EXCITED BARYON PRODUCTION AND DECAYS

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We consider decays of the lowest-lying positive parity (56-plet) and negative parity (70-plet) excited baryons. For the 70-plet, we include both single-quark and two-quark decay operators, and find, somewhat mysteriously, that the two-quark operators are not phenomenologically important. Studies of decays $70 \to \Delta^+\gamma$ may strengthen or vitiate this observation. For the 56-plet decays, now using only the single-quark operator, we can predict many strong decays after fitting parameters on the assumption that the Roper is a $3q$ state. Comparison of these predictions to experiment can verify the structure of the 56-plet. As a sidelight, we show a large $N_c$ derivation of the old Gürsey-Radicati mass formula.

1. Introduction

I should note that I have written on large $N_c$ topics in collaboration with the chief organizer of this conference and with the chairman of this session. Those works [1] concerned mass operators for excited baryons using large $N_c$ to segregate terms by size, and a continuation of the mass analysis work is reported elsewhere in this conference [2]. This talk is not about that work.

This talk will discuss the decays of excited baryons, in particular strong decays of the excited positive parity states [3] and electromagnetic decays of the negative parity states [4] (work from a similar viewpoint on strong decays of the latter are reported in [5]). Large $N_c$ ideas have contributed directly to the analysis we will report on by categorizing operators by order in $N_c$. In addition, we will show a large $N_c$ derivation of an old mass formula [6]. We need this mass formula because not all interesting states are yet found experimentally, and masses and phase space are crucial to calculating decay rates. It also happens that the large $N_c$ derivation of the mass formula is new and interesting.

As mentioned, we have looked at two sets of states. One set is the lowest-lying positive parity excited states of the baryons. Baryons in the ground state form a 56-dimensional representation of $SU(6)$. The excited positive parity states also form a 56-plet, the 56'. The other set is the non-strange members of the lowest lying negative parity baryon excitations, which are members of a 70-plet in $SU(6)$.

Regarding the 70-plet and the electromagnetic decays, the old and well known [7, 8] treatment identifies three independent operators that involve a single quark. The 19 possible decay amplitudes have been measured [9], and one

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1Invited talk at the workshop on The Phenomenology of Large $N_c$ QCD, held at Arizona State University, Tempe, AZ, USA, 9-11 January 2002.
may try to fit them in terms of matrix elements of the three single quark operators, with the overall coefficient for each operator fit to the data. The resulting fit is not awful, although with a $\chi^2$ about 4 per independent data point, one might hope for better. There are additional operators that involve other quarks as well, and one should not a priori neglect them in a low momentum transfer strong interaction process. We examined the 2-body operators [4] and found many that are not, contrary to our hope, suppressed by large $N_c$ arguments. Regarding how much including them improved the fit to the data, we will let the reader find in Section 2. A hint may be found by scanning the conclusions in Section 5.

Preceding the decay discussion for the 56' will be a seemingly tangential discussion of the Gürsey-Radicati mass formula [6]. This is a 4-parameter mass formula for a 56-plet of SU(6) (which has 8 independent masses, not counting isospin splittings). It applies to the ground state baryons, but also to the positive parity excited 56'. There are two reasons for including the Gürsey-Radicati mass formula discussion. One is that the original 1964 derivation is not to the modern taste, whereas a derivation using large $N_c$ ideas is both clear and illustrative of how large $N_c$ ideas let us estimate the sizes of terms [10], allowing us to keep the few largest mass operators to produce a mass relation, and to use the largest omitted term to estimate its error. The second reason is that not all the 56' states have been experimentally found, and we need the mass formula to get the masses of the missing states. The mass formula is discussed in Section 3.

Were one to select a prime motive for the 56' strong decay discussion, it would be to help determine if the Roper be dominantly an ordinary 3q baryon state [11, 12], or something more exotic such as a hybrid q^3g state [13], or something not really a baryon state at all, but just an enhancement in some partial wave in a scattering process [14].

We shall be treating the Roper as if it is a 3q member of a 56-plet of 3q states, and calculate decay rates based on assuming the dominance of the (only possible) single quark decay operator [4], an assumption that follows from the more detailed analysis made in the context of the negative parity 70-plet. The details of the analysis and predictions for many decays are presented in Section 4.

Conclusions, again, are in Section 5.

2. Radiative Decays of the 70

The work described here follows upon the large $N_c$ revival of the middle 1990's [15, 16, 17, 18]. Most applications were to the ground state [10], but the excited baryons were not entirely neglected [19, 20]. In particular, strong decays of the 70-plet were studied [5] in a manner one can recognize from the study of the electromagnetic decays that follows (though familiarity with the earlier work is not assumed in what is written here).

One does know [8] that the 70-plet of SU(6) has one unit of orbital angular momentum ($L = 1$) and negative parity. It has 20 states that are not strange, and if one lumps the isospin partners together, one is left with 7 independent states.
Two of the states are $\Delta$’s, characterized by $I = 3/2$, and denoted $\Delta_J$, 

$$\Delta_{1/2, 3/2} = \left| I = \frac{3}{2}, S = \frac{1}{2}, L = 1, J = \frac{1}{2}, \frac{3}{2} \right>.$$  

(1)

Five of the states are nucleons, with $I = 1/2$, and distinguished by whether the core quark spins are combined to give $S = 1/2$ or $3/2$,

$$N_{1/2, 3/2} = \left| I = \frac{1}{2}, S = \frac{1}{2}, L = 1, J = \frac{1}{2}, \frac{3}{2} \right>,$$

$$N'_{1/2, 3/2, 5/2} = \left| I = \frac{1}{2}, S = \frac{3}{2}, L = 1, J = \frac{1}{2}, \frac{3}{2}, \frac{5}{2} \right>.$$  

(2)

Since nucleon states with the same $J$ can mix, the physical $N_{1/2, 3/2}$ states are the linear combinations

$$\begin{bmatrix} N_{1535} \\ N_{1650} \\ N_{1520} \\ N_{1700} \end{bmatrix} = \begin{bmatrix} \cos \theta_{N1} & \sin \theta_{N1} \\ -\sin \theta_{N1} & \cos \theta_{N1} \\ \cos \theta_{N3} & \sin \theta_{N3} \\ -\sin \theta_{N3} & \cos \theta_{N3} \end{bmatrix} \begin{bmatrix} N_{1/2} \\ N'_{1/2} \\ N_{3/2} \\ N'_{3/2} \end{bmatrix}. $$

(3)

Thus the non-strange sector of the 70-plet has 2 mixing angles and 7 masses.

Consider decays in the $N^*$ rest frame (where $N^*$ is a member of the 70-plet). Using the outgoing photon direction as the helicity direction for the $N^*$, one has

$$\lambda = \lambda_\gamma - \lambda_N = \pm \frac{3}{2}, \pm \frac{1}{2};$$  

(4)

giving the helicity $\lambda$ of the $N^*$ also specifies the helicity $\lambda_\gamma$ of the photon and of the ground state exiting nucleon $\lambda_N$.

The decay amplitudes $A_\lambda$ are matrix elements [21] of decay operators $O_i$

$$A_\lambda = \left< N_i, \gamma \left| \sum O_i \right| N^*, \lambda \right>, $$

(5)

and by parity, $A_{3/2}$ and $A_{1/2}$ suffice [21].

We have to consider how to make the operators $O_i$. We can use the

- quark charge matrix $Q_i$
- photon polarization $\vec{A} \times \vec{\xi}$
- photon momentum $\vec{k}$ (or derivative $\vec{\nabla}$)
- P-wave quark polarization $\vec{\varepsilon}$
- spin operator for quark $\vec{\sigma}$

With these ingredients, there are 3 one-body operators $O_i$,

$$a_1 Q_0 \left< \vec{\varepsilon} \cdot \vec{A} \right>, $$

$$b_1 Q_0 \left< \vec{\varepsilon} \cdot \vec{k} \right> (\vec{\sigma} \cdot \vec{k} \times \vec{A}), $$

$$b_2 Q_0 \left< \vec{\sigma} \cdot \vec{k} \right> \left< \vec{\varepsilon} \cdot \vec{k} \times \vec{A} \right>.$$  

(6)
which are equivalent to the textbook [8] SU(6) one-body operators. The star means an operator acting on the initial P-wave quark.

Examples of two-body operators include

\[
\begin{align*}
&\frac{c_3}{N_c} \left( \sum_{\alpha \neq \star} Q_\alpha \bar{\sigma}_\alpha \right) \cdot \bar{\sigma}_\star (\bar{\epsilon} \cdot \bar{A}), \\
&\frac{d_2}{N_c} \left( \sum_{\alpha \neq \star} Q_\alpha \bar{\sigma}_\alpha \right) \cdot \bar{k} (\bar{\epsilon} \cdot \bar{k}) (\bar{\sigma}_\star \cdot \bar{A}), \\
&\frac{d_3}{N_c} \left( \sum_{\alpha \neq \star} Q_\alpha \bar{\sigma}_\alpha \right) \cdot \bar{k} (\bar{\sigma}_\star \cdot \bar{k}) (\bar{\epsilon} \cdot \bar{A}), \\
&\frac{d_4}{N_c} \left[ \left( \sum_{\alpha \neq \star} Q_\alpha \bar{\sigma}_\alpha \right) \times \bar{\sigma}_\star \cdot \bar{k} \right] (\bar{\epsilon} \cdot \bar{k} \times \bar{A}).
\end{align*}
\]

(7)

There are 8 leading-order (in \(N_c\)) 2-body operators, which are listed in [4]. The explicit factors of \(N_c\) are included, so that one may expect that the \(a_i, b_i, c_i,\) and \(d_i\) are all \(O(N_c^0)\).

But now comes an important observation. The 2-body operators have matrix elements \(\propto N_c\) (since they get one contribution from each on the non-excited quarks in the baryon). Hence their contributions to the decay rates are not suppressed by large \(N_c\) arguments as \(N_c \to \infty\), compared to the 1-body operators.

Following this statement comes a surprise. The 2-body operators are empirically unimportant. Fitting data with 1-body operators alone gives \(\chi^2 = 52.97\) (with mixing angles fixed at \(\theta_{N1} = 0.61\) and \(\theta_{N3} = 3.04\), which were obtained from an earlier analysis of the hadronic decays of the 70-plet [5]; we can also do the fit letting the mixing angles be fit using the electromagnetic data only, and get the same mixing angles within the uncertainty limits).

Now include one 2-body operator with the three one-body operators. The resulting \(\chi^2\) depends on which 2-body operator is chosen, as tabulated here:

<table>
<thead>
<tr>
<th>2-body op.</th>
<th>(c_1)</th>
<th>(c_2)</th>
<th>(c_3)</th>
<th>(c_4)</th>
<th>(d_1)</th>
<th>(d_2)</th>
<th>(d_3)</th>
<th>(d_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\chi^2)</td>
<td>52.85</td>
<td>52.04</td>
<td>39.21</td>
<td>52.91</td>
<td>48.38</td>
<td>52.81</td>
<td>52.23</td>
<td>52.59</td>
</tr>
</tbody>
</table>

One does best using \(c_3\), and the result including all 8 two-body operators simultaneously is not notably better than using \(c_3\) alone. Including the 2-body operators does not significantly improve the fit to the data.

The dynamics will have to be understood better if we are to understand why the 2-body operators enter with small effect. Large \(N_c\) is not violated, in that no coefficient is forced to be larger than expected, but it does appear that the 2-body terms can have coefficients that are smaller that they have to be from large \(N_c\) ideas.

One should say that there is an experimental opportunity here. Current data on \(N^*\) “decays” really come from pion photoproduction, using time-reversal invariance. One then understands why all the decays that have been discussed have a ground-state nucleon in the final state. One can anticipate that modern
machines (for example, Jefferson Lab) can produce enough $N^*$’s to see radiative decays directly. That means that one can also think about decays into $\Delta \gamma$ final states.

There are 24 (non-isospin related) amplitudes for the decays

$$ N^* \text{ or } \Delta^* \rightarrow \Delta \gamma ; \quad (8) $$

the subscripts on the amplitudes $A_\lambda$ can now run from $-1/2$ to $5/2$ (with parity invariance allowing us to fix $\lambda_\gamma = 1$).

The 1-body operators and their coefficients are the same as for the decays into $N\gamma$; only the evaluation of the matrix elements changes. Hence each of the 24 decays is predicted [3, 22]. We can see if invariance of the 1-body operators will persist.

In addition, we have made predictions [3] using the 1-body operators plus the $c_3$ operator and can say which of the 70-plet decays are among the most sensitive to including the previous best-fit value of the $c_3$ term. Here are some examples, good and bad [decay amplitudes are all in (GeV)$^{-1/2}$]:

$$ A_{1/2}[\Delta^+(1620) \rightarrow \Delta \gamma] = \begin{cases} 0.073 \pm 0.006 & \text{1-body,} \\ 0.074 \pm 0.006 & \text{1-body + } c_3 \end{cases} $$

$$ A_{1/2}[p(1675) \rightarrow \Delta \gamma] = \begin{cases} -0.019 \pm 0.009 & \text{1-body,} \\ -0.060 \pm 0.013 & \text{1-body + } c_3 \end{cases} $$

$$ A_{5/2}[\Delta^+(1675) \rightarrow \Delta \gamma] = \begin{cases} -0.258 \pm 0.012 & \text{1-body,} \\ -0.337 \pm 0.020 & \text{1-body + } c_3 \end{cases} \quad (9) $$

3. Mass Formula

We have explained in the Introduction why we are going to discuss a mass formula in the midst of this baryon decay talk: the derivation is clear and illustrative of large $N_c$ methods, the original derivation is dated, and we need the result.

We looking at a 56-plet of baryons where the spatial state, and so also the spin-flavor state, is totally symmetric. There are 56 totally symmetric 3-quark states that one can make from $u_\uparrow, u_\downarrow, d_\uparrow, d_\downarrow, s_\uparrow, s_\downarrow$, where the arrows indicate the spin projection. The ground states form the 56, and the radially-excited states form the 56’. The states are the $N, \Lambda, \Sigma, \Xi, \Delta, \Sigma^*, \Xi^*$, and $\Omega$.

The mass operators for these states are built from the spin $S^i = \sum_\alpha \sigma^i_\alpha / 2$ (the sum is over the quarks $\alpha$), the flavor operators $T^a = \sum_\alpha \tau^a_\alpha / 2$ (where the $\tau^a$ are a set of $3 \times 3$ matrices), and the SU(6) operators

$$ G^{ia} = \sum_\alpha \frac{1}{2} \sigma^i_\alpha \cdot \frac{1}{2} \tau^a_\alpha. \quad (10) $$

Terms in mass operators must be rotationally symmetric, and flavor symmetric to leading order. Not all terms should be included. For example, in
symmetric states matrix elements of $T^2$ and $G^2$ are linearly dependent on those of $S^2$ and the unit operator [10].

Flavor symmetry is not exact. The mass of the strange quark allows non-flavor symmetric terms in the effective mass operator, visible as unsummed flavor indices $a = 8$ below. The effective mass operator is

$$H_{\text{eff}} = a_1 1 + \frac{a_2}{N_c} S^2 + \epsilon a_3 T^8 + \frac{\epsilon}{N_c} a_4 S^i G^{i8}$$

$$+ \frac{\epsilon}{N_c^2} a_5 S^2 T^8 + \frac{\epsilon^2}{N_c} a_6 T^8 S^8$$

$$+ \frac{\epsilon^2}{N_c^2} a_7 T^8 S^i G^{i8} + \frac{\epsilon^3}{N_c^2} T^8 T^8 S^8 . \quad (11)$$

There is an $\epsilon$ for each violation of flavor symmetry, where $\epsilon \approx 1/3$. Also, a term that is a product of two or three operators comes from an interaction that has at least one or two gluon exchanges, and the strong coupling falls with number of colors as $g^2 \sim 1/N_c$. (A crucial theorem is that no perturbation theory diagrams fall slower in $1/N_c$ than the lowest order ones [10]).

Keeping the first four terms, taking the matrix elements, and reorganizing leads to

$$M = A + BN_s + C \left[ I(I + 1) - \frac{1}{4} N_s^2 \right]$$

$$+ DS(S + 1), \quad (12)$$

where $N_s$ is the number of strange quarks. This is the Gürsey-Radicati [6] mass formula. We use it to predict masses of 4 undiscovered members of the 56′, given that 4 are known.

We can estimate the error of the formula. The constants $a_i$ above are nominally about 500 MeV. The first term omitted is hence nominally of order 500 MeV/3 or about 20 MeV, and this is the estimated error in the Gürsey-Radicati mass formula.

4. The Decays 56′ → 56 + Meson

Four of the 8 states in the 56′ are undiscovered or unconfirmed, and existing measurements have large uncertainty. However, we anticipate new results soon from the CLAS detector at CEBAF. One member of the 56′ is the Roper or N(1440), whose composition has been debated. Might it be a $q^3 g$ state [13], a non-resonant cross-section enhancement [14], or just a 3-quark radial excitation [11, 12]? Recent lattice calculations, which seem to find the negative-parity excited baryons with about the right mass, still find the positive-parity excitation at a fairly heavy mass [23]. Our predictions depend upon the 3-quark possibility.

We assume that only single-quark operators are needed. Two-quark operators were studied for decays of orbitally-excited states [4], and found unneces-
sary. There is only one single quark operator here, so

$$H_{\text{eff}} \propto G^a k^i \pi^a,$$  \hspace{1cm} (13)

where \( k^i \) is the meson 3-momentum and \( \pi^a \) is a meson field operator.

One gets for the decay widths,

$$\Gamma = \frac{M_f}{6\pi M_i} k^3 f(k)^2 \sum |\langle B_f | G_{ja} | B_i \rangle|^2,$$  \hspace{1cm} (14)

where \( f(k) \) parameterizes the momentum dependence of the amplitude. For the 7 measured decays it is well fit by \( f = (2.8 \pm 0.2)/k \). With this in hand, we can predict the widths for 22 decays. The detailed results are in [3]. The success of our predictions would bolster the view of the Roper as a 3-quark state.

5. Conclusions

Here is a summary of our conclusions in list form:

1. For 70-plet decays to \( N\gamma \), single quark-operators are not favored by large \( N_c \), yet they suffice to explain the data. Adding extra operators does not materially improve the fit. Why this should be so remains a mystery.

2. Decays of the 70-plet to \( \Delta\gamma \) add 24 new decay channels to test large \( N_c \) ideas or the approximation of using the single-quark operators only.

3. For 56'-decays (to \( N\pi \)), the mass spread necessitated thinking about the momentum dependence of the matrix elements. We got masses of the as yet unfound members of the 56' from the Gürsey-Radicati mass formula—for which we have a modern large \( N_c \) derivation.

4. Seven 56' decays into baryon plus meson have been observed, the remaining 22 being predicted. We assumed the states, including the Roper, were 3-quark states. The success of the predictions could shed light on whether this is true.

Acknowledgments

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Note added: New data is coming in from JLab, and since the Workshop there has been an analysis [24], finding good results also for electroproduction (photon off-shell) with only single quark amplitudes.
References


