral corrections of the order \( \sim \log(m_\pi) \) [3, 4] in accordance with the theorem of Beg and Zepeda [6], definitely improves the agreement with data and one should have in mind this type of correction in other applications of the approach outlined.

We turn now to sum rules for meson resonances in photon-photon collisions. Varying the polarizations of colliding photons, one can show that the linear combination of certain \( \gamma\gamma \to q\bar{q} \) cross-sections will dominantly collect, at low and medium energies most
important for saturation of the integral sum rules considered, the $q\bar{q}$ states with definite
spin-parity and hence, by the adopted quark-hadron duality, the meson resonances with
the same quantum numbers. The polarization structure of the transition matrix element
$M(J^{PC} \leftrightarrow 2\gamma)$ for the meson resonance with the spin-parity $J^{PC} = 0^+, 0^{++}, 2^{++}$ is taken as follows:

$$M(0^{++} \leftrightarrow 2\gamma) = G_S[(\epsilon_1\epsilon_2)(k_1k_2) - (\epsilon_1k_1k_2)],$$  \hfill (5)
$$M(2^{++} \leftrightarrow 2\gamma) = G_{T0}[(\epsilon_1\epsilon_2)(k_1k_2) - (\epsilon_1k_1k_2)]k_1^\mu k_2^\nu \epsilon^{\mu\nu} + G_{T2}[(\epsilon_1k_1k_2)]k_1^\mu k_2^\nu \epsilon^{\mu\nu} - G_{T2}[(\epsilon_1k_1k_2)]k_1^\nu k_2^\mu \epsilon^{\mu\nu},$$  \hfill (6)
$$M(0^{-+} \leftrightarrow 2\gamma) = G_{PS}\epsilon_{\mu\nu}\lambda\sigma k_1^\mu k_2^\nu \epsilon_1^\lambda \epsilon_2^\sigma,$$  \hfill (7)
$$M(2^{-+} \leftrightarrow 2\gamma) = G_{PT}\epsilon_{\mu\nu}\lambda\sigma k_1^\mu k_2^\nu \epsilon_1^\lambda \epsilon_2^\sigma k_1^\alpha k_2^\beta \epsilon^{\alpha\beta},$$  \hfill (8)

where $k_i^\mu$ are the momenta of photons, $\epsilon_i^\nu (\epsilon^{\alpha\beta})$ - polarization vector (tensor) of the photon
(the tensor meson), $G$ - corresponding coupling constants, $G_{T\lambda}$ being the tensor meson
coupling constants with the $z$-projection of the total angular momentum $\lambda = 0$ or $2$,
respectively.

It follows then that the combinations of the integrals over the bremsstrahlung-weighted
and polarized $\gamma\gamma \rightarrow q\bar{q}$ cross-sections, $I_\perp - (1/2)I_p$, $I_\parallel - (1/2)I_p$. $I_p$ will be related
to low-mass meson resonances having spatial quantum numbers $J^{PC} = 0^-$ and $2^+$,
$0^{++}$ and $2^{++}(\lambda = 0)$, $2^{++}(\lambda = 2)$, if we confine ourselves to the mesons with spins $J \leq 2$
for further discussion.

The $\gamma\gamma$ - cross-sections $\sigma_{\perp(\parallel)}$ (and the integrals thereof) refer to plane-polarized
photons with the perpendicular (parallel) polarizations, and $\sigma_p$ corresponds to circularly
polarized photons with parallel spins.

Evaluating cross-sections and elementary integrals we get the sum rules for radiative
widths of resonances with different values of $J^{PC}$

$$\sum_i \frac{\Gamma(PS_i \rightarrow 2\gamma)}{m_{PS_i}^3} + 5 \sum_j \frac{\Gamma(PT_j \rightarrow 2\gamma)}{m_{PT_j}^3} \simeq \frac{3}{16\pi^2} \sum_q \langle Q(q) \rangle^2 \frac{2\pi\alpha^2}{m_q^2}$$  \hfill (9)

$$\sum_i \frac{\Gamma(S_i \rightarrow 2\gamma)}{m_{S_i}^3} + 5 \sum_j \frac{\Gamma(T0_j \rightarrow 2\gamma)}{m_{T0_j}^3} \simeq$$

$$\simeq \frac{3}{16\pi^2} \left[ \sum_q \langle Q(q) \rangle^2 \frac{5\pi\alpha^2}{9m_q^2} + \sum_q \langle Q(q\bar{q}) \rangle^2 \frac{2\pi\alpha^2}{9m_{qq}^2} \right]$$  \hfill (10)

$$5 \sum_i \frac{\Gamma((T2)_i \rightarrow 2\gamma)}{m_{T2_i}^3} + (2J + 1) \sum_{R(J \geq 2, \lambda = 2)} \frac{\Gamma(R \rightarrow 2\gamma)}{m_R^3} \simeq$$

$$\simeq \frac{3}{16\pi^2} \sum_q \langle Q(q) \rangle^2 \frac{14\pi\alpha^2}{9m_q^2}$$  \hfill (11)

All the integrals over the resonance cross sections are taken in the narrow width approxima-
tion and, further, the contributions of the states with $J \geq 3$ will be neglected. For
generality of the consideration, we included the last term in (10) that corresponds to a
possible role of scalar diquarks as constituent partons composing, at least in part, the scalar meson nonets (for discussion of this acute problem in hadron spectroscopy see, e.g., minireview in [7] and references therein). Assuming no mixing between light and heavy quark sectors, one can read every sum rule separately for mesons constructed of the u-, d-, s-, and c-quarks. Moreover, one can split sum rules into the isovector and isoscalar resonance parts in the light quark sector, which enables one to carry a more detailed comparison with experimental data. For numerical estimation of radiative widths we use the mass values \( m \) as expressed via the strangeness-changing GT-relation, \( m_s \approx 350 \) MeV, and for the diquark masses we take the values following from the extended chiral model including scalar diquarks into the low-energy effective Lagrangian approach [8]: \( m_{qq} \approx 310 \div 330 \) MeV, \( m_{q_s} \approx 545 \div 570 \) MeV, where \( q = u, d \).

In the charm sector, from our sum rules we have obtained \( \Gamma_{\gamma\gamma}(\eta_c) \approx 7.4 (6.9\pm 1.7\pm 0.8) \), \( \Gamma_{\gamma\gamma}(\chi_{c0}) \leq 6.1 (3.76\pm 0.65\pm 1.81) \), \( \Gamma_{\gamma\gamma}(\chi_{c2}) \leq 3.9 [(0.53\pm 0.15\pm 0.23)] \div [(1.76\pm 0.47\pm 0.37)] \) where the most recent results on the two-photon widths of resonances in the units \( \text{keV} \) from [9] are given in parentheses and a smaller (larger) value for the width \( \Gamma_{\gamma\gamma}(\chi_{c2}) \) refers to the registered \( \chi_{c2} \) - decay final states \( 2\pi^+ 2\pi^- (l^+ l^- \gamma) \). The closeness of \( \Gamma_{\gamma\gamma}^{\text{exp}}(\eta_c) \) to the value calculated from sum rules points to relative smallness of the higher resonance part of the spectrum, e.g., the radial excitations of pseudoscalar charmonia compared to the ground state contribution. The situation with the scalar charmonium seems to signal about a more important role of higher radial excitations. The cross section of the transition \( \gamma\gamma \rightarrow q\bar{q}(J^{PC} = 2^{++}) \) is known [10] to fall much slower with rising photon energies as compared to the transitions to the states with \( J^{PC} = 0^{++} \), and this is reflected by rather a large contribution from the higher resonances with larger masses and spins. The situation with the light tensor mesons is largely the same, and we shall instead focus on the pseudoscalar and scalar meson sum rules in the light quark sector.

Remarkably enough, the small upper bound of the radiative width of the pseudotensor \( \pi_2(1670) \)-resonance, reported recently by the L3 and ARGUS Collaborations [11, 12] together with the known two-photon widths of \( \pi^0 \), \( \eta \), and \( \eta' \)-mesons provide the fulfillment of the pseudoscalar sum rule within experimental uncertainties.

Concerning the nature of the low-lying scalar mesons, there is still no general agreement on where are the \( q\bar{q} \) states, whether there is a glueball among the light scalars, and whether some of the many scalars are multiquark, or meson-meson bound states. One of the most economic viewpoints would be to try to identify low-mass scalars with the ground and radial excited \( q\bar{q} \) states and the glueball mixed with quarkonia-type states [13, 14]. Another popular scheme consists in the hypothesis that above 1 GeV the scalar states form a conventional \( q\bar{q} \) nonet mixed with the glueball, suggested by lattice QCD, while below 1 GeV the states also form a nonet, but of a more complicated nature [15, 16, 17]. Namely, as implied by the attractive forces of QCD, the diquark configurations of the type \( (qq)_3(q\bar{q})_3 \) in S-wave, possibly, with some \( q\bar{q} \) admixture in P-wave, can form a number of broad and low-lying resonances, which can also be identified with observed states. We apply the sum rule (12) for light scalar mesons to test both the afore-mentioned pictures of two nonets of scalar mesons. For the isovector mesons \( a_{S_1} \equiv a_0(980) \) and \( a_{S_2} \equiv a_0(1450) \) we have

\[
\sum_{i=1,2} \frac{\Gamma_{\gamma\gamma}(S_i)}{m(S_i)^3} \simeq \frac{a^2}{96\pi} \left( \frac{5}{9m_{q_s}^2} + \frac{2}{9m_{qs}^2} \right)
\]
We take $\Gamma_{\gamma\gamma}(a_0(980)) \simeq 0.3$ keV \cite{1}, and $m_{q_s} = 246(980)$ MeV for a further numerical calculation. Let us consider first the “ground + radial-excited” $q\bar{q}$-option for $a_0(980)$ and $a_0(1450)$-resonances. It follows then from the sum rule (12) with the second term (i.e., the diquark term, containing $m_{q_s}$) omitted in the right-hand side:

$$\frac{\Gamma_{\gamma\gamma}(a_0(1450))}{\Gamma_{\gamma\gamma}(S_1)} \approx 2.9 \text{ keV}$$

$$\frac{\Gamma_{\gamma\gamma}(S_1)}{m(S_1)^3} : \frac{\Gamma_{\gamma\gamma}(S_2)}{m(S_2)^3} \simeq 1 : 4.34 \quad (13)$$

While the absolute value of $\Gamma_{\gamma\gamma}(a_0(1450))$ looks reasonable, the ratio of the “reduced couplings” squared (the second line in Eq.(13)) is not, in our opinion. In the potential nonrelativistic models, this ratio can essentially be interpreted as a ratio of derivatives of the meson radial wave function at the “zero” interquark distance: $|R_{S_1}^\prime(0)|^2 : |R_{S_2}^\prime(0)|^2$, and the situation of the ground-to-(1st)radial-state ratio such as given by Eq.(13) looks disfavouring (for example, in heavy quark systems obeying the Schrödinger equation with the QCD-motivated potentials $|R_{1P}^\prime(0)|^2 : |R_{2P}^\prime(0)|^2 \simeq 1 : 1.2$ for the nP-wave states \cite{18}).

Following another option, we identify $a_0(980)$ and $a_0(1450)$ as generally mixed configurations $(1/\sqrt{2})(u\bar{u} - d\bar{d})$ in P-wave and $(1/\sqrt{2})((us)(\bar{u}s) - (ds)(\bar{d}s))$ in S-wave. First, we note that if one takes $a_0(980)$ as the pure $2q\bar{2}q$ and $a_0(1450)$ as the pure $q\bar{q}$-configuration, then one can obtain from the sum rule (12) $\Gamma_{\gamma\gamma}(a_0(980)) \leq 0.12$ keV and $\Gamma_{\gamma\gamma}(a_0(1450)) \leq 5.2$ keV. The two-photon decay width of $a_0(980)$ turns out essentially lower than accepted value 0.3 keV in that case. Therefore, it appears reasonable to correct the situation with the help of the mixing of two parton configurations. Tentatively, we accept the simplest case of the orthogonal mixing of two isovector $|q\bar{q}\rangle$ and $|(qs)(\bar{q}s)\rangle$ basis states, assuming also $\Gamma_{\gamma\gamma}(a_0(980)) = 0.3$ keV. Irrespectively of the mixing, the value $\Gamma_{\gamma\gamma}(a_0(1450)) \simeq 3.5$ keV follows just from the sum rule. Solving the following equation:

$$\frac{\Gamma_{\gamma\gamma}(S_1)}{m(S_1)^3} : \frac{\Gamma_{\gamma\gamma}(S_2)}{m(S_2)^3} = \left[ \frac{\sqrt{2}}{3m_{q_s}} \cos \theta + \frac{\sqrt{5}}{3m_q} \sin \theta \right]^2 : \left[ -\frac{\sqrt{2}}{3m_{q_s}} \sin \theta + \frac{\sqrt{5}}{3m_q} \cos \theta \right]^2, \quad (14)$$

for the mixing angle $\theta$, one obtains two solutions: $\theta \simeq 12^\circ$ and $\theta \simeq 137^\circ$, where the first (second) corresponds to the positive (negative) relative sign of the amplitudes $A(a_0(980) \rightarrow 2\gamma)$ and $A(a_0(1450) \rightarrow 2\gamma)$. The quark-flavour structure of physical resonance states is then

$$a_0(980) = 0.691((us)(\bar{u}s) - (ds)(\bar{d}s)) + 0.147(u\bar{u} - d\bar{d})$$

$$a_0(1450) = -0.147((us)(\bar{u}s) - (ds)(\bar{d}s)) + 0.691(u\bar{u} - d\bar{d}), \quad (15)$$

for the positive sign, and

$$a_0(980) \approx \frac{1}{2}(-(us)(\bar{u}s) + (ds)(\bar{d}s)) + u\bar{u} - d\bar{d})$$

$$a_0(1450) \approx -\frac{1}{2}((us)(\bar{u}s) - (ds)(\bar{d}s) + u\bar{u} - d\bar{d}), \quad (16)$$

for the negative one.
A smaller value of the mixing angle \( \theta \) seems to conform better to the analysis of the strong decays of light scalars\[16\]. The predicted large radiative width \( \Gamma_{\gamma\gamma}(a_0(1450)) \) may present a potential difficulty for the explanation of the apparent absence of a resonance effect due to the excitation of \( a_0(1450) \) in the mass spectrum of the \( K_S^0K_S^0 \)-pairs produced in the \( \gamma\gamma \)-reaction \[19\]. However, the detailed analysis of this situation requires simultaneous consideration of excitation of both isovector and closely lying isoscalar scalar resonances, which are expected (by analogy with the known effect of the tensor \( a_2(1320) \) - and \( f_2(1270) \) - resonance interference) to interfere destructively in the \( K^0\bar{K}^0 \) decay channel. Unfortunately, it is presently impossible to analyse, via analogous sum rules, the very interesting and important isoscalar sector of light scalar mesons due to the lack of needed data on radiative widths.

3 Concluding remarks

We believe that the sum rules for resonance \( \gamma\gamma \) interaction give, on the average, a good evidence for the relevance of quark- hadron duality in the considered context.

1. The application of the developed approach to derive and check the \( \gamma\gamma \)-sum rules leads to especially good results for pseudoscalar mesons in both light and charm quark sector. A rapid decrease in the corresponding polarized \( \gamma\gamma \)-cross section in the integral sum rule explains the sum rule saturation by the ground states of pseudoscalar mesons.

2. As it follows from sum rules to be saturated by the light scalar mesons, the \( \gamma\gamma \)-resonance couplings can give important information on the flavour and parton content of these states. One can hope that such information on the scalar mesons will be an essential part of interpreting these states. Further, the \( Q^2 \) dependence of the reaction \( \gamma^*\gamma \rightarrow f_0/a_0 \) on the photon virtuality could probe the spatial dependence of the wave function of these states.

3. The measurements of radiative decays of higher spin meson resonances would be very desirable and interesting in view of a demonstrated stronger dependence of the sum rule, including the higher spin, \( J \geq 2 \), on higher mass intermediate states and photon energies.

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References


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