Gravitational diffusion in the intra-cluster medium

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ABSTRACT

We revisit the process of gravitational sedimentation of helium and heavy elements in the intra-cluster medium. We find that helium applies an inward drag force on heavy elements, boosting their sedimentation speed to nearly half its own. This speed is almost independent of the mass and the electric charge of heavy elements. In the absence of small-scale magnetic fields, helium sedimentation can increase the He/H abundance ratio in the cores of hot clusters by three orders of magnitude. It also steepens the baryonic density profile yielding a higher X-ray luminosity, which offers an explanation of the observed luminosity-temperature relation.

If the primordial He/H ratio is assumed, then the gas density inferred from the observed X-ray emissivity might be underestimated by 30% in the cores of clusters and overestimated by 7% in the outer regions. The dark matter density on the other hand might be overestimated by a factor of 8/3 in the cores and underestimated by 18% in outer regions.

Key words: cosmology: theory – dark matter – baryons – galaxies: clusters: general – X-rays: galaxies

1 INTRODUCTION

In the equilibrium state of a multi-component plasma the number density, \( n_i \), of particles of mass \( m_i \), is \( n_i \propto e^{-m_i\phi(r)/kT} \), where \( \phi(r) \) is the gravitational potential. In the high temperature intra-cluster medium (ICM), light elements, helium (He) and hydrogen (H), can diffuse fast enough to reach their equilibrium distribution in a Hubble time. Larger frictional drag forces work on heavier elements, but still we expect, at least partial segregation as a result of diffusion.

Diffusion can have important consequences. Fabian & Pringle (1977) suggested diffusion as a possible explanation for the observed gradients in the iron abundance inside galaxy clusters. In their calculations iron ions sediment by a distance comparable to the radius of the cluster within a Hubble time. Larger frictional drag forces work on heavier elements, but still we expect, at least partial segregation as a result of diffusion.

In this paper we revisit the calculation of element sedimentation in the ICM. We confirm the claim by Rephaeli that helium drag on iron is comparable to that of protons. However, instead of hindering the sedimentation of iron and other heavy elements we show that helium acts as a catalyst. Our analysis includes electric fields that are inevitably generated by segregation of charged elements. Although these fields reduce the diffusion rate, the sedimentation time-scales of heavy elements are still several times shorter than previous estimates.

The paper is organized as follows. In §2 we present the basic equations and estimate the sedimentation speeds and time-scales. In §3 we discuss the equilibrium distribution as a limiting case of element sedimentation. We conclude in §4.

2 THE EQUATIONS OF ELEMENT DIFFUSION

We write the equations governing the evolution of individual species in an ICM of any composition. We do not con-
sider magnetic fields in this paper, but include electric fields which must exist in any ionized plasma in a gravitational field (Eddington 1926). Let the ICM be made of any number of species each made of particles characterized by mass \( m_i \) and electric charge \( q_i = eZ_i \). We denote the local number density and velocity of a patch of matter of each species by \( n_i \) and \( V_i \). Then, the mass density is \( \rho = \sum n_i m_i \) and the pressure is \( P_i = n_i k T_i \), where we have assumed that the ICM is in local thermal equilibrium so all species share the same temperature \( T \). With this notation the equations are

\[
\sum V_i \cdot \nabla V_i = - \frac{\nabla P_i}{\rho} + \frac{q_i e}{m_i} \mathbf{g} + \sum_j (V_i - V_j) / \tau_{ij},
\]

where \( \mathbf{E} \) is the electric field, and \( \mathbf{g} \) is the gravitational field (force per unit mass). The constants \( \tau_{ij} \) are the time-scales for the drag forces acting on the species \( i \) as a result of Coulomb interactions with species \( j \). Momentum conservation implies that \( \tau_{ij} = (\rho_j / \rho_i) \tau_{ji} \). If \( m_i \gg m_j \) then \( \tau_{ij} \) can be approximated by (Spitzer 1968)

\[
\tau_{ij} = \frac{3(2n_i)^{7/2}}{16\pi} \frac{n_j}{Z_i Z_j} \frac{m_i}{m_j} \left( \frac{T_i}{10^4 K} \right)^{3/2} \left( \frac{\ln \Lambda}{\ln n_j} \right)^{-3/2},
\]

where \( \ln \Lambda \) is the Coulomb logarithm. The equations (1) must be supplied by additional relations that specify the electric field. To create an electric force comparable with gravitational and pressure forces a tiny fractional charge excess \( (Gm_e^2 / e^2 \sim 10^{-36}) \) is sufficient. The additional relations can then be obtained by assuming charge neutrality and zero electric currents, i.e.,

\[
\sum_i n_i q_i = 0, \quad \text{and} \quad \sum_i n_i q_i V_i = 0.
\]

### 2.1 Sedimentation speeds and time-scales

Multiplying the equations (1) by \( n_i m_i / \sum n_i m_i \) and summing over all species yields

\[
\frac{\nabla P}{\rho} = \tilde{\mathbf{g}}, \tag{4}
\]

where \( \rho = \sum n_i m_i \) is the total local density of all species, \( \mathbf{V} = \sum \rho_i V_i / \rho \) and \( P = \sum P_i \) are, respectively, the mass-weighted mean velocity and total pressure, and \( \tilde{\mathbf{g}} = \mathbf{g} - \frac{\partial \mathbf{V}}{\partial t} - \mathbf{V} \cdot \nabla \mathbf{V} \) is the gravitational field in the frame of reference moving with the velocity \( \mathbf{V} \). The terms involving the electric and drag forces have disappeared by virtue of charge neutrality and momentum conservation, respectively. Since \( |V_i - V| \) is much smaller than the thermal velocities, the velocity dispersion term \( \sum \rho_i (V_i - V) \nabla (V_i - V) / \rho \) is negligible compared to the pressure and gravity terms we have not included it in the equation (4).

At the initial stage of the diffusion process the dominant light species have the same distribution, which gives \( \nabla (n_i k T_i) / (n_i m_i) = (\mu_{m_i} / m_i) \tilde{\mathbf{g}} \), where \( \mu = (1/m_\mu) \sum n_i m_i / \sum n_i \) is the mean molecular weight. Thus equations (1) yield

\[
(\mu_{m_p} / m_p - 1) \tilde{\mathbf{g}} + \frac{q_i e}{m_i} + \sum_j (V_i - V_j) / \tau_{ij} = 0. \tag{5}
\]

Using equations (3) and (5) we find that the electric field is \( \mathbf{E} = \mu_{m_p} \tilde{\mathbf{g}} / e \). From (2) \( \tau_{ij} \propto n_j^{-1} m_j^{-1/2} \), so because of the low abundance of heavy ions and the low mass of electrons, only the fast species dominate all others in (6). Thus we are left with

\[
(1 + Z_i / A) (\mu - 1) \tilde{\mathbf{g}} + \frac{V_p - V_i}{\tau_p} + \frac{V_e - V_i}{\tau_e} = 0. \tag{6}
\]

We can now estimate the velocity of each species relative to \( V_p \), the velocity of the proton fluid at each point. The advantage of using velocities relative to protons is the independence of the results from physical processes other than diffusion (radiative cooling, heating by supernovae, etc.). Though these processes can have important dynamical effects on the cluster, they do not affect the expressions we develop for the relative velocities and element abundance.

From (6) the helium velocity is

\[
V_e - V_p = (3\mu/4 - 1) \tilde{\mathbf{g}} (\mu_{m_p} / m_p) \approx -0.56 \tilde{\mathbf{g}}. \tag{7}
\]

The relative sedimentation speed of heavy elements, \( V_a - V_p \), can easily be related to that of helium \( V_e - V_p \). Heavy elements experience an upward proton drag and inward helium drag forces. According to (2), \( \tau_{ij} \propto Z_i^{-2} \) and for large \( Z_i \) these drag terms dominate all others in (6). Thus we are left with

\[
(V_e - V_i)/\tau_e \approx (V_e - V_a)/\tau_a. \tag{8}
\]

Substituting \( \tau_a \) and \( \tau_p \) from (2) in the last equation yields

\[
\frac{V_e - V_i}{V_e - V_p} \approx \left( \frac{1}{1 + n_{a_p} / n_{a_p}} \right)^{-1} = 0.4, \tag{9}
\]

where we have taken \( n_{a_p} / n_p = 0.08 \). Thus, contrary to the prediction of Fabian & Pringle (1977), who by neglecting helium drag obtained \( V_e - V_i \propto A Z_i^{-3} \), the diffusion speeds are approximately the same for all heavy elements. Including all forces in the calculation increases slightly this result (by a factor of \( \sim 1 + 4/Z_i \))

\[
\frac{V_e - V_i}{V_e - V_p} \approx 0.4 \left[ 1 + \frac{A}{2Z_i^2} - 1.8 \left( \frac{1}{Z_i} + \frac{1}{Z_i^2} \right) \right]. \tag{10}
\]

There is also a correction of a similar magnitude if the initial distribution of heavy elements is different from that of light elements.

The relation (10) is independent of the physical state of the ICM. To obtain the relative velocities we have to know the temperature, the density and the gravitational acceleration. Taking in (2) \( \ln \Lambda \approx 40 \) as the typical value for the ICM, we find the helium velocity relative to protons to be

\[
V_e - V_p = 5 \times 10^{4} \tilde{\mathbf{g}} \times 10^{-9.5} \rho_p^{-1} T_s^{-1} m^{-1}, \tag{11}
\]

where \( \tilde{\mathbf{g}} \times 10^{-9.5} \rho_p^{-1} T_s^{-1} \) and \( T_s = T / 10^8 \) K. This is lower by \( \sim 40\% \) than the result
of Qin & Wu (2000), the main difference is the inclusion of
electric field in our calculation. As seen from equation
(11) the diffusion speeds depend on the local density and
temperature. However, a single diffusion time-scale is ob-
tained if the gas and dark matter both follow an isothermal
spherical density profile $\rho(R) \propto R^{-2}$ and the gas is in hy-
drostatic equilibrium $(\ddot{g} = g)$ with constant $T$. In this case
$V_1 - V_i \propto g/n_p \propto R$, which together with continuity equa-
tion, $\frac{d}{dt}(n_i) = -3 \Omega_R^2 \frac{\partial}{\partial R}(R^2 V_i)$, yields,
$$\frac{d}{dt} \ln \left( \frac{n_i}{n_0} \right) = \frac{3(V_i - V_1)}{R}.$$ (12)

This motivates us to define the time-scale for diffusion be-
tween two species as
$$\tau_D = \frac{R}{3(V_i - V_1)},$$ (13)
which for helium relative to protons gives
$$\tau_D = 3 \times 10^9 \left( \frac{f_h}{0.1} \right) \left( \frac{T}{10^8 K} \right)^{3/2} \text{Yr},$$ (14)
where $f_h$ is the baryonic mass fraction in the cluster.

3 THE EQUILIBRIUM DISTRIBUTION

The diffusion time-scale of helium as seen from (14) is com-
parable to the Hubble time. It is thus prudent to examine in
detail the equilibrium distribution of hydrogen and helium,
which is the final product of diffusion.

We assume a spherically symmetric cluster in which the
dark mass inside a radius $r$ is given by (Navarro, Frenk &
White 1997)
$$M_{dm}(r) = 4\pi r^3 \rho_s \left[ \ln \left( 1 + r/r_s \right) + \frac{1}{1 + r/r_s} \right],$$ (15)
where $\rho_s$ and $r_s$ are constants. The gas density profile, $\rho$, is
determined by the equation of hydrostatic equilibrium (see eq.
4),
$$\frac{kT}{\mu m_p} \frac{d}{dr} \ln \left( \frac{\rho}{\mu} \right) = -\frac{GM_{dm}(r)}{r^2},$$ (16)
where we have neglected the contribution of baryons to grav-
it and assumed constant temperature, $T$, throughout the
ICM. In the absence of sedimentation $\mu$ is constant and the
solution to (16) is (Makino, Sasati, & Suto 1998)
$$\rho(r) = \rho(0) e^{-\mu r} \left( 1 + r/r_s \right)^{\mu r/s},$$ (17)
where $\eta = 4\pi G m_p \rho_s r_s^2 / kT$. Sedimentation introduces a
dependence of $\mu$ on $r$ and the above analytical solution is
no longer valid. The abundances of the various elements are
then determined by the equation of hydrostatic equili-
trum for each element separately and the condition for local charge neutrality,
$$\frac{kT d \ln m_i}{m_i} dr = -\frac{GM_{dm}(r)}{r^2} - \frac{q_i E(r)}{m_i},$$ (18)
$$\sum n_i(r) q_i = 0.$$ (19)

Taking $m_\alpha = 4m_p$ and neglecting the electron mass, these
equations have the solution
$$n_p = 6C_1 \left[ f(r) + f^{-1}(r) - 1 \right] / h^2(r),$$ (20)
$$n_\alpha = C_1 \left[ f(r) + f^{-1}(r) - 1 \right]^2 / h^2(r),$$ (21)
where $f(r) = \left[ C_2 h^2(r) - 1 + \sqrt{(C_2 h^2(r) - 1)^2 - 1} \right]^{1/3}$,
$h(r) = (1 + r/r_s)^{\eta r/s}$, and $C_1$ and $C_2$ constants. The
behavior of the solution in the inner and outer regions is easily
understood as follows. In the inner regions helium is domi-
nant, then $\mu = 4/3$ and $E = \mu g/e = 4g/3e$. From this follows
$n_p \propto (1 + r/r_s)^{-n/3}$ and $n_\alpha \propto (1 + r/r_s)^{n/3}$. Note that, since
the total force felt by protons becomes repulsive $(eE > mg)$,
their density is falling towards the center. The outer regions
consist almost entirely of hydrogen plasma, thus $\mu = 1/2$
and $E = \mu g/e = g/2e$. This gives $n_p \propto (1 + r/r_s)^{n/2}$ and
$n_\alpha \propto (1 + r/r_s)^{3n/2}$, and so $n_\alpha \propto n_p^3$ in agreement with Gil-

The constants $C_1$ and $C_2$ in the analytic solution (21)
are fixed by the boundary conditions imposed on the abun-
dances. Here we require that the ratio of total helium to
hydrogen abundances inside the virial radius is equal to the
primordial value ($\sim 0.08$). We will present results for a clus-
ter with gas temperature such that $\eta = 10$ and a virial
radius equal to $3r_s$ in agreement with observations and N-
body simulations of massive clusters (NFW 1997; Ettori &
Fabian 1999). In Fig. 1 we show as the solid line the baryonic
density obtained from the analytic solution. For comparison
we also plot as the dashed curve the profile (17) which cor-
responds to equilibrium without diffusion. A proper estima-
tion of the density profile from the observed X-ray emissiv-
ity $(\chi \sum n_i Z_i^2 \sum n_i Z_i)$ should take into account variations of
the He/H abundance ratio throughout the ICM. Assuming
constant He/H ratio can yield a biased estimate of the pro-
file. To demonstrate this we plot as the dotted line in fig. 1
the estimated profile if a constant abundance ratio were
assumed. In fig. 2 we show the baryonic mass fraction as
a function of radius for the same three cases as in fig. 1. We
see (solid line) that diffusion introduces distinct features in
the behavior of the baryonic fraction as a function of ra-
dius. Finally fig. 3 shows the number density of helium and
hydrogen in the case with diffusion.

4 CONCLUSIONS

We have seen that diffusion steepens the gas density pro-
file near the center of hot clusters, increasing their X-ray
luminosity (by factor of 5 in our example). In colder clus-
ters ($T \sim 10^7$ K) the diffusion time-scale is larger than the
Hubble time and their luminosity remains unchanged. This
may explain the observed discrepancy between the observed
$L \propto T^3$ (Mushotzky 1984; Edge & Stuwart 1991; David et
al. 1993) and $L \propto T^2$ which is expected from self-similar
arguments (e.g., Kaiser 1986).

If the inner regions of clusters are dominated by he-
lium then the baryonic mass density as inferred from the
X-ray emissivity can be underestimated by $\sim 30\%$ if con-
stant He/H abundance ratio is assumed, while in the helium-
Figure 1. Gas density profiles. The solid and dashed lines correspond to equilibria with and without diffusion, respectively. The dotted line corresponds to the profile inferred from the X-ray emissivity by assuming constant abundances in the case with diffusion. The density is normalized so that the total baryonic mass within $3r_s$ is unity.

Figure 2. The same as the previous figure, but for the baryonic mass fraction (divided by the primordial value).

Figure 3. The number density of hydrogen (solid) and helium (dashed) as a function of radius. The curves are normalized so that the total number of particles inside $3r_s$ is unity.

deficient outer region it can be overestimated by $\sim 7\%$. Estimates of of the dark matter density can be affected by even a larger factor. These estimates assume hydrostatic equilibrium (eq. 16), so by taking $\mu = 0.59$ (cosmic abundance) instead of 0.5 (pure hydrogen plasma) in the outer regions, we underestimate the total mass by $\sim 18\%$ (Qin & Wu 2000). In the helium dominated core the mass would be overestimated by a factor of 2.3.

Our estimates of the sedimentation time-scales have neglected magnetic fields. Magnetic fields with coherence length comparable to the size of the cluster force the ions to move on longer orbits defined by the field lines. This can increase the sedimentation time-scales by factor of a few. Small-scale magnetic fields, however, can increase the time-scales by a factor ranging from a few in some estimates (Narayan & Medvedev 2001; Malyshkin 2001) to 100-1000 in others (Chandran & Cowley 1998).

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