1 Accretion in Compact Binaries  page 1
1.1 Introduction 1
1.2 Accretion disc theory 1
1.3 Dwarf novae: the nature of the outbursts 7
1.4 Dwarf novae: the occurrence of the outbursts 8
1.5 Soft X–ray transients: the nature of the outbursts 12
1.6 Soft X–ray transients: the occurrence of the outbursts 17
1.7 Quiescent transients and black hole horizons 31
1.8 Ultraluminous X–ray sources 32
1.9 Conclusions 40
   References 41
1

Accretion in Compact Binaries

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1.1 Introduction

Compact binaries have long been a paradigm for accretion theory. Much of our present view of how accretion occurs comes directly from the comparison of theory with observations of these sources. Since theory differs little for other objects such as active galaxies, increasing efforts have recently gone into searching for correspondences in observed behaviour. This chapter aims at giving a concise summary of the field, with particular emphasis on new developments since the previous edition of this book.

These developments have been significant. Much of the earlier literature implicitly assumed that accreting binaries were fairly steady sources accreting most of the mass entering their vicinity, often with main–sequence companions, and radiating the resulting accretion luminosity in rough isotropy. We shall see that in reality these assumptions fail for the majority of systems. Most are transient; mass ejection in winds and jets is extremely common; a large (sometimes dominant) fraction of even short–period systems have evolved companions whose structure deviates significantly from the zero–age main sequence; and the radiation pattern of many objects is significantly anisotropic. It is now possible to give a complete characterization of the observed incidence of transient and persistent sources in terms of the disc instability model and formation constraints. X–ray populations in external galaxies, particularly the ultraluminous sources, are revealing important new insights into accretion processes and compact binary evolution.

1.2 Accretion disc theory

Essentially all of the systems discussed here accrete via discs. Accretion disc theory is the subject of many books and reviews (see e.g. Frank et al., 2002 and Pringle, 1981). Accordingly this section simply summarizes the main results without giving detailed derivations.

1.2.1 Disc formation

Matter accreting on to a mass \( M_1 \) forms a disc if its specific angular momentum \( J \) is too large for it to impact the object directly. We define the circularization radius

\[
R_{\text{circ}} = \frac{J^2}{GM_1},
\]
which is where the matter would orbit if it lost energy but no angular momentum. The condition for disc formation is typically that $R_{\text{circ}}$ should exceed the effective size of the accretor (a parabolic orbit with specific angular momentum $J$ would reach a minimum separation $0.5R_{\text{circ}}$). This effective size is identical with the radius of a non–magnetic white dwarf or neutron star, but is of order the magnetospheric radius if there is a dynamically significant magnetic field. For a black hole the effective size is the radius of the last stable circular orbit. If mass transfer occurs via Roche lobe overflow, $J$ is comparable with the specific orbital angular momentum of the binary, $R_{\text{circ}}$ is large, and the condition for disc formation is almost always satisfied in a compact binary. The exceptions are some cataclysmic variables where the white dwarf accretor is strongly magnetic, and double white dwarf systems with companion/accretor mass ratios $M_2/M_1$ larger than about 0.15 (see e.g. Nelemans et al., 2001).

In the usual case that matter can indeed orbit at $R_{\text{circ}}$, disc formation will follow under the assumption that energy is lost through dissipation faster than angular momentum is redistributed. Since the orbit of lowest energy for a given angular momentum is a circle, matter will follow a sequence of circular orbits about the compact accretor. The agency for both energy and angular momentum loss is called viscosity. For many years the nature of this process was mysterious, but recently a strong candidate has emerged, in the form of the magnetorotational instability (MRI: Balbus & Hawley, 1991). Here a comparatively weak magnetic field threading the disc is wound up by the shear, and transports angular momentum outwards. Reconnection limits the field growth and produces dissipation. Numerical simulations show that this is a highly promising mechanism, and will shortly reach the point of allowing direct comparison with observations.

1.2.2 Thin discs

While viscosity transports angular momentum and thus spreads the initial ring at $R_{\text{circ}}$ into a disc, the nature of this accretion disc is determined by the efficiency with which the disc can cool. In many cases this is high enough that the disc is thin: that is, its scaleheight $H$ obeys

$$H \simeq \frac{c_s}{v_K} R << R$$

at disc radius $R$, where $c_s$ is the local sound speed, and

$$v_K = \left(\frac{GM}{R}\right)^{1/2}$$

is the Kepler velocity, with $M$ the accretor mass. In this state the azimuthal velocity is close to $v_K$, and the radial and vertical velocities are much smaller. The properties of being thin, Keplerian and efficiently cooled are all equivalent, and if any one of them breaks down so do the other two.

If the thin disc approximation holds, the vertical structure is almost hydrostatic and decouples from the horizontal structure, which can be described in terms of its surface density $\Sigma$. If the disc is axisymmetric, mass and angular momentum conservation imply that the latter obeys a nonlinear diffusion equation.
1.2 Accretion disc theory

\[ \frac{\partial\Sigma}{\partial t} = \frac{3}{R} \frac{\partial}{\partial R} \left( R^{1/2} \frac{\partial}{\partial R} \left[ \nu \Sigma R^{1/2} \right] \right). \]  

Here \( \nu \) is the kinematic viscosity, which is usually parametrized as

\[ \nu = \alpha c_s H. \]  

where \( \alpha \) is a dimensionless number. In a steady state this gives

\[ \nu \Sigma = \frac{\dot{M}}{3\pi} \left[ 1 - \beta \left( \frac{R_{in}}{R} \right)^{1/2} \right], \]  

where \( \dot{M} \) is the accretion rate and the dimensionless quantity \( \beta \) is specified by the boundary condition at the inner edge \( R_{in} \) of the disc. For example, a disc ending at the radius \( R_* \) of a non–rotating star has \( R_{in} = R_* \). In a steady thin disc dissipation \( D(R) \) per unit surface area is also proportional to \( \nu \Sigma \), i.e.

\[ D(R) = \frac{9}{8} \nu \Sigma \frac{GM}{R^3} \left[ 1 - \beta \left( \frac{R_{in}}{R} \right)^{1/2} \right], \]  

so that the surface temperature \( T \) is independent of the viscosity \( \nu \) despite being entirely generated by it:

\[ T = T_{visc} = \left\{ \frac{3GM\dot{M}}{8\pi R^3 \sigma} \left[ 1 - \beta \left( \frac{R_*}{R} \right)^{1/2} \right] \right\}^{1/4}. \]

1.2.3 Disc timescales

Equation (1.4) shows that \( \Sigma \) changes on a timescale

\[ t_{visc} \sim \frac{t^2}{\nu} \]  

if its spatial gradient is over a lengthscale \( l \). Hence we would expect a disc to make significant changes in its surface density and thus its luminosity on a timescale \( \sim R^2/\nu \), where \( R \) is its outer radius. We can use this fact to get an idea of the magnitude of the viscosity in observed discs. In dwarf novae, which are short–period white–dwarf binaries, the disc size is \( R \sim 1 - 3 \times 10^{10} \text{ cm} \), and surface density changes take a few days. This suggests that \( \alpha \sim 0.1 \). Encouragingly, numerical simulations of the MRI give comparable answers. There are two other obvious timescales in a disc. The first is the dynamical timescale

\[ t_{dyn} \sim \frac{R}{v_K} = \left( \frac{R^3}{GM} \right)^{1/2}, \]  

characterizing states in which dynamical equilibrium is disturbed; note that vertical hydrostatic balance is resored on a timescale

\[ t_z \sim \frac{H}{c_s} = \frac{R}{v_K} = t_{dyn}, \]  

where we have used eqn (1.2). The second is the thermal timescale
Accretion in Compact Binaries

\[ t_{\text{th}} = \frac{\Sigma c_s^2}{D(R)} \sim \frac{R^3 c_s^2}{GM \nu} = \frac{c_s^2}{\nu \Omega_K} \frac{R^2}{\nu} = \left( \frac{H}{R} \right)^2 t_{\text{visc}} \]  

(1.12)

where we have used eqn (1.7). The alpha–disc parametrization (1.4) can be used to show that

\[ t_{\text{visc}} \sim \frac{1}{\alpha} \left( \frac{H}{R} \right)^2 t_{\text{dyn}} \]  

(1.13)

so we finally have the ordering

\[ t_{\text{dyn}} \sim t_z \sim \alpha t_{\text{th}} \sim \alpha \left( \frac{H}{R} \right)^2 t_{\text{visc}}, \]  

(1.14)

i.e. dynamical < thermal < viscous.

1.2.4 Breakdown of the thin disc approximation

The thin disc approximation discussed above requires the accreting matter to cool efficiently. However flows with low radiative efficiency on to a black hole can in principle occur, for at least two reasons: the accretion rate \( \dot{M} \) may be so low that the inflowing gas has low density and thus a long cooling time, or conversely \( \dot{M} \) may be so large that the flow is very optically thick, and radiation is trapped and dragged down the hole. As energy is advected inwards, these flows are called ADAFs (advection–dominated accretion flows). If the accretor is not a black hole, the advected energy must be released near the surface of the accretor. This effect has been invoked to explain observations of quiescent transients (see Section 1.7 below).

Considerable theoretical effort has gone into trying to understand such radiatively inefficient flows. Clearly the flows cannot be geometrically thin, making analytic treatments difficult. Such studies assume a discontinuous change from a thin Keplerian disc to an ADAF at some ‘transition radius’ \( R_{\text{tr}} \), whereas it is easy to show that any transition region must be extended itself over a size several times the assumed \( R_{\text{tr}} \). Moreover the difficulty of applying a predictive theory for the disc viscosity means that \( R_{\text{tr}} \) is taken as a free parameter in attempts to fit observations, a freedom that would not be present in reality. Recent numerical studies of accretion at low radiative efficiency (e.g. Stone, Pringle & Begelman, 1999, Stone & Pringle, 2001) find that very little of the matter flowing in at large radius actually accretes to the black hole. Instead the time–averaged mass inflow and outflow rates both increase strong with radius, and almost cancel. The simulations have not so far been run for long enough to reach any kind of steady state: one obvious possibility is that the density may eventually reach values at which radiative cooling does become efficient, leading to a thin disc phase.

1.2.5 Warping of discs

An important effect in disc physics, discovered only recently (Pringle, 1996) is that accretion discs tend to warp if exposed to irradiation from a source at their centres, which may be the accretion flow itself. The origin of the warping is that the disc must scatter or reradiate the incident radiation, which results in a pressure force normal to its surface. If the surface is perturbed from complete axisymmetry the force can increase or decrease in such a way as to cause the perturbation to grow. As
the gravitational potential is close to spherical symmetry near the accreting object, it is quite possible for disc material to orbit at angles to the binary plane. A full perturbation analysis (Pringle 1996) shows that the condition for warping is

\[ L \gtrsim 12\pi^2 \nu_2 \Sigma \varphi c, \quad (1.15) \]

where \( \nu_2 \) is the vertical kinematic viscosity coefficient.

This inequality shows that we can expect warping in discs in sufficiently luminous systems. We can re-express (1.15) by defining the ratio of the vertical viscosity coefficient \( \nu_2 \) to the usual radial one \( \nu \) as \( \psi = \nu_2/\nu \). Then (1.6) allows us to write (1.15) as

\[ L \gtrsim 12\pi^2 \psi \Sigma \varphi c = 4\pi \psi \dot{M} \varphi c \quad (1.16) \]

for a steady disc. Now

\[ \varphi = \left( \frac{GM}{R} \right)^{1/2} = \left( \frac{R_{\text{Schw}}}{2R} \right)^{1/2} c, \quad (1.17) \]

where \( R_{\text{Schw}} = 2GM/c^2 \) is the Schwarzschild radius of the central star, so combining with (1.16) gives the condition

\[ L \gtrsim 4\pi \psi \dot{M} c^2 \left( \frac{R_{\text{Schw}}}{2R} \right)^{1/2}. \quad (1.18) \]

If we finally assume that the central luminosity \( L \) comes entirely from accretion at the steady rate \( \dot{M} \) on to a compact object of radius \( R_\ast \), we can write

\[ L \approx \dot{M} c^2 \frac{R_{\text{Schw}}}{R_\ast}, \quad (1.19) \]

and use (1.18) to give

\[ \frac{R}{R_\ast} \gtrsim 8\pi^2 \psi^2 \frac{R_\ast}{R_{\text{Schw}}}. \quad (1.20) \]

Equation (1.20) now tells us if warping is likely in various systems. First, it is clearly very unlikely in accretion–powered white dwarf binaries such as CVs, since with \( M = 1M_\odot, R_\ast = 5 \times 10^8 \text{ cm} \) and \( \psi \sim 1 \) we find the requirement \( R \gtrsim 7 \times 10^{13} \text{ cm} \), demanding binary periods of several years. (Warping may occur in some supersoft X–ray binaries, where the energy source is nuclear burning rather than the gravitational energy release of the accreted matter.) For neutron stars and black holes by contrast warping is probable if the disc is steady, since with \( M = 1M_\odot, R_\ast = 10^6 \text{ cm} \), \( \psi \sim 1 \) we find \( R \gtrsim 3 \times 10^8 \text{ cm} \), while for a black hole with \( R_\ast = R_{\text{Schw}} \) warping will occur for \( R \gtrsim 8\pi^2 R_{\text{Schw}} \sim 2.4 \times 10^7(M/1M_\odot) \text{ cm} \). It thus seems very likely that discs in LMXBs are unstable to warping, at least for persistent systems. A warped disc shape therefore offers an explanation for the observed facts that persistent LMXB discs are both irradiated and apparently have large vertical extent. The X–ray light curves of persistent LMXBs are strongly structured, implying that the X–rays are scattered by the accretion flow White & Holt (1982).

Of course, the discussion above only tells us about the possible onset of warping. To see what shape the disc ultimately adopts one must resort to numerical calculations.
These have so far only been performed in a highly simplified manner, but do provide suggestive results. For example, one might imagine that warps would be self-limiting, in that a significant warp would geometrically block the very radiation driving the warp. Numerical calculations show that this does not occur: the reason is that warping always starts at large disc radii (cf 1.20), where the mean disc plane is perturbed away from the original one. Matter flowing inwards from these radii has angular momentum aligned with the perturbed disc plane, and so transfers a ‘memory’ of it to the inner disc. One can show that a warped disc always has a line of nodes following a leading spiral, with the result that the disc can become markedly distorted from its original plane shape without shadowing large areas and arresting the growth of the warp. Of course these calculations are highly simplified, and one might in reality expect the warps to be limited (perhaps by tidal torques, before attaining such distorted shapes. However it is clear that radiation-induced warping is a promising mechanism for making persistent LMXB discs deviate from the standard picture. An attractive feature of this explanation is that, although the discs deviate globally from the standard picture, the thin disc approximation nevertheless continues to apply locally, in the sense that the warping is always over lengthscales much larger than the local scaleheight $H$.

1.2.6 Accretion disc stability

Many accreting sources are observed to vary strongly. The clearest examples are dwarf novae (DN), which are binaries in which a white dwarf accretes from a low mass star, and soft X-ray transients (SXTs), where a black hole or neutron star accretes from a low-mass companion. In both cases the system spends most of its time in quiescence, with occasional outbursts in which it is much brighter. However, beyond this qualitative similarity, there are very clear quantitative differences. In dwarf novae the typical timescales are: quiescence $\sim$ weeks – months, outburst $\sim$ days, and the system luminosity typically rises from $\sim 10^{32}$ erg s$^{-1}$ to $\sim 10^{34}$ erg s$^{-1}$. In SXTs, the corresponding numbers are quiescence $\sim 1–5$ yr or more, outburst $\sim$ months, system luminosity rises from $\sim 10^{32}$ erg s$^{-1}$ to $10^{38}–10^{39}$ erg s$^{-1}$. Remarkably, it is possible to explain both types of system with a similar model: the SXT version contains only one extra ingredient over that currently accepted for DN.

The basic model at work in both cases is the disc instability picture. There is a huge literature on this subject: Lasota (2001) and Frank et al. (2002) give recent reviews. The fundamental idea behind the model is that in a certain range of mass transfer rates, the disc can exist in either of two states: a hot, high viscosity state (outburst) and a cool, low viscosity state (quiescence). In practice these two states correspond to hydrogen existing in ionized or neutral states respectively. The very steep dependence of opacity on ionization fraction and thus temperature makes any intermediate states unstable, and the disc jumps between the hot and cool states on a thermal timescale. In each of these two states it evolves on a viscous timescale. The hierarchy (1.14) shows that this pattern does qualitatively reproduce the observed behaviour of long quiescence, short outburst, with rapid transitions between them. Since the basic cause of instability is hydrogen ionization we can immediately deduce the condition for a disc to be stable: it must have no ionization zones. Thus a sufficient condition for suppressing outbursts and making a system persistent is that
1.3 Dwarf novae: the nature of the outbursts

its surface temperature \( T \) should exceed some value \( T_H \) characteristic of hydrogen ionization (a typical value for \( T_H \) is 6500 K, depending somewhat on the disc radius). We can assume that this condition is also necessary if the system is to be persistent, i.e. a system is persistent if and only if

\[
T_{\text{visc}} > T_H
\]  

(1.21)

throughout its accretion disc. Since \( T \) decreases with disc radius (cf eqn 1.8) this condition is most stringent at the outer disc edge \( R = R_{\text{out}} \), so we require

\[
T_{\text{visc}}(R_{\text{out}}) > T_H
\]  

(1.22)

for stability. In principle there is another family of persistent sources where the opposite condition

\[
T_{\text{visc}} < T_H
\]  

(1.23)

holds throughout the disc; the condition is tightest close to the inner edge of the disc, where \( T \) has a maximum value. There is clearly a strong selection effect against finding such systems, which must be inherently faint, and do not call attention to themselves by having outbursts.

In the following sections we shall see that the occurrence of outbursts implies powerful constraints on the evolution of both CVs and X-ray binaries.

1.3 Dwarf novae: the nature of the outbursts

The disc instability picture described above works quite well when applied to dwarf novae (see the review by Lasota, 2001), given the limitations of current treatments of disc viscosity. There is good reason to hope that improvements here will refine the picture further. In particular, two- and three-dimensional disc simulations have given a realistic picture of many disc phenomena which remained obscure in the early one-dimensional calculations. The first example of this was Whitehurst’s (1988) investigation of superhumps. Superhumps are a photometric modulation at a period slightly longer than the spectroscopically determined orbital period. They occur in a subclass of short-period dwarf novae during particularly long outbursts called superoutbursts. They had defied many attempted explanations until Whitehurst’s simulations of discs residing in the full Roche potential, rather than simply the field of the accreting star. These revealed that in binaries with sufficiently small secondary-to-primary mass ratios \( q = M_2/M_1 \lesssim 0.25 \) the disc becomes eccentric and precesses progradely within the binary. Tidal stressing of this disc causes dissipation and thus the superhump modulation.

This picture explains many of the observed superhump properties. Because the modulation results from intrinsic tidal stressing rather than geometrical effects, it is independent of the system inclination. Given that white dwarfs in CVs have masses \( M_1 \) lying in a small range \( 0.6M_\odot \approx 1.0M_\odot \) the restriction to small mass ratios \( q \) means that \( M_2 \) must be small \( (\lesssim 0.15M_\odot \approx 0.25M_\odot) \), accounting for the fact that almost all of the superoutbursting systems have periods below the 2–3 hr CV gap. The reason for the restriction to small \( q \) was subsequently traced to the fact that the superhump phenomenon is driven by the 3:1 orbital resonance (Whitehurst & King,
1991; Lubow, 1991); only for small ratios can the disc get large enough to access this resonance.

For some time there were competing explanations for the superoutbursts themselves. One (Vogt, 1983) invoked enhanced mass transfer from the secondary triggered by a normal thermal–viscous instability. In contrast Osaki (1989) suggested that in a series of normal outbursts the disc grows in size because accretion on to the white dwarf successively removes matter of low angular momentum. After several such episodes the disc reaches the 3:1 resonant radius, where tides remove angular momentum very effectively and cause more prolonged accretion of a significant fraction of the disc mass. Recent 2–D simulations (Truss et al., 2001) show that the latter model does function as suggested, and agrees with observation. Superoutbursts are a direct result of tidal instability. No enhanced mass transfer from the secondary is required to initiate or sustain either the superoutburst or the superhumps, provided that the mass ratio is small enough that the disc can grow to the 3:1 resonant radius.

1.4 Dwarf novae: the occurrence of the outbursts

1.4.1 Short–period dwarf novae

An important question for the disc instability idea is whether it correctly divides observed CV systems into dwarf novae and persistent (novalike) systems. In particular all non–magnetic CVs with periods below the well–known gap at 2–3 hr are dwarf novae. To answer this question we have to predict conditions such as mass transfer rate $-\dot{M}_2$ as a function of binary period $P$, and then check condition (1.21) by setting $\dot{M} = -\dot{M}_2$ in (1.8). Evidently the inequality (1.21) should fail for systems below the gap. In the standard view of CV evolution the secondary stars are assumed to be completely unevolved low–mass main sequence stars, and CV binaries evolve under angular momentum loss via gravitational radiation and magnetic stellar wind braking (see e.g. King, 1988 for a review). This picture leads to mass transfer rates $-\dot{M}_2(P)$ which fulfil our expectation above (see Fig. 1.2): the disc instability picture correctly predicts that short–period non–magnetic CVs are dwarf novae.

However things are clearly more difficult for systems above the gap. Here there is a mixture of dwarf novae and novalikes, strongly suggesting that the simple recipe described above for checking (1.21) does not capture the essence of the situation. As the disc instability picture seems to describe outbursts quite well, given a suitable mass transfer rate $-\dot{M}_2$, the most likely resolution of the problem is that the adopted relation for $-\dot{M}_2(P)$ is too simple. This problem is not confined to dwarf novae, but appears to be generic to all CVs above the period gap.

One possibility uses the fact that $-\dot{M}_2(P)$ is the average mass transfer rate, taken over timescales $> 10^5$ yr. Fluctuations on timescales shorter than this could still be unobservable in any individual system, but lead to an effective spread in instantaneous mass transfer rates. This suggestion has the virtue of leaving intact the existing picture of long–term CV evolution (e.g. the period histogram), which uses $\dot{M}_2(P)$. The fluctuations might themselves result from cycles driven by irradiation of the companion star (King et al., 1995). The main problem for this approach is that it tends to predict an almost bimodal distribution of instantaneous mass transfer.
1.4 Dwarf novae: the occurrence of the outbursts

rates, with the low state too low to give dwarf nova properties in good agreement with observation.

A second quite different idea (King & Schenker, 2002; Schenker & King, 2002) uses the observed fact that many CV secondaries have spectral types significantly later than would be expected for a ZAMS star filling the Roche lobe (Baraffe & Kolb, 2000). The idea here is that the spread in \(-\dot{M}_2\) reflects real differences in the nature of the secondary star; in many cases this has descended from a star which was originally more massive than the white dwarf, allowing significant nuclear evolution (I shall use the term ‘evolved’ to describe any star whose internal structure deviates significantly from the zero–age main sequence, even if the exterior appearance resembles a ZAMS star.) Moreover the large mass ratio means that the donor star’s Roche lobe tends to contract as it loses mass, leading to mass transfer on its thermal timescale.

Schenker et al. (2002) give an explicit example of this type of evolution, modelling the CV AE Aquarii. This system is not a dwarf nova, but instead has a rapidly spinning magnetic white dwarf which centrifugally expels most of the matter transferred to it. The fact that the spin has not attained some kind of equilibrium in which accretion is allowed strongly suggests that the mass transfer rate has dropped very sharply in the recent past, on a timescale short compared with the observed spindown timescale of \(\sim 10^7\) yr. This is highly suggestive of the ending of thermal–timescale mass transfer when the binary mass ratio becomes sufficiently small; the current masses are \(M_1 \simeq 0.9M_\odot\), \(M_2 \simeq 0.6M_\odot\), Welsh et al., 1995; Casares et al., 1996. These masses themselves are suggestive: the white dwarf may have accreted some matter from the companion, implying steady nuclear burning and thus a thermal–timescale mass transfer rate, while the secondary is clearly larger than its main–sequence radius given the current binary period \(P = 9.88\) hr (Welsh et al., 1993). Finally AE Aqr’s ultraviolet spectrum (Jameson et al., 1980) shows extremely strong NV\(_\lambda 1238\), but the usual corresponding resonance line CIV\(_\lambda 1550\) is completely undetectable. As these lines have virtually identical ionization and excitation conditions this is strongly suggestive of an abundance anomaly in which nitrogen is enhanced at the expense of carbon. This is turn is a signature of CNO processing, which clearly requires the star to have been more massive (\(\gtrsim 1.5M_\odot\)) in the past.

Schenker et al (2002) show that descent from thermal–timescale evolution gives a consistent picture of AE Aqr (see Figure 1.1). This route therefore offers a way of introducing an intrinsic spread in \(-\dot{M}_2(P)\). Generally the mass transfer rates are lower than for ZAMS secondaries, which may explain some of the dwarf novae observed at periods above the gap (Fig 1.2). In this picture many dwarf novae should show signs of chemical evolution. Evidently a good way of checking for this is to use the ratio of the ultraviolet resonance lines of carbon and nitrogen (CIV\(_\lambda 1550\) to NV\(_\lambda 1238\)) as explained above.

Descendants of thermal timescale mass transfer are expected on phase–space grounds to make up as much as 50% of short–period CVs (see Schenker et al, 2002) and so may account for the incidence of dwarf novae at such periods. The main difficulty for this type of explanation is that as seen from Fig. (1.2) there is a tendency for CVs with evolved secondaries to fill the CV period gap. A possible resolution of this difficulty is that the system moves very rapidly through such periods, but more work is needed to substantiate this.
Evolution of a model AE Aqr progenitor system. For the assumed parameters and $M_{1,\text{now}} = 0.89M_\odot$ various tracks are shown, starting from a 1.6$M_\odot$ star which has almost reached its maximum MS radius. The three panels show the evolution of the orbital period, secondary radius and WD mass for different cases of mass loss $\dot{M} = -\eta \dot{M}_2$ (dotted line: $\eta = 1$ – full line: $\eta = 0$ – dashed line: $\eta = 0.3$). The diamond marks the current position of AE Aqr. Note that the conservative model ($\eta = 1$) cannot meet the requirements for $M_1$: even starting from the lowest possible WD mass, it has grown well beyond the Chandrasekhar limit before reaching the current $P_{\text{orb}}$ of AE Aqr. The shaded area in the middle panel marks the radii of single stars during their main-sequence life as labelled. (From Schenker et al., 2002).
1.4 Dwarf novae: the occurrence of the outbursts

1.4.2 Long–period dwarf novae

We have so far discussed only short–period ($P \lesssim 12$ hr) CVs. There is a small number of systems at longer periods, the best–studied being GK Per ($P = 48$ hr). All of these systems appear to have outbursts. This is exactly what we would expect from the stability condition (1.22). As the period of a CV increases, so does the disc size $R_{\text{out}}$, roughly as the binary separation $a \propto P^{2/3}$. From (1.8, 1.22) a stable disc then requires

$$-\dot{M}_2 > M_{\text{crit}} \propto R_{\text{out}}^3 \propto P^{2}.$$  \hspace{1cm} (1.24)

At such periods mass transfer is driven by nuclear expansion of the secondary, at a rate

$$-\dot{M}_2 \propto P$$  \hspace{1cm} (1.25)

roughly (see eq.1.39 below). Hence at sufficiently long periods all systems are likely to be dwarf novae. The coefficients in (1.24, 1.25) show that this will hold for $P \gtrsim 1$ d, in agreement with observation.
Accretion in Compact Binaries

It is interesting to ask why relatively few CVs are seen at these periods. There are several effects selecting against finding them, the strongest probably being that they would only be found in outbursts, which may be rather rare. Nevertheless these systems may be important, as they may offer a channel for making Type Ia supernovae (King, Rolfe & Schenker, in prep.)

1.5 Soft X–ray transients: the nature of the outbursts

The comparative success of the disc instability idea in explaining dwarf novae, and the qualitative similarities with soft X–ray transients, make it natural to ask if the accretion discs in SXTs are subject to the same instability. The major obstacle here is the vastly different timescales noted earlier. One way of accommodating this mathematically is simply to reduce the disc viscosity in SXTs compared with dwarf novae. However there is no physical motivation for this step, which cannot be regarded as plausible.

A more likely explanation of the long timescales in SXTs uses the observed fact that the accretion discs in SXT outbursts (and indeed in persistent LMXBs) are heavily irradiated. Van Paradijs & McClintock (1994) show that the optical brightness of outbursting SXTs and persistent LMXBs correlates strongly with their X–ray luminosity, and is far greater than would be expected from local viscous dissipation within the disc (cf eqn 1.8). Irradiation thus raises the surface temperature of the disc, and may potentially stabilize it by removing its ionization zones. The early work on irradiated discs all followed van Paradijs (1996) in using the formula

\[ T_{\text{irr}}(R) = \frac{\eta M_c c^2 (1 - \beta_a)}{4 \pi \sigma R^2} \left( \frac{H}{R} \right)^n \left[ \frac{d \ln H}{d \ln R} - 1 \right], \]

(1.26)

where \( \eta \) is the efficiency of rest–mass energy conversion into X–ray heating, \( M_c \) is the central accretion rate, \( H \) the disc scaleheight at disc radius \( R \), \( \beta_a \) is the albedo of the disc faces, and the factor in square brackets lies between 1/8 and 2/7. The index \( n = 1 \) or 2 depending on whether there is a central irradiating point source or not; this is discussed further below. The ratio \( H/R \) is roughly constant in a disc, so \( T_{\text{irr}} \) falls off as \( R^{-1/2} \). Thus for a large enough disc, \( T_{\text{irr}} \) dominates the disc’s own effective temperature \( T_{\text{visc}} \), which goes as \( R^{-3/4} \). This agrees with our expectations above.

Constructing a self–consistent irradiated disc is a difficult theoretical problem, and attempts to date do not produce results in agreement with observation. In particular calculations of axisymmetric discs with irradiation (e.g. Kim et al., 1999; Dubus et al., 1999; Tuchman et al., 1990) tend to show that the inner parts of the disc expand and shadow the outer disc from the irradiation. As most of the disc mass is at large radii, this effect would prevent irradiation having a significant effect on the outbursts. However since there is abundant observational evidence that LMXB discs are strongly irradiated, it is safe to assume this; we shall return later to the question of why axisymmetric disc calculations have difficulty in reproducing this result.

Now let us consider an outburst in a soft X–ray transient. This will be triggered by the ionization instability at some disc radius, and eventually lead to matter accreting strongly on to the black hole or neutron star at the disc centre. At this point the X–ray outburst begins: the central X–ray emission irradiates the disc, and heats it. If
the disc is small enough the whole of it will now have a surface temperature $T_{\text{irr}}$ above $T_H$ and be in the hot, high-viscosity state. This reinforces the tendency of mass to accrete inwards, and reduces the local surface density $\Sigma$ on a viscous timescale. If there were no irradiation, as in a dwarf nova, $\Sigma$ would eventually at some radius reach the value $\Sigma_{\text{min}}$ where the disc must jump back (on a thermal timescale) to the cool, low-viscosity state. This in turn would trigger a cooling wave to move across the disc and return it all to the low state, ending the outburst. However if the disc is irradiated by the central X-ray source this cannot happen: the local temperature is fixed not by local viscous dissipation, but by irradiation. As this is fuelled by the central accretion rate it is a globally rather than locally determined quantity. The disc is trapped in the hot state everywhere until the central accretion rate declines to the point where irradiation can no longer keep the disc in this state. But if there is no cooling wave, the central accretion rate can only decline as a result of the accretion of a significant fraction of the disc mass. It follows that irradiation is likely to prolong disc outbursts and make them use up more of the disc mass. For a given mass transfer rate, the latter effect will lengthen the quiescent intervals in which the disc mass is rebuilt by accretion from the companion star.

This line of reasoning allowed King & Ritter (1998) to give a simple explanation of why SXTs have much longer outburst and quiescent timescales. Their treatment also shows that the X-ray light curves are likely to have certain characteristic shapes. An outbursting disc is approximately in a steady state with surface density

$$\Sigma(R) \simeq \frac{\dot{M}_c}{3\pi \nu}$$

(1.27)

where $\dot{M}_c$ is the central accretion rate. Integrating this gives the total initial mass of the hot zone as

$$M_h = 2\pi \int_0^{R_h} \Sigma R dR \simeq \frac{\dot{M}_c R_h^2}{3\nu}$$

(1.28)

since the inner disc radius is much smaller than the outer radius $R_h$ reached by the heating front. In (1.28) $\nu$ is some suitable average of the kinematic viscosity in the disc, close to its value near $R_h$. Note that (1.28) is effectively a dimensional relation, and simply asserts the obvious fact that the mass of a steady disc is given by the product of the accretion rate and the viscous time at its outer edge: it does not for example assume that $\nu$ is constant through the disc.

As we reasoned above, the only way in which the mass of the hot zone can change is through central accretion, so we have $\dot{M}_c = -\dot{M}_h$, and

$$-\dot{M}_h = \frac{3\nu}{R_h^2} M_h$$

(1.29)

or

$$\dot{M}_h = M_0 e^{-3\nu t / R_h^2},$$

(1.30)

where $M_0$ is the initial mass of the hot zone. This in turn implies that the central accretion rate, and thus the X-ray emission, decays exponentially, i.e.

$$\dot{M}_c = \frac{R_h^2 M_0}{3\nu} e^{-3\nu t / R_h^2},$$

(1.31)
Note that the peak accretion rate at the start of the outburst can be expressed as 
\( \frac{\text{disc mass}}{\text{viscous time of the entire hot disc}} \). If the quiescent disc is close to the 
maximum mass allowed before becoming unstable, this peak rate depends only on 
the disc size: King & Ritter (1998) find
\[
\dot{M}_c(\text{peak}) \simeq 4.8 \times 10^{-8} R_{11}^{7/4} M_\odot \text{ yr}^{-1}
\]  
for small (fully irradiated) discs, and
\[
\dot{M}_c(\text{peak}) \simeq 4.1 \times 10^{-8} R_{12}^{7/4} M_\odot \text{ yr}^{-1}
\]  
for large discs, where \( R_{11}, R_{12} \) are the disc radii in units of \( 10^{11} \text{ cm} \) and \( 10^{12} \text{ cm} \) respectively. These expressions agree with the fact that most SXT outbursts are 
observed to be close to the Eddington luminosity, even for cases where the accretor 
is a \( \sim 10 M_\odot \) black hole. The decay constant for outbursts is
\[
\tau \simeq 40 R_{11}^{5/4} \text{ d} \simeq 2 R_{12}^{5/4} \text{ yr},
\]  
showing that outbursts can be very prolonged in wide systems.

Eventually \( \dot{M}_C \) drops to the point that irradiation cannot keep the outer edge of the 
disc in the hot state. The outburst proceeds nevertheless, as the central irradiation 
keeps the inner disc regions ionized. A simple calculation shows that the X–rays then 
decay linearly rather than exponentially, still on the hot–state viscous timescale. A 
sufficiently large disc cannot be kept in the hot state by irradiation even at the start 
of the outburst, and so is always in the linear regime.

A more exact treatment (King, 1998) solves the diffusion equation (1.4) for an 
irradiated disc. The solutions predict steep power–law X–ray decays \( L_X \sim (1 + 
t/t_{visc})^{-4} \), changing to \( L_X \sim (1 - t/t'_{visc})^{4} \) at late times, where \( t_{visc}, t'_{visc} \) are viscous 
timescales. These forms closely resemble the approximate exponential and linear 
decays inferred above in these two regimes. It is important to realise that the decays 
are quite different than for unirradiated discs because the viscosity is a function of 
the central accretion rate rather than of local conditions in the disc.

Since disc size scales with the binary separation, and thus as \( P^{2/3} \) by Kepler’s law, 
we arrive at a simple picture in which SXT outbursts in short–period systems begin 
as exponential, becoming linear near the end of the outburst. In long–period systems 
the outbursts may be linear throughout. These basic features are generally, but not 
universally, found in observations. Thus short–period systems such as A0620–00 
(\( P = 7.8 \text{ hr} \)) have classic ‘FRED’ (fast rise, exponential decay) light curves (see 
Ch ** of this book), whereas GRO J1744–28 (with \( P = 11.8 \text{ d} \) one of the longest– 
period SXTs) has an entirely linear decay (Giles et al., 1996). The exponential–linear 
dichotomy is examined in detail by Shahbaz et al. (1998), and is in good agreement 
with observation.

While this simple picture is largely correct, there are a number of complications 
which we should address. We return first to the problem mentioned above, namely 
that axisymmetric calculations of irradiated discs tend to produce configurations 
which are strongly self–shadowed, and thus very unlike what is observed. The resolu-
tion of this difficulty comes from the realisation (Pringle, 1996) that as discussed 
in Section 2.5 above, strong self–irradiation causes an accretion disc to warp in a
1.5 Soft X–ray transients: the nature of the outbursts

Fig. 1.3. Simulated outburst of a soft X–ray transient showing a secondary maximum, here after \( \sim 26 \) d (Truss et al., 2002). Note that the slopes of the light curve are similar each side of this maximum.

non–axisymmetric fashion, with irradiation possible at all radii. Even though this irradiation is relatively patchy (see Fig. 7 of Pringle, 1997) it is likely to keep the disc ionized if it is intense enough, i.e. if \( T_{\text{irr}} > T_H \), because recombination times are long compared with the dynamical time. This suggests a reason why irradiation seems to be unreasonably effective, as discussed above. A second consequence of warping will be very important later on, namely that the radiation field must become quite anisotropic. This follows from considering the disc angular momentum vectors \( j(R) \) measured wrt the accreting object. An unwarped disc has all the \( j(R) \) vectors parallel to that of the binary, \( J_{\text{orb}} \). However, the \( j(R) \) are clearly scrambled in many directions once warping has taken place. Since matter joined the disc with \( j(R) \) parallel to \( J_{\text{orb}} \), the radiation field must have changed its own (originally zero) angular momentum to compensate. This must mean that it is anisotropic, and indeed this is what Pringle’s calculations reveal (Pringle, 1997; Fig. 7). In general the radiation field at infinity is confined to a fairly narrow double cone.

In addition to the exponential or linear X–ray light curves discussed above, many SXT outbursts show a secondary maximum (factors of a few) in their lightcurves once the X–rays have declined by a few e-folds. This increase signals the accretion of a new source of mass, which is rather smaller than the original heated disc region giving rise to the basic light curve shape. There have been several attempted explanations of this, sometimes invoking extra mass transfer from the secondary star, but a recent
Accretion in Compact Binaries

2-D simulation of SXT outbursts (Truss et al., 2002) reveals a likely cause. The simulations confirm the simple picture described above as the cause of the main outburst. The secondary maximum results from two effects not included in the arguments above. First, even in a disc irradiated to its outer edge, some of the matter at the outer disc rim will remain in the cool state, because the outer edge of the disc flares and shields it. Second, the mass ratios in all SXTs allow the disc to reach the 3:1 resonant radius. This is fairly obvious for black-hole systems, where \( M_1 \sim 5 - 10 M_\odot \), but we will see shortly that it is true of neutron-star transients also.

The result of including these two effects in the simulations is to produce a second, superoutburst-like increase in the accretion rate through the disc (Fig. 1.3), aided by the strong tidal torque at the resonant radius.

The main effect causing a deviation from linear decays in long-period systems is that by definition, such systems can have a large fraction of their disc mass permanently in the low state. This mass reservoir can give rise to hysteresis effects, distorting the simple picture predicting linear decays. The mass reservoir can also allow bursts to recur more frequently than might be expected. For example, GRO J1744–28 was completely undetected in \( \sim 30 \) yr of X-ray astronomy until its first outburst, but then had another outburst only a few months later. For very wide systems the outbursts can also be very long, and indeed some LMXBs usually classified as 'persistent' may actually be in outbursts which have lasted for the entire history of X-ray observations. Proof of this comes from the longest-period SXT currently known, GRS 1915+105, which again was not detected until it went into outburst in 1992. With some variability it has remained bright ever since, showing that outbursts in wide systems can certainly last for at least a decade.

The binary period of 33.5 d and mass \( \sim 15 M_\odot \) (Greiner et al., 2001) show that the disc radius here must be \( \sim 4 \times 10^{12} \) cm, implying from eq. (1.38) below that \( M_{\text{disc, max}} \sim 7 \times 10^{29} \) g. Such a disc could supply the inferred outburst accretion rate \( \sim 10^{19} \) g s\(^{-1}\) for more than \( 10^3 \) yr. This is far longer than the decay constant \( \tau \sim 10 \) yr (eq. 1.34). GRS 1915+105 provides an explicit example of the complex behaviour of long-period SXTs, as it is observed to vary on the timescale \( \tau \) but maintains its outburst for longer.

An obvious corollary of these ideas is that there must exist a large unseen population of quiescent transients which have never been observed to have an outburst. In the next subsection we will see that there is strong evidence for the existence of such objects. Taken together, all these effects show that there is a general tendency for outbursts in long-period systems to show a more complex variety of behaviours than in short-period systems.

As mentioned above, all SXTs have mass ratios allowing the disc to reach the 3:1 resonant radius, and thus one might expect superhumps to appear. A survey of the optical data on these systems (O'Donoghue & Charles, 1996) concluded that indeed superhumps are observed in them. However at first sight this seems paradoxical, as we have asserted above that superhumps are a modulation of the viscous dissipation in precessing discs, but also that irradiation completely outweighs this dissipation in LMXB (including SXT) discs. The resolution of this problem is that the superhump modulates not only the disc dissipation, but also the disc area (Haswell et al., 2001).
Since a larger area intercepts and reradiates a large fraction of the central X-rays, there should indeed be a superhump modulation of the optical light. In fact for constant X-ray luminosity the optical light curve should simply map the variation of the disc area (Fig. 1.4). The resulting curve agrees well with observation. In general the optical light curve is the convolution of the X-ray light curve with this area variation.

1.6 Soft X-ray transients: the occurrence of the outbursts

We should next consider the question of when SXT outbursts occur. Here there have been major advances since the last edition of this book. It is now clear
that outbursts are extremely prevalent among LMXBs, so that if anything, persistent systems are rather the exception. This realization has in turn had important consequences for our understanding of compact binary evolution.

These advances stem from van Paradijs’s (1996) realisation that the correct condition for LMXBs to be stable against outbursts (and thus appear as persistent systems) is

\[ T_{\text{irr}}(R_{\text{out}}) > T_H \]

rather than (1.22). In his paper van Paradijs (1996) used the observed X–ray luminosities \( L_X \) to replace the combination \( \eta M_c c^2 \) in the expression (1.26) for \( T_{\text{irr}} \) (with \( n = 1 \), appropriate for a strong central irradiating source). He was able to demonstrate that indeed the condition (1.35) correctly divides persistent LMXBs from transient systems.

This success means that we can now use the condition (1.35) with \( T_{\text{irr}} \) calculated from the evolutionary mean mass transfer rate rather than the observed \( L_X \), i.e. with \( \dot{M}_c \) replaced by \( -\dot{M}_2 \). The results are revealing.

1.6.1 Short–period SXTs

For short orbital periods \( P < 12 \) hr mass transfer must be driven by angular momentum loss, and we might expect the secondary stars to be unevolved low–mass main–sequence stars. Now Fig. (1.5) shows that if the secondaries in either neutron–star or black–hole LMXBs are unevolved main–sequence stars, all systems are predicted to be persistent. But observation (see Ch. **) shows that most, if not all, short–period black–hole LMXBs are transient, and there are also a number of neutron–star SXTs at such periods.

One possible way of avoiding this conclusion, at least for black-hole systems, is the idea that the irradiation effect might be weaker for a black–hole accretor. Shakura & Sunyaev (1973) pointed out that if there is no strong central source, the irradiation comes only from central disc regions lying in the orbital plane, reducing its effect by a second projection angle \( \sim H/R \). Thus the value \( n = 2 \) would be appropriate in (1.26). King et al. (1997) found that indeed this would make black–hole LMXBs transient even with unevolved secondary stars. However the X–ray spectra of most black–hole systems have a strong power–law component, which is usually thought to come from a corona. In this case it is unlikely that the weaker irradiation law with \( n = 2 \) in (1.26) is appropriate for determining disc stability.

If this explanation is abandoned, it seems that the inevitable conclusion (King et al., 1996) is that despite appearances, the secondary stars are actually chemically evolved in a large fraction of short–period LMXBs – if not a majority or even a totality in the black–hole case. This must mean that they descend from stars with initial masses greater than \( 0.8M_\odot \), and have had time to evolve away from the ZAMS before mass mass transfer driven by angular momentum loss has pulled them in to short orbital periods. Figures 1.6 and 1.7 show explicitly that this kind of evolution does produce short–period transient systems.

This shift of view parallels the similar shift in our view of CVs (see subsection (1.4.1) above), where we now believe that there is a significant admixture (\( \lesssim 50\% \)) of evolved secondaries in CVs which have descended via the thermal–timescale mass
1.6 Soft X–ray transients: the occurrence of the outbursts

Fig. 1.5. Mass transfer rate versus orbital period for LMXBs with various primary masses and an unevolved secondary of initial mass $1M_\odot$ (King et al., 1996). The evolution is driven by magnetic braking as long as the secondary has a convective core. The dotted and dashed lines are the critical mass transfer rates given by (1.22) and (1.35) for standard and irradiated discs respectively. Systems with lower mass transfer rates are transient. 

- Top panel: $1.4M_\odot$ neutron star primary.
- Middle panel: $10M_\odot$ BH primary.
- Bottom panel: $50M_\odot$ primary. All systems are stable according to the irradiated-disc criterion (1.35). This shows that short–period transients must have evolved secondaries.
Accretion in Compact Binaries

Fig. 1.6. Mass transfer rate $-\dot{M}_2$ versus orbital period $P$ for neutron–star LMXBs ($M_1 = 1.4M_\odot$). The curve starting near $P = 8.5$ h represents evolution under magnetic braking and gravitational radiation, beginning with an unevolved secondary star. The mass transfer rate falls to zero in a period gap $2.3 \lesssim P \lesssim 2.8$ h.

The second $-\dot{M}_2(P)$ curve begins with a slightly evolved secondary, initially more massive (2.35$M_\odot$) than the neutron star. This undergoes thermal–timescale mass transfer (the horizontal line shows the Eddington limit) before reaching short periods with a lower mass transfer rate than the standard evolution. The dashed curves show the disc stability criterion $T_{\text{rim}}(R_{\text{out}}) > T_H$. These differ as the two evolutions have different $R_{\text{out}}$. The unevolved system is always persistent, while the system with the evolved secondary is transient in the period range $5 - 10$ hr. Compare Fig. (1.2). Figure by Klaus Schenker.

transfer route. But the shift for short–period LMXBs may be more drastic: it could be that all black–hole LMXBs have evolved secondaries, as so far there is no convincing evidence that any black–hole LMXB has an unevolved secondary. Black–hole systems with main–sequence companions would appear among the handful of persistent LMXBs which do not have Type I X–ray bursts (signalling the presence of a neutron star). They are clearly difficult to identify, as in persistent systems one cannot measure a mass function and so get a dynamical mass estimate. However we will see that even some persistent LMXBs probably have evolved secondaries.

An independent line of argument leading to a similar conclusion comes from the period distribution of LMXBs (Fig 1.8). This shows no sign of the familiar CV period gap between 2 and 3 hours orbital period. This suggests that in LMXBs accretion either does not cease at 3 hr, or does so in a narrower period range, which may also differ for individual systems. This is just what we see in Figs. (1.6) and (1.7),
1.6 Soft X-ray transients: the occurrence of the outbursts

Fig. 1.7. As for Figure 1.6, but for black-hole LMXBs \( (M_1 = 7M_\odot) \). The curve starting near \( P = 11.8 \) h represents evolution under magnetic braking and gravitational radiation, beginning with an unevolved secondary star of mass \( 1.5M_\odot \). The curve starting near \( P = 13 \) h shows the evolution starting with a significantly nuclear-evolved secondary of \( 1M_\odot \). As before the dashed curves are the stability criterion \( T_{\text{irr}}(R_{\text{out}}) > T_{\text{H}} \) in the two cases. Again only the system with the evolved secondary is transient. Figure by Klaus Schenker.

showing the mass transfer rates for neutron-star and black-hole LMXBs which have reached short periods with secondaries having some degree of nuclear evolution.

Confirmation of these ideas comes from observations of the short-period black-hole SXT XTE J1118+480 (Haswell et al., 2002). The Hubble Space Telescope ultraviolet spectrum of this object shows extremely strong NV\( \lambda 1238 \), but CIV\( \lambda 1550 \) is completely undetectable. As with AE Aqr (see above) this is a clear sign of CNO processing. The secondary star must have been more massive \( (\gtrsim 1.5M_\odot) \) in the past than the present binary period (4.1 hr) would suggest. It was evidently somewhat chemically evolved when mass transfer began. For much of its life this would not have been obvious from the composition of its outer layers and thus of the mass visible in emission lines: Fig. (1.9) shows that this would have appeared completely normal until the secondary mass got low enough for convection to mix the processed layers into the envelope.

Given the pervasive evidence that short-period SXTs have evolved companions we should now ask why this is so. In the black-hole case the answer seems to go back to evolution of the evolution of the binary as the black-hole progenitor expands off the main sequence. There are two cases. The binary may be so wide at this stage that the companion never interacts with the primary’s envelope as this expansion occurs. Such systems clearly do not reach contact and produce a BHLMXB until
the companion has expanded off the main sequence, if at all. These systems thus cannot produce such binaries with unevolved companions. If on the other hand the binary is relatively narrow, so that the primary’s envelope engulfs the companion as it expands, we must ask if the latter can survive this common-envelope (CE) evolution. Friction between the star and the envelope will remove orbital energy and help to unbind the envelope. If the black-hole progenitor, its compact He core and its envelope have masses $M_{1p}$, $M_c$, $M_e$, and $R_{1p}$ is its radius, then equating the loss of orbital energy to the binding energy of the envelope in the standard way shows that

Fig. 1.8. The cumulative period histogram below $P = 12$ hr for LMXBs (solid) compared with that for CVs (dashed). Also shown are the histograms for black hole LMXBs (dotted) and neutron-star LMXBs (dot-dashed). The CV histogram clearly shows a flatter slope between period $P = 3$ and 2 hours, corresponding to the well-known period gap. There is no significant evidence for such a feature in the LMXB histogram, which is consistent with a uniform distribution. Neutron-star and black-hole systems appear to have significantly different period distributions. Data from Ritter & Kolb, 2003. Figure by Klaus Schenker.
1.6 Soft X–ray transients: the occurrence of the outbursts

Fig. 1.9. Evolution of the surface abundance ratio C/N for a binary beginning mass transfer from a 1.5\(M_\odot\) main sequence star on to a 7\(M_\odot\) black hole. The C/N ratio remains at the cosmic value (\(\log(C/N) \simeq 0.5\)) until the companion mass \(M_2\) is reduced below 0.8\(M_\odot\). Until this point chemical changes are confined to the interior: at the turn–on period of 15 hr the core hydrogen fraction had already been reduced to 28 %. The current period of XTE J1118+480 is reached at a mass of 0.33\(M_\odot\), indicated by the left vertical line, while the other near 0.6\(M_\odot\) shows the period of A0620-00. The transferred mass has been accreted by the black hole which has grown beyond 8\(M_\odot\). Figure from Haswell et al., 2002.)

\[ \frac{GM_{1p}M_e}{\lambda} = \alpha \left( \frac{GM_cM_{2i}}{2a_f} - \frac{GM_{1p}M_2}{2a_0} \right) \] (1.36)

where \(M_{2i}\) is the companion mass at this point, \(a_0, a_f\) are the initial and final orbital separations, and \(\lambda, \alpha\) are the usual weighting and efficiency parameters. For a low–mass companion we have \(M_{2i} \ll M_{1p}, M_c, M_e\), and the final separation obeys

\[ a_f < \frac{\alpha \lambda}{2} \frac{M_c}{M_e} \frac{M_{2i}}{R_1} \ll R_1. \] (1.37)
Accretion in Compact Binaries

Fig. 1.10. The evolution of short–period neutron–star LMXBs. The initial companion mass is $2.35\text{M}_\odot$, with core hydrogen fraction 35%. The system first goes through a phase of thermal–timescale mass transfer ($M_2 \gtrsim 1.5\text{M}_\odot$ in lower rh panel, $P \sim 17 - 20$ hr in lower lh panel). After this phase the mass transfer rate drops below the Eddington limit and the neutron star begins to grow significantly in mass (upper rh panel). At periods between 10 and 5 hr the system becomes transient (shaded region in lh panels). The spectral type of the secondary (solid curve in top lh panel is close to that which a ZAMS secondary would have (light curve) at these periods. Varying the mass and initial degree of evolution of the secondary at the onset of mass transfer produces a family of curves displaced horizontally from the solid curve. The squares show persistent neutron–star LMXBs and the diamonds are neutron–star transients.

In general the final separation is too small for the companion star to fit inside it, so the stars merge rather than forming a binary. The small mass ratio meant that there was insufficient orbital energy available to eject the black–hole progenitor’s envelope. Black–hole X–ray binaries must have initial mass ratios $q_i = M_2 i / M_1$ above some limiting value, and thus post–CE secondary star masses $M_2 i$ above some minimum value ( The uncertainties inherent in any discussion of CE evolution do not allow a precise answer, but it is plausible that this effect limits $q_i$ to values $\gtrsim 0.1$ and thus $M_2 i$ to values $\gtrsim 1\text{M}_\odot$. This creates a real possibility that this star has time to undergo significant nuclear evolution before getting into contact and starting to transfer mass (see Fig. 1.11). The first paper to consider this type of evolution was by Pylyser & Savonije (1988).

After the CE phase, the fate of the binary is determined by a competition between various processes which all tend to bring the secondary into contact. The first is nuclear evolution: the star is changing its core composition and eventually increasing its radius on a nuclear timescale $t_{\text{nuc}}$. The other timescales $t_{\text{MB}}, t_{\text{GR}}$ describe
1.6 Soft X–ray transients: the occurrence of the outbursts

Fig. 1.11. Comparison of timescales for a 7\textit{M}_\odot black hole primary. The full curve gives the nuclear timescale as a function of the secondary mass. At various points the orbital period for a ZAMS star filling its Roche lobe is indicated. The other 3 curves show the various relevant angular momentum loss timescales \((\frac{-d\ln J}{dt})^{-1}\): for gravitational radiation (GR, dashed) and magnetic braking according to Mestel & Spruit (M&S, dash-dotted) and Verbunt & Zwaan (V&Z, dash-triple-dotted), all in the version of Kolb (1992) Magnetic braking is assumed to be quenched in stars which have no convective envelopes, i.e for \(M_2 \gtrsim 1.5\textit{M}_\odot\).

orbital angular momentum loss via magnetic braking and gravitational radiation respectively, which shrink the binary separation. If \(t_{\text{nuc}}\) is always shorter than the other timescales, the companion reaches contact as it expands away from the main sequence, and the binary evolves towards longer periods \(P \gtrsim 1–2\ d\) with mass transfer driven by nuclear expansion. This is the origin of the long–period LMXBs we shall discuss in the next Section. If instead the timescale for orbital angular momentum loss (in practice \(t_{\text{MB}}\)) is shorter than the nuclear timescale \(t_{\text{nuc}}\), the binary shrinks and reaches contact at a period of a few hours. It becomes a short–period LMXB, with mass transfer driven by angular momentum loss as discussed earlier in


Accretion in Compact Binaries

this Section. However \( t_{\text{nuc}} \) is only slightly longer than \( t_{\text{MB}} \) for the masses \( M_{2i} \gtrsim 1M_\odot \) we have inferred for black–hole binaries. (Note that magnetic braking is probably ineffective [i.e. \( t_{\text{MB}} \rightarrow \infty \)] for masses \( M_{2i} \gtrsim 1.5M_\odot \).) If the post–CE separation is narrow (i.e. just wide enough to avoid a merger) the binary will reach contact with the secondary still on the main sequence, and produce a persistent \( \sim \text{BH + MS system} \) (Fryer & Kalogera, 2001). Such special initial conditions must make these systems intrinsically rare. In addition they will be very hard to identify, as it is difficult to measure a dynamical mass in a persistent system; they could lurk undetected among LMXBs which do not show Type I X–ray bursts. For the majority of systems, the shrinkage towards contact after the CE phase takes a noticeable fraction of \( t_{\text{nuc}} \), and the companion is likely to be significantly nuclear–evolved when it comes into contact. At this point further nuclear evolution is frozen, as the star is losing mass on a timescale \( (t_{\text{MB}} \approx t_{\text{GR}}) \) which is shorter than \( t_{\text{nuc}} \). This argument suggests that we should expect most short–period black–hole LMXBs to have chemically evolved secondaries, accounting for the high fraction of transients among them.

For short–period neutron star LMXBs the question is obviously more delicate: a majority of them appear to be persistent, and thus could have completely unevolved companions, but there is a non–negligible fraction of transients. It is now well understood (Kalogera & Webbink, 1996) that the formation of a neutron–star LMXB is an extremely rare event, requiring highly constrained initial parameters. This results chiefly from the requirement to keep the binary intact when the neutron–star progenitor explodes. King & Kolb (1997) nevertheless found that a significant fraction of short–period neutron–star binaries would have evolved companions and thus be transient if the supernova explosion was assumed fairly symmetrical, and Kalogera et al. (1998) extended this result to the case of significantly anisotropic supernovae in which the neutron star receives a kick.

1.6.2 Post–minimum SXTs

We have seen that a large fraction of neutron–star LMXBs apparently have fairly unevolved companions, and are persistent systems in the \( \sim 2–10 \, \text{hr} \) period range as expected from Fig. (1.5). However this Figure also shows that such systems are likely to become transient after passing the predicted minimum period \( \sim 80 \, \text{min} \) for this type of ‘CV–like’ binary evolution. In the CV case, post–minimum systems are generally regarded as undetectable owing to their faintness. However straightforward application of the ideas of the last two Sections shows that post–minimum NSLMXBs could very well be detectable (King, 2000). They are likely to have outbursts reaching \( \sim 10^{37} \, \text{erg s}^{-1} \) before declining on an e–folding timescale of a few days. In effect, these systems bring themselves to our notice by saving up their rather feeble mass transfer rates \( \sim 10^{-11}M_\odot \, \text{yr}^{-1} \) and using them to produce X–rays for only about 1% of their lifetimes, but with of course 100 times the luminosity. The population of faint transients found by BeppoSAX in a \( 40^\circ \times 40^\circ \) field around the Galactic centre may be drawn from this group. Almost all of these are known to contain neutron stars, signalled by Type I X–ray bursts. Further systematic study of this population may have much to tell us about binary evolution in the Galaxy.
1.6 Soft X–ray transients: the occurrence of the outbursts

1.6.3 On/off transients

Some short–period neutron–star SXTs show variability unlike any other LMXBs. In particular at least two short–period transients, EXO 0748–68 and GS 1826–24 seem essentially to have simply ‘turned on’, i.e. they were undetected for the first ∼ 30 yr of X–ray astronomy, but have remained ‘on’ ever since their discovery. In compensation, several short–period transients, e.g. X2129+470, X1658-298, have also been observed to turn off during the same time. These on/off transitions cannot result from disc instabilities, as the mass involved is too great. Taking the bolometric luminosity of X0748-678 as \( \sim 10^{37} \) erg s\(^{-1}\), a 10% efficiency of rest–mass energy conversion requires its neutron star to have accreted \( \sim 5 \times 10^{25} \) g since 1985 when it was observed to turn on. However the maximum surface density allowing a disc to remain in the quiescent state leads to a maximum possible quiescent disc mass

\[
M_{\text{disc}, \text{max}} \sim 10^{-8} R^3 \text{ g}
\] (1.38)

(e.g. King & Ritter, 1998) where \( R \) is the outer disc radius in cm. The 3.82 hr period implies a total binary separation of only \( 9 \times 10^{10} \) cm, and thus \( R \lesssim 5 \times 10^{10} \) cm (assuming a disc filling 90% of the Roche lobe, with a conservative mass ratio \( M_2/M_1 > 0.1 \)). From (1.38) this gives \( M_{\text{disc, max}} \lesssim 1.3 \times 10^{24} \) g, far smaller than the mass accreted since 1985. This can only have come from the companion star, implying stable disc accretion during the ‘on’ state.

The fact that these systems are all known (from the presence of Type I X–ray bursts) to contain neutron stars offers a suggestive answer. We see from Fig. (1.10) that even neutron–star systems with quite evolved secondaries only just contrive to lower their mass transfer rates sufficiently to become transients, typically in the period range 5 – 10 hr. Evidently some of these systems make transitions across this stability curve for intervals which are shortlived in evolutionary terms, but long enough to account for the observed turn–on as a persistent LMXB. This is quite similar to the behaviour of a group of CVs known as Z Cam stars, where dwarf nova behaviour is from time to time suspended as the systems enters a ‘standstill’ (see e.g. Warner, 1995). This behaviour can plausibly be ascribed (e.g. King & Cannizzo, 1998) to starspot activity on the secondary star; during outbursting epochs, starspots block enough of the mass transfer region near the inner Lagrange point that the mass transfer rate is reduced to a value slightly below the critical one for disc instability (for the unirradiated discs in CVs). During standstills, enough of these starspots disappear that the mass transfer rate now reaches the regime for stable disc accretion.

The secondaries in short–period LMXBs are probably magnetically active like those in CVs, so it is plausible that a similar effect could cause LMXBs to move between transient and persistent behaviour. However, since such a standstill can only occur after an outburst has triggered the transition to the hot disc state, they must be separated by the usual very long quiescent intervals. If the standstills are themselves also very prolonged, and the SXT outbursts during the low mass transfer rate phase are rare or faint, the long–term X–ray behaviour of these systems will consist essentially of ‘off’ and ‘on’ states, with only short transitions (‘outbursts’ and ‘decays’) between them. This on–off behaviour is of course just what observed
Accretion in Compact Binaries

Table 1.1. Transient and persistent behaviour among LMXBs as predicted by the disc instability picture

<table>
<thead>
<tr>
<th>accretor</th>
<th>companion</th>
<th>$P \lesssim 12$ hr</th>
<th>$P \gtrsim 12$ hr</th>
</tr>
</thead>
<tbody>
<tr>
<td>neutron star unevolved</td>
<td>persistent NSLMXB, faint SXT after minimum period</td>
<td></td>
<td></td>
</tr>
<tr>
<td>neutron star evolved</td>
<td>persistent NSLMXBs plus some SXTs and on/off transients for $P \sim 5 - 10$ hr</td>
<td>SXT unless $M_2 \gtrsim 0.8 - 0.9M_\odot$, progenitors of wide PSR binaries</td>
<td></td>
</tr>
<tr>
<td>black hole unevolved</td>
<td>formation very rare; persistent BHLMXB</td>
<td></td>
<td></td>
</tr>
<tr>
<td>black hole evolved</td>
<td>SXT</td>
<td>SXT, microquasar</td>
<td></td>
</tr>
</tbody>
</table>

for several of these systems. Accordingly we identify the on–off transients of Table 1 as the LMXB analogues of the Z Cam systems.

1.6.4 Long–period SXTs

For orbital periods $\gtrsim 1 - 2$ d mass transfer in LMXBs must be driven by the nuclear expansion of the secondary. This star is a low–mass subgiant or giant whose radius and luminosity are determined almost purely by the mass of its helium core, quite independently of its total mass. A simple analytic prescription for this (Webbink, Rappaport & Savonije, 1983; King, 1988; see also Ritter, 1999) shows that the mass transfer rate is

$$-\dot{M}_2 \simeq 4.0 \times 10^{-10} P_0^{0.93} m_2^{1.47} M_\odot \text{ yr}^{-1}$$

(1.39)

where $P_0$ is the orbital period measured in days and $m_2$ the total secondary mass in $M_\odot$. Using this together with the stability criterion (1.35) King et al. (1997) showed that most such systems must be transient according to the criterion (1.35): see Fig. 1.12. The only exceptions to this statement are neutron–star systems where the secondary has lost relatively little of its envelope and still has a mass $M_2 \gtrsim 0.8 - 0.9M_\odot$. The reason for this propensity of long–period systems to be transient is clear: the accretion disc is so large that central irradiation cannot keep its outer edge ionized.

This result has a number of important consequences. First, taken together with the results of the previous subsection, it shows that persistent sources are if anything the exception among LMXBs, being largely confined to a group of short–period neutron–star sources with fairly unevolved companions (see Table 1.1).

Second, we see from (1.39) that essentially all LMXBs with periods longer than a day or so must have mass transfer rates $\gtrsim 10^{-10}M_\odot \text{ yr}^{-1}$. In transients, almost all of this mass must attempt to accrete on to the neutron star or black hole during
outbursts. With typical duty cycles $\lesssim 10^{-2}$ it follows that the outburst accretion rates must be $\gtrsim 10^{-8} M_\odot \text{yr}^{-1}$, and so at or above the Eddington limit for a neutron star. For longer orbital periods or smaller duty cycles the rates are still higher, and will in general reach those corresponding to the Eddington limit for a black hole. These predictions agree with observations of SXT outbursts: for example
V404 Cygni ($P_d = 6.47$) had a peak outburst luminosity $\sim 10^{39}$ erg s$^{-1}$ (see Tanaka & Lewin, 1995).

Third, the endpoint of this binary evolution is detectable in many cases, and gives us a test of the theory outlined here. As the companion burns more hydrogen in the shell source surrounding its helium core, the mass of the latter grows, expanding the envelope, increasing the binary period, and driving mass transfer at an increasing rate (cf eq 1.39). Eventually all of the envelope mass will be used up, some added to the helium core but most transferred to the compact accretor. We are left with a binary consisting of the low–mass helium white dwarf core of the companion in a wide orbit with the ‘bare’ accretor. The orbital period is given directly by the helium core mass, which specifies the envelope size just before the latter was lost, and thus the period via Roche geometry (Savonije, 1987). If the accretor was a neutron star it may have been spun up by accreting angular momentum along with mass. This spinup can cause the neutron star to turn on again as a radio pulsar, a process known as recycling (Radhakrishnan & Srinivasan, 1982). However if almost all the mass reaches the neutron star in super–Eddington outbursts, the efficiency of both mass and angular momentum gain will be extremely low. This effect may prevent the neutron star spinning up to millisecond periods in systems with a final period $\gtrsim 100$ d if the duty cycle is $\lesssim 10^{-2}$ (Li & Wang, 1998; Ritter & King, 2001). This may account for the otherwise surprisingly slow spin rates of some pulsars in long–period binaries. In addition it appears that there are no millisecond pulsars in wide circular binaries with periods $\gtrsim 200$ d (see e.g. Table 1 in Taam et al., 2000). However the dearth of such binaries may also reflect formation constraints (Willems & Kolb, 2002).

Fourth, from the arguments above it appears that radio pulsars in wide circular binaries must descend from transients with shorter orbital periods. Yet although at least 10 pulsar binaries with periods longer than 50 d are known (Taam et al., 2000), we do not know of a single neutron–star SXT with an orbital period longer than 11.8 d (GRO J1744-28), even though we would have expected X–ray satellites to see outbursts from such systems anywhere in the Galaxy within the last 30 yr. One obvious reason for this is that the outbursts are very rare. If this is the sole reason for our failure to see outbursts, Ritter & King (2001) estimate that the recurrence times of the outbursts must be at least 300 yr, and probably considerably longer. The idea of such long recurrence times gets strong independent support from observations of the quiescence of the long–lasting transient KS 1731-260 (Wijnands et al. 2001, Rutledge et al. 2002). Here the neutron star is seen to be so cool that a considerable time, perhaps $10^3$ yr, must elapse between outbursts.

1.6.5 Transient outbursts in high–mass systems

So far this Section has dealt with outbursts in LMXBs. In general outbursts do not occur in high–mass X–ray binaries. The obvious reason for this is that the companion star is itself a potent ionization source, and is able by itself to keep the accretion disc in the hot state. If it has effective temperature $T_*$ and radius $R_*$ then the irradiation temperature on the surface of a disc element at distance $R >> R_*$ from it is
\[ \left( \frac{T_{\text{irr}}}{T_*} \right)^4 \approx \frac{2}{3\pi} \left( \frac{R_*}{R} \right)^3 (1 - \beta) \]  

(1.40)

where \( \beta \) is the albedo (e.g. Frank et al., 2002, eq. 5.103; note that the star is an extended source of irradiation since its radius \( R_* \) is much larger than the local scaleheight \( H \) of the disc). For a disc around a compact star orbiting the massive star in a circular orbit this gives

\[ T_{\text{irr}} = 6900T_{30}^{3/4} R_{10}^{1/4} M_{10}^{-1/4} P_{10}^{-1/2} \text{ K} \]  

(1.41)

where \( T_{30}, R_{10}, M_{10}, P_{10} \) are \( T_*, R_*, \) and the binary total mass \( M \) and period \( P \) in units of \( 3 \times 10^4 \) K, \( 10R_\odot \), \( 10M_\odot \) and \( 10 \) d respectively, and we have taken \( (1 - \beta)^{1/4} \approx 1 \).

Comparing \( T_{\text{irr}} \) with \( T_H \approx 6500 \) K, this equation suggests that outbursts will be suppressed in HMXBs with O or early B primaries \( (T_* \gtrsim 30,000 \) K, \( R_* \sim 20-30R_\odot) \) unless the binary period is longer than \( \sim 10 \) d. This agrees with the fact that outbursts are not seen in most supergiant X–ray binaries with known orbital periods.

However outbursts may be possible for systems with longer orbital periods or high eccentricities \( e \), since at apastron a factor \( (1 + e)^{-3/4} \) appears on the rhs of eq. (1.41). Both possibilities occur in Be X–ray binaries. Here the accretion disc is replenished by a burst of mass transfer as the accretor (apparently always a neutron star) passes close to the Be star’s equatorial disc. This burst, or the change in the gravitational potential felt by the disc, is probably the cause of the outbursts usually observed near periastron. Evidently there might in some cases be a second outburst near apastron as the disc is allowed to cool. However the accretion disc here is clearly not in a steady state, so numerical simulations will be needed to check this idea.

### 1.7 Quiescent transients and black hole horizons

One of the main motivations for studying accretion flows is to learn more about the accreting objects. We have seen in the earlier Sections that there are very strong indications that many compact binaries, particularly SXTs, do contain black holes. However all of this evidence is indirect: black holes are a consistent solution, rather than a required one. It would be very interesting to discover direct evidence for the defining property of a black hole, namely the lack of a stellar surface. If the systematic difference \( (n = 2 \) or \( 1) \) in the irradiation law (eq. 1.26) for black–hole and neutron–star SXTs had held up this would have provided such evidence. However we saw in Subsection 1.6.1 that there is little reason to believe this.

A quite separate argument for a systematic BH/NS difference (Narayan et al., 1997; Garcia et al., 2001) uses the idea (see Subsection 1.2.4) that an ADAF on to a black hole will be inherently fainter than the same flow on to a neutron star, because the advected energy is released at the stellar surface in the latter case. As we have seen, dynamical mass determinations suggest that some SXTs contain black holes, while others contain neutron stars. There is some observational evidence that the former systems are systematically fainter than the latter in quiescence, as expected if indeed the two groups have similar ADAFs in this phase.

However the last requirement is very strong, even granted that ADAFs actually occur in quiescence, which is not entirely settled. We have seen in Section 1.6 that
there are systematic differences between black–hole and neutron–star SXTs evolution. Even at similar orbital periods, the two groups probably have different mean mass transfer rates. This in turn may lead to differing outburst/quiescent behaviour, undermining the assumption of similar ADAFs in the two cases. It is clear that much more work is needed on these effects if quiescent transients are to provide direct evidence for black hole horizons.

1.8 Ultraluminous X–ray sources

It has been known for more than 20 years that some external galaxies contain X–ray sources outside their nuclei whose luminosities exceed the Eddington limit for a $1 M_\odot$ object (Fabbiano, 1989). These ultraluminous X–ray sources (ULXs) have attracted considerable interest in recent years (see e.g. Makishima et al., 2000 and references therein) partly because one simple way of evading the Eddington limit constraint is to assume larger black hole masses than are generally found as the endpoints of stellar evolution (e.g. Colbert & Mushotzky, 1999; Ebisuzaki et al., 2001; Miller & Hamilton, 2002). Such intermediate–mass ($\sim 10^2 - 10^4 M_\odot$) black holes are an ingredient of some pictures of galaxy formation (e.g. Madau & Rees, 2001), and thus raised the hope that ULXs might represent such a population.

However it now appears that although individual ULXs might conceivably harbour intermediate–mass black holes, this cannot be true of the class as a whole. Instead ULXs are probably in the main X–ray binaries in rather extreme evolutionary phases. They offer exciting insight into many of the topics discussed in this book.

1.8.1 The nature of the ULX class

There are several lines of argument suggesting that the ULX class involves stellar–mass accretors. These are both negative and positive. The negative arguments concern the difficulties of forming and then feeding intermediate–mass objects. King et al. (2001) summarize several of these. A black hole of $10^2 - 10^4 M_\odot$ cannot result from current stellar evolution, as stars of $\gtrsim 100 M_\odot$ are subject to huge mass loss if they have any significant metal content, and rapidly reduce their masses to quite modest values before producing black holes. Primordial stellar evolution (i.e. with hydrogen and helium alone) can produce black holes with such masses, but then the question of feeding the hole with accretion becomes critical. As we have seen in Subsection (1.6.1), black–hole binaries must either be born with separations so wide that reaching contact at all is problematical, or have initial mass ratios above a minimum value $q_i \gtrsim 0.1$. With black–hole masses $M_1 > 100 M_\odot$ the companion must have $M_2 i \gtrsim 10 M_\odot$ and thus long ago have become a compact object itself. Various other routes to making intermediate–mass black holes have been suggested, often invoking mergers within globular clusters. Again the process is rather delicate, as the merged object must not attain the rather low space velocity required to escape the cluster before its mass has built up to the required value. ULXs are not observed to be members of globular clusters, and indeed their incidence is often associated with recent star formation, so the hole must eventually be ejected from the cluster. There is then again the problem of finding a companion to supply the hole with mass: the hole apparently did not achieve this feat in the cluster, even given the high stellar density, but must nevertheless manage it in the field. At the very least
these difficulties suggest that the efficiency of finding a companion and thus turning the system on as a ULX must be rather low. The observed numbers of ULXs found in star–forming systems such as the Antennae ($\sim 10$) therefore demand rather high formation rates for intermediate–mass black holes if they are to explain the ULX class as a whole.

In addition to these negative arguments, there are some positive arguments favouring a stellar–mass black hole origin for ULXs. First, in most cases their X–ray spectra are consistent with thermal components at $kT \sim 1 – 2$ keV. This is a natural temperature for a stellar–mass object (see Fig. 1.15). In addition X-ray spectral transitions typical of such sources are observed in ULXs (e.g. Kubota et al., 2001). Second, many ULXs, though not all, are close to regions of star formation (Zezas, Georgantopoulos & Ward 1999; Roberts and Warwick 2000, Fabbiano, Zezas & Murray, 2001; Roberts et al., 2002). This is consistent with ULXs being the extreme end of an HMXB population formed in such regions. Third, optical identifications (e.g. Goad et al., 2002) are consistent with HMXBs.

On the basis of these arguments King et al. (2001) suggested that most ULXs were probably mildly (factors $\lesssim 10$) anisotropically emitting X–ray binary systems accreting at close to the Eddington value. This allows apparent luminosities up to $\sim 10^{40}$ erg s$^{-1}$, compatible with the great majority of claimed ULXs. At first sight this seems to require large numbers of unseen sources, and thus a high birthrate. However this is not so, as the following analysis shows.

We assume that a compact object of mass $M_1$ accretes from a mass reservoir (e.g. a companion star) of mass $M_2$. We denote the mean observed number of ULXs per galaxy as $n$, the beaming factor as $b$ ($= \Omega / 4\pi$, where $\Omega$ is the solid angle of emission), the duty cycle (= time that the source is active as a fraction of its lifetime) as $d$, and define an ‘acceptance rate’ $a$ as the ratio of mass accreted by $M_1$ to that lost by $M_2$, i.e. the mean accretion rate $\dot{M}_1 = a(-\dot{M}_2)$. We further define $L_{\text{sph}}$ as the apparent X–ray (assumed bolometric) luminosity of a source, given by the assumption of isotropic emission, and let $L_{40} = L_{\text{sph}}/10^{40}$ erg s$^{-1}$. From these definitions it follows that the luminosity

$$L = bL_{\text{sph}} = 10^{40}bL_{40} \text{ erg s}^{-1} \quad (1.42)$$

and the minimum accretor mass if the source is not to exceed the Eddington limit is

$$M_1 \gtrsim 10^2 bL_{40} M_\odot. \quad (1.43)$$

The total number of such sources per galaxy is

$$N = \frac{n}{bd} \quad (1.44)$$

with a minimum mean accretion rate during active phases of

$$\dot{M}_{\text{active}} = \frac{M_1}{d} = \frac{-\dot{M}_2a}{d} > 10^{-6}bL_{40} M_\odot \text{ yr}^{-1}. \quad (1.45)$$

The mass loss rate from $M_2$ is thus

$$-\dot{M}_2 > 10^{-6} \frac{bd}{a}L_{40} M_\odot \text{ yr}^{-1}, \quad (1.46)$$
and the lifetime of a source is
\[ \tau = -\frac{M_2}{M_2} \lesssim 10^6 \frac{m_2 a}{bdL_{40}} \text{ yr}, \]
with \( m_2 = M_2/M_\odot \), leading to a required birthrate per galaxy
\[ B = \frac{N}{\tau} \gtrsim \frac{n}{bd} \frac{bdL_{40}}{10^9 m_2 a} = 10^{-6} \frac{nL_{40}}{m_2 a} \text{ yr}^{-1}. \]

The important point to note here is that the required birthrate is independent of beaming (and duty cycle): the greater intrinsic source population \( N \) required by \( bd < 1 \) (cf eq. 1.44) is compensated by their longer lifetimes (cf eq. 1.47).

A possible alternative to the idea of mild anisotropy as an explanation for ULXs was proposed by Begelman (2002), who suggested that a magnetized accretion disc might allow luminosities which were genuinely super–Eddington by factors up to \( \sim 10 \) (see also Shaviv, 1998, 2000). An observationally–motivated objection to this is the existence of neutron stars which have apparently passed through phases of super–Eddington mass transfer without showing signs of significant mass or angular momentum gain, as we might expect if super–Eddington accretion were allowed. The difficulty in spinning up neutron stars in wide circular binaries (Subsection 1.6.4 above) is an example.

Strong confirmation of the idea the ULXs represent a population of stellar–mass X–ray binaries comes from work by Grimm et al. (2002). They show that the cumulative luminosity functions of nearby starburst galaxies, as well as the Milky Way and Magellanic Clouds, can be fitted by a single form normalized by the star formation rate (SFR), as measured by various conventional indicators. The form
\[ N(>L) = 5.4 \times SFR \times (L_{38}^{0.61} - 210^{-0.61}) \]
is used in Figures (1.13, 1.14) below, where \( L_{38} \) is the X–ray luminosity in units of \( 10^{38} \text{ erg s}^{-1} \).

These results mean that the ULX population must be some kind of extension of the HMXB/LMXB populations contributing to the luminosity function at lower luminosity. Note that this result is asserted only for the ULX as a class. King et al (2001) point out that all the arguments above still allow the possibility that individual ULXs could involve intermediate–mass black holes. However it is probably fair to say that at the time of writing no convincing example is known.

1.8.2 Models for ULXs

If ULXs are X–ray binaries, we should ask what causes their unusual appearance, and in particular their defining feature, the apparent super–Eddington luminosity. The immediate cause appears to be a highly super–Eddington mass inflow rate near the accretor, leading to three characteristic features: (i) the total accretion luminosity is of order \( L_{\text{Edd}} \), (ii) this is confined to a solid angle \( 4\pi b \lesssim 4\pi \), making the source apparently super–Eddington when viewed from within this solid angle (even if it is not genuinely super–Eddington), and (iii) the bulk of the super–Eddington mass inflow is either accreted at low radiative efficiency, or more probably, ejected in the form of a dense outflow, probably including relativistic jets.
A suggested accretion flow with these features (Paczyński & Wiita, 1980; Jaroszynski et al., 1980; Abramowicz et al., 1980) postulates an accretion disc whose inner regions are geometrically thick, and a central pair of scattering funnels through which the accretion radiation emerges. Note that this form of 'beaming' does not involve relativistic effects, although Doppler boosting in a relativistic jet has also been suggested as a way of explaining the high luminosities (Koerding et al., 2001, Markoff et al., 2001). The thick-disc plus funnels anisotropy mechanism explicitly requires a high mass inflow rate near the black hole or neutron star accretor, much of which must be ejected, probably some of it in the form of a jet. (In fact the motivation
of the original papers (Jaroszynski et al., 1980, Abramowicz et al., 1980) was to produce a geometry favouring jet production.)

The identification with super–Eddington mass inflow rates made above allows us to identify the likely ULX parent systems. There are two situations in which X–ray binaries naturally have such rates: phases of thermal–time mass transfer, and bright SXT outbursts. The first of these is considered extensively by King et al. (2001) and its main features can be summarized briefly here.

Thermal–timescale mass transfer occurs in any Roche–lobe–filling binary where the ratio \( q \) of donor mass to accretor mass exceeds a critical value \( q_{\text{crit}} \sim 1 \). Thus all high–mass X–ray binaries will enter this phase once the companion fills its Roche lobe, either by evolutionary expansion, or by orbital shrinkage via angular momentum loss. Depending on the mass and structure of the donor, extremely high mass transfer rates \( \dot{M}_{\text{tr}} \sim 10^{-7} - 10^{-3} M_\odot \text{yr}^{-1} \) ensue. SS433 is an example of a system...
1.8 Ultraluminous X–ray sources

currently in a thermal–timescale mass transfer phase (King et al., 2000) which has
descended by this route. The idea that SS433 itself might be a ULX viewed ‘from the
side’ provides a natural explanation of its otherwise puzzlingly feeble X–ray emission
\(L_x \sim 10^{36}\) erg s\(^{-1}\), Watson et al., 1986).

The binary probably survives the thermal–timescale phase without entering common–
envelope (CE) evolution provided that the donor’s envelope is largely radiative (King
& Begelman, 1999). Observational proof of this is provided by Cygnus X–2 (King &
Ritter, 1999; Podsiadlowski & Rappaport, 2000), whose progenitor must have been
an intermediate–mass binary (companion mass \(\sim 3M_\odot\), neutron star mass \(\sim 1.4M_\odot\)).
CE evolution would instead have engulfed the binary and extinguished it as a high–
energy source. The binary would probably have merged, producing a Thorne–Żytkow
object.

The birthrates of intermediate and high–mass X–ray binaries are compatible with
the observed numbers of ULXs: King et al. (2001) show that the birthrates required
to explain the latter are independent of the dimensionless beaming and duty–cycle
factors \(b, d\). For massive systems the thermal timescale lasts longer than the preceding
wind–fed X–ray binary phase; the fact that there are far fewer observed ULXs
than massive X–ray binaries must mean that the beaming and duty–cycle factors
obey \(bd << 1\). This picture also explains the observed association of ULXs with star
formation. There is in addition some evidence that the ULXs in the Antennae are on
average slightly displaced from star clusters, suggesting that they have acquired sig-
ificant space velocities as a result of a recent supernova explosion, just like HMXBs
(Zezas et al., 2002). If this is correct it is a direct demonstration that the masses of
ULX systems are not unusually high.

While thermal–timescale mass transfer probably accounts for a significant fraction
of observed ULXs, bright SXT outbursts will also produce super–Eddington accre-
tion rates, and are the only possibility for explaining the ULXs observed in elliptical
galaxies (King, 2002). SXT outbursts in long–period systems are an attractive can-
didate because they are both bright and long–lasting.

SXT outbursts in systems with such periods are complex because of the large
reservoir of unheated mass at the edge of the disc, which can eventually contribute
to the outburst (see Section 1.5). Full numerical calculations will be needed to
describe this process. However the trends with increasing \(P\) are clear: the outbursts
become longer (several decades) and involve more mass, but the quiescent intervals
increase more rapidly (several \(\gtrsim 10^3\) yr) so that the outburst duty cycle \(d\) decreases
(Ritter & King, 2001). This results in inflow rates which become ever more super–
Eddington at large \(P\). Spectacular evidence of super–Eddington accretion is provided
by GRS 1915+105, which has been in effectively continuous outburst since 1992. The
observed X–ray luminosity \(L_x \gtrsim 7 \times 10^{39}\) erg s\(^{-1}\) implies that at least \(\sim 10^{-6}\)M\(_\odot\)
has been accreted over this time, requiring a large and massive accretion disc. In
line with this, it appears that the binary is wide (\(P \approx 33\) d; Greiner, Cuby &
McCaughearan, 2001). At the reported accretor mass \(M_1 = (14 \pm 4)M_\odot\) (Greiner et
al., 2001) there is little doubt that the current mass inflow near the black hole is
highly super–Eddington. Evolutionary expansion of the donor will drive a persistent
mass transfer rate \(\dot{M}_2 \sim 10^{-9}(P/d)\)M\(_\odot\)yr\(^{-1}\) \(\sim 3 \times 10^{-8}\)M\(_\odot\)yr\(^{-1}\) (King, Kolb &
Burderi, 1996) which is already close to the Eddington rate \(\dot{M}_{\text{Edd}} \sim 10^{-7}\) M\(_\odot\)yr\(^{-1}\).
Given an outburst duty cycle $d \ll 1$, the mean inflow rate $\dot{M}_2/d$ is $\gg \dot{M}_{\text{Edd}}$. Note that we definitely do not look down the jet in GRS 1915+105, which is at about 70° to the line of sight (Mirabel & Rodríguez, 1999), so it is quite possible that the apparent luminosity in such directions is much higher than the observed $L_x$.

The observed ULX population of a given galaxy is a varying mixture of these thermal–timescale and transient types, depending on the star formation history of that galaxy. Thermal–timescale SS433–like systems should predominate in galaxies with vigorous star formation, such as the Antennae, while ULXs in elliptical galaxies must be of the microquasar transient type, as there are no high–mass X–ray binaries. We therefore expect ULXs in ellipticals to be variable. However the microquasar systems most likely to be identified as ULXs are clearly those with the brightest and longest outbursts, so baselines of decades may be needed to see significant numbers turning on or off. There is some evidence of such variability from the differences between ROSAT and Chandra observations of the same galaxies. The fact that none of the SXTs found in the Galaxy has turned out to be a ULX suggests that the beaming factor $b$ must be $\lesssim 0.1$ for this mode of accretion. This agrees with our conclusion above that $b \ll 1$ for the ULXs in ellipticals.

Evidence that the two suggested classes of ULXs do resemble each other in similarly super–Eddington accretion states comes from Revnivtsen et al. (2002), who report RXTE observations of an episode of apparently super–Eddington accretion in the soft X–ray transient V4641 Sgr. Revnivtsen et al. remark on the similarity of the object’s appearance to SS433 in this phase. One might be discouraged by the apparent suppression of X–rays in this state. However we are presumably outside the beam of most intense X–ray emission in both cases: neither should actually appear as a ULX. More work is needed on whether the X–ray spectra from these objects are consistent with X–rays leaking sideways from the assumed accretion geometry. Direct evidence that X–ray emission in ULXs is anisotropic is perhaps understandably meagre, but may be suggested by the comparison of optical and X–ray data in NGC 5204 X–1 (Roberts et al., 2002), where low–excitation optical spectra are seen from regions close to the ULX.

Both SS433 and the microquasars are distinguished by the presence of jets, at least at some epochs. In SS433 the jets precess with a 164–day period, presumably because of disc warping (Pringle, 1996). If looking closely down the jet is required in order to see high luminosities one might expect to see such periods in a class of ULXs. This effect could for example explain the $\sim 106$ d modulation seen in the bright source in M33 (M33 X–8, $L_x \sim 10^{39}$erg s$^{-1}$) by Dubus et al. (1999). However a beam as narrow as commonly inferred ($\lesssim 1^\circ$) for SS433 would give an unacceptably short duty cycle. If instead it is not necessary to look down the jet to see a high luminosity, this would rule out Doppler boosting as the cause of the latter, and ULXs would not be direct analogues of BL Lac systems.

### 1.8.3 Black–hole blackbodies

The tentative conclusion at the end of the last subsection suggests another. X–ray binaries and active galactic nuclei share the same basic model, and so far have shown a fairly good correspondence in their modes of behaviour. If as suggested above ULXs do not correspond to BL Lac systems, this may mean that we are currently
missing a class of each type: there should exist an apparently super–Eddington class of AGN, and a set of X–ray binaries with Doppler–boosted X–ray emission.

A tentative answer to one of these questions has recently emerged. Many black–hole sources emit a substantial fraction of their luminosities in blackbody–like spectral components. It is usual to assume that these are produced in regions at least comparable in size to the hole’s Schwarzschild radius, so that a measure of the emitting area provides an estimate of the black hole mass \( M \). However there is then no guarantee that the source luminosity (if isotropic) obeys the Eddington limit corresponding to \( M \). King & Puchnarewicz (2002) show that the apparent blackbody luminosity \( L_{\text{sph}} \) and temperature \( T \) must obey the inequality

\[
L_{\text{sph}} < L_{\text{crit}} = 2.3 \times 10^{44} (T/100 \text{ eV})^{-4} \text{ erg s}^{-1},
\]

(1.50)

(where \( T_{100} \) is \( T \) in units of 100eV) for this to hold. This limit is shown in Fig. 1.15. Sources violating it must either be super–Eddington, or radiate anisotropically, or
radiate from a region much smaller than their Schwarzschild radii. Not suprisingly, some ULXs appear above the limit. (Note that they are not required to do this to qualify as ULXs: the defining characteristic is simply that their ‘Eddington masses’ [rh scale of Figure 1.15] are $>> 10M_\odot$.) The large group of AGN violating the limit are the so-called ultrasoft AGN, which may thus be the AGN analogues of the ULXs. The second question remains: a search for for Doppler–boosted X–ray binaries among the ULXs may be rewarding.

1.8.4 Supersoft ULXs

Very recently a number of ULXs have been observed with very low spectral temperatures ($\sim 50 – 100$ eV) and consequent photospheric sizes ($\sim 10^9$ cm) much larger than the Schwarzschild radius of a stellar–mass object (e.g. Mukai et al., 2002). At first sight these might at least appear as strong evidence for the long–sought intermediate–mass black holes. However Mukai et al. (2002) have pointed out that accretion at rates comparable to Eddington must lead to outflow, and shown that the opacity of the resulting wind does imply supersoft emission with a photospheric size of this order. The M101 source studied by Mukai et al (2002) has a supersoft luminosity of order $10^{39}$ erg s$^{-1}$ and so does not require anisotropic emission for a black hole mass $\gtrsim 10M_\odot$. However their analysis is easily extended to the case that an Eddington–limited source blows out a wind confined to a double cone of total solid angle $4\pi b$ about the black hole axis. Since this wind is the path of lowest optical depth through the accretion flow, the radiation will escape this way also, implying anisotropic emission once again. Mukai et al (2002) assume a constant velocity for the outflowing material as this is likely to achieve escape velocity and coast thereafter. This leads to an equivalent hydrogen column from radius $R$ to infinity of $N_H = \dot{M}_{\text{out}}/4\pi bvR$ and thus (assuming Compton scattering opacity) a photospheric radius

$$R_{\text{ph}} = \frac{3 \times 10^8}{b v_9} \dot{M}_{19} \text{ cm}$$ (1.51)

where $v_9$ is $v$ in units of $10^9$ cm s$^{-1}$ and $\dot{M}_{19}$ is the outflow rate in units of $10^{19}$ gs$^{-1}$, the Eddington accretion rate for a $10M_\odot$ black hole. Clearly we can again interpret such supersoft ULXs in terms of stellar–mass black holes.

It is worth noting that the presence of a photosphere of this kind seems inevitable in any source accreting significantly above the Eddington accretion rate $\dot{M}_{\text{Edd}}$. A completely general calculation (Pounds et al., 2003) shows that

$$\frac{R_{\text{ph}}}{R_a} = \frac{1}{2 \eta b} \frac{c \dot{M}_{\text{out}}}{v \dot{M}_{\text{Edd}}} \approx \frac{5 c}{b} \frac{\dot{M}_{\text{out}}}{\dot{M}_{\text{Edd}}}$$ (1.52)

where we have taken the accretion efficiency $\eta \approx 0.1$ at the last step. Since $b \leq 1$, $v/c < 1$ we see that $R_{\text{ph}} > R_a$ for any outflow rate $\dot{M}_{\text{out}}$ of order $\dot{M}_{\text{Edd}}$. In other words, any black hole source accreting at above the Eddington rate is likely to have a scattering photosphere at several $R_a$. 
1.9 Conclusions

Our picture of accretion in compact binary systems has advanced considerably over recent years. In the past it was common to think of these sources as relatively steady systems which accreted most of the mass transferred to them, often from main–sequence companions, and radiated roughly isotropically. It now seems that none of these implicit assumptions is really justified. Transient behaviour is extremely widespread, to the point that persistent sources are rather exceptional. Much of the transferred mass is not accreted at all, but blown away from the accretor: jets are only the most spectacular manifestation of a very widespread trait. Even short–period systems often have significantly evolved companions, and there is little evidence for a period gap for short–period LMXBs. Disc warping and other effects can apparently cause many sources to radiate with significant anisotropy.

Despite these complicating effects there are reasons for optimism. One can now give a complete characterization of the observed incidence of transient and persistent sources in terms of the disc instability model and formation constraints. X–ray populations in external galaxies, particularly the ultraluminous sources, are revealing important new insights into accretion processes and compact binary evolution.

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Accretion in Compact Binaries


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